1.

B

$$y''(e^x+1)=e^x y' \qquad y'=z \quad 1P$$

$$z'(e^x+1)=e^x\cdot z \qquad \text{SZEPARÁLHATÓ}$$

$$z'\cdot\frac{1}{z}=\frac{e^x}{e^x+1} \quad 1P$$

$$\int\frac{1}{z}dz=\int\frac{e^x}{e^x+1}dx \quad 1P$$

$$\ln|z|=\ln|e^x+1|+c \quad 1P$$

$$z=(e^x+1)\cdot K_1 \quad 1P$$

$$y'=(e^x+1)\cdot K_1 \Rightarrow \boxed{y=(e^x+x)\cdot K_1+K_2} \quad 1P \quad 1P$$

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2.

$$\cos^8 x\, y'+\frac{\cos^9 x}{\sin x}y=1$$

$$\cos^8 x\neq 0 \quad 1P$$

$$y'+\frac{\cos x}{\sin x}y=\frac{1}{\cos^8 x}$$

$$v=e^{\int P(x)dx}=e^{\int\frac{\cos x}{\sin x}}=e^{\ln|\sin x|}=\sin x \quad 1P \quad 1P \quad 1P$$

$$(y\cdot v)'=\frac{1}{\cos^8 x}\cdot v \quad 1P$$

$$y\cdot v=\int\frac{1}{\cos^8 x}\cdot\sin x\,dx \quad 1P$$

HOMOGÉN EGYENLET

$$y'=\frac{-\cos x}{\sin x}y$$

$$\int\frac{1}{y}dy=\int\frac{-\cos x}{\sin x}dx \quad 1P$$

$$\ln|y|=-\ln|\sin x|+c \quad 1P$$

$$y \cdot v = \int \cos^{-8} x \cdot \sin x \, dx$$

$$y \cdot v = \frac{\cos^{-7} x}{-7} + c$$

(1P) (1P)

$$y = \frac{\cos^{-7} x}{-7 \cdot \sin x} + \frac{c}{\sin x}$$ (1P)

KEZDETIÉRTÉK-PROBLÉMA

$$y\left(\frac{\pi}{4}\right) = 1736$$

$$1736 = \frac{\cos^{-7}\left(\frac{\pi}{4}\right)}{-7 \sin\left(\frac{\pi}{4}\right)} + \frac{C}{\sin\left(\frac{\pi}{4}\right)}$$ (1P)

$$1736 = \frac{\sqrt{2}^{7}}{-7 \cdot \frac{1}{\sqrt{2}}} + C \cdot \sqrt{2}$$ (1P)

$$1736 = \frac{16}{-7} + C \cdot \sqrt{2}$$

$$C = \left(1736 + \frac{16}{7}\right) \cdot \frac{1}{\sqrt{2}}$$ (1P)

$$y = \frac{K}{\sin x}$$ (1P)

INHOMOGÉN MO:

$$y = K(x) \cdot \frac{1}{\sin x}$$

$$K'(x) \cdot \frac{1}{\sin x} + \cancel{K(x)\left(\frac{1}{\sin x}\right)' + \frac{\cos x}{\sin x} \cdot K(x) \frac{1}{\sin x}} = \frac{1}{\cos^{8} x}$$ (1P)

$$K'(x) \cdot \frac{1}{\sin x} = \frac{1}{\cos^{8} x}$$

$$K(x) = \int \cos^{-8} x \cdot \sin x \, dx$$ (1P)

$$K(x) = \frac{\cos^{-7} x}{-7}$$ (1P)

$$y = \frac{\cos^{-7} x}{-7 \sin x} + \frac{K}{\sin x}$$ (1P)

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3. $$y'' + 6y' + 9y = 2 \operatorname{sh} 3x$$

$$\lambda^2 + 6\lambda + 9 = 0$$ (1P)

$$\lambda_1 = \lambda_2 = -3$$ (1P)

$$y_H = C_1 \cdot e^{-3x} + C_2 \cdot x \cdot e^{-3x}$$

(1P) (1P)

KÜLSŐREZONANCIA: $2\,\mathrm{sh}\,3x = 2\,\dfrac{e^{3x}-e^{-3x}}{2} = e^{3x}-e^{-3x}$ (1P)

$$y = A\cdot e^{3x} + Bx^2 e^{-3x}$$
(1P) (1P)

$$y' = 3A e^{3x} + 2Bx e^{-3x} - 3Bx^2 e^{-3x}$$ (1P)

$$y'' = 9A e^{3x} + 2B e^{-3x} - 6Bx e^{-3x} - 6Bx e^{-3x} + 9Bx^2 e^{-3x}$$ (2P)

$$y'' + 6y' + 9\cdot y = e^{3x} - e^{-3x}$$

$9A + 18A + 9A$	1	e^{3x}	(1P)
$2B$	1	e^{-3x}	(1P)
0	0	$x e^{-3x}$	(1P)
0	0	$x^2 e^{-3x}$	

$36A = 1 \Rightarrow A = \frac{1}{36}$ (1P)

$2B = 1 \Rightarrow B = \frac{1}{2}$ (1P)

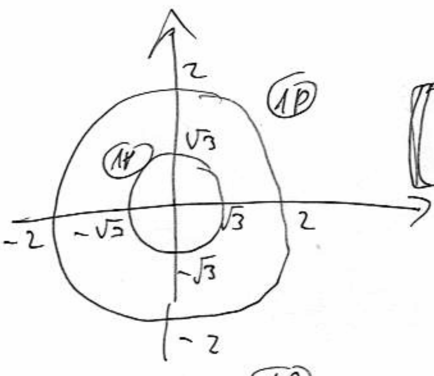
$$y = C_1\cdot e^{-3x} + C_2\cdot x e^{-3x} + \frac{1}{36}\cdot e^{3x} + \frac{1}{2}x^2 e^{-3x}$$ (1P)

$\boxed{4.}$ $y' = \sqrt{x^2 + y^2 - 3}$

$\boxed{8\ \text{PONT}}$

HA $K = 0$ AKKOR $x^2 + y^2 = 3$ (1P)

HA $K = 1$ AKKOR $x^2 + y^2 = 4$ (1P)



$P(\sqrt{3};0)$: $y'' = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2x + 2y\cdot y') = \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\cdot 2\sqrt{3} > 0$ (1P)

(2P)

LOK MIN (1P)

$\boxed{5.}$

a)
$$\sum_{n=1}^{\infty} \sqrt{\frac{n^2+1}{n^3}} \cdot (-1)^n$$

KONVERGENS, MERT LEIBNIZ-SOR (1P)

→ VÁLTAKOZÓ ELŐJELŰ (1P)

→ $\sqrt{\frac{n^2+1}{n^3}} \to 0$ (1P)

→ $\sqrt{\frac{n^2+1}{n^3}} = \sqrt{\frac{n^2}{n^3} + \frac{1}{n^3}} = \sqrt{\frac{1}{n} + \frac{1}{n^3}}$ (1P)

SZIG. MON CSÖKK

b)
$$\sum_{n=1}^{\infty} \frac{n^2+2}{9n+n^2}$$

$$\lim_{n\to\infty} \frac{n^2+2}{9n+n^2} = \lim_{n\to\infty} \frac{1+\frac{2}{n^2}}{\frac{9}{n}+1} = 1$$ (1P)

(1P)

A SOR DIVERGENS, MERT $\lim_{\infty} a_n \neq 0$ (1P)

$\boxed{7 \text{ PONT}}$