Információfeldolgozás „hivatalos puska”
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$$P\left(|x-\mu_x| \le c\sigma_x\right) \ge 1 - \frac{1}{c^2} \tag{1}$$

$$f(\boldsymbol{x}) = \frac{1}{(\sqrt{2\pi})^N |\boldsymbol{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_x)^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_x)}$$

$$\mathcal{E}\{x_1x_2x_3x_4\} = \mathcal{E}\{x_1x_2\}\mathcal{E}\{x_3x_4\} + \mathcal{E}\{x_1x_3\}\mathcal{E}\{x_2x_4\} + \mathcal{E}\{x_1x_4\}\mathcal{E}\{x_2x_3\} - 2\mu_1\mu_2\mu_3\mu_4 \tag{2}$$

$$\mathcal{E}\{\,\xi_1|\,\xi_2 = y\} = \mu_1 + r_{12}\frac{\sigma_1}{\sigma_2}(y-\mu_2), \qquad \mathrm{var}\,\{\,\xi_1|\,\xi_2 = y\} = \sigma_1^2(1-r_{12}^2) \tag{3}$$

$$\Phi_x(u) = \mathcal{E}\left\{e^{jux}\right\} = \int_{-\infty}^{\infty} f(x)e^{jux}dx, \qquad \Phi_x(u) = e^{ju\mu - \frac{\sigma^2u^2}{2}}, \qquad \Phi_{\boldsymbol{x}}(\boldsymbol{u}) = e^{j\boldsymbol{u}^T\boldsymbol{\mu} - \frac{\boldsymbol{u}^T\boldsymbol{\Sigma}\boldsymbol{u}}{2}} \tag{4}$$

$$\mathcal{E}\left\{x^n\right\} = \frac{1}{j^n}\left.\frac{d^n\Phi_x(u)}{du^n}\right|_{u=0}, \qquad \Phi_x(u) = \sum_{n=0}^{\infty}\alpha_n\frac{(ju)^n}{n!}, \qquad \alpha_n = \mathcal{E}\left\{x^n\right\} \tag{5}$$

$$\chi_n^2 = z_1^2 + z_2^2 + \ldots + z_n^2, \qquad \mathcal{E}\left\{\chi_n^2\right\} = n, \qquad \mathrm{var}\left\{\chi_n^2\right\} = 2n, \qquad f_{\chi_2^2}(x) = \frac{1}{2}e^{-x/2} \text{ ha } x > 0 \tag{6}$$

$$s^{*2} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}x_i^2 - n(\overline{x})^2\right) = \frac{\sigma^2}{n-1}\chi_{n-1}^2 \tag{7}$$

$$\hat{y} = \hat{a}x + \hat{b}$$

$$\hat{a} = \frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sum_{i-1}^{n}(x_i-\overline{x})^2} = \frac{s_{xy}}{s_x^2}, \qquad \hat{b} = \overline{y} - \hat{a}\overline{x}, \qquad \hat{\rho}_{xy} = \frac{s_{xy}}{s_xs_y} = \sqrt{\hat{a}_x\hat{a}_y} \tag{8}$$

$$\mathrm{sinc}(x) = \frac{\sin x}{x}, \qquad \mathrm{rect}(x) = \begin{cases} 1 & \text{ha } |x| < 0,5 \\ 0 & \text{egyébként} \end{cases} \tag{9}$$

$$w(t) = \mathrm{rect}\left(\frac{t}{T} - 0,5\right) \qquad W(f) = e^{-j\pi fT}T\mathrm{sinc}(\pi fT) \tag{10}$$

$$H(f) = H_0\mathrm{rect}\left(\frac{f}{2B}\right) \qquad R(\tau) = H_0 2B\mathrm{sinc}(2\pi B\tau) \tag{11}$$

$$\mathrm{var}\{\hat{\mu}_x\} = \frac{1}{T}\int_{-T}^{T}\left(1-\frac{|\tau|}{T}\right)C_x(\tau)d\tau \approx \frac{1}{T}S_{xc}(0) = \frac{\sigma^2}{2B_nT}, \qquad B_n = \frac{\int_{-\infty}^{\infty}S_{xc}(f)df}{2S_{xc}(0)} \tag{12}$$

$$H(f) = \frac{1}{1+j2\pi Kf}, \qquad \mathcal{E}\left\{\hat{\mu}_x(t)\right\} = \mu_x\left(1-e^{-\frac{t}{T}}\right), \qquad \mathrm{var}\left\{\hat{\mu}_x(t)\right\} \approx \frac{\sigma^2}{4BK} \tag{13}$$

$$C(\tau) = \sigma^2e^{-\frac{|\tau|}{K}} \qquad S_c(f) = \frac{\sigma^2 2K}{1+(2\pi fK)^2} \tag{14}$$

$$\mathrm{var}\left\{\hat{\Psi}_x^2\right\} = \frac{2}{T}\int_{-T}^{T}\left(1-\frac{|\tau|}{T}\right)\left(C_x^2(\tau) + 2\mu_x^2C_2(\tau)\right)d\tau \approx \underbrace{\frac{2}{T}\int_{-\infty}^{\infty}S_{xc}^2(f)df}_{\frac{\sigma^4}{B_sT}} + \frac{4}{T}\mu_x^2S_{xc}(0) \tag{15}$$

$$B_s = \frac{\left(\int_{-\infty}^{\infty}S_{xc}(f)df\right)^2}{2\int_{-\infty}^{\infty}S_{xc}^2(f)df} \tag{16}$$

$$S_{xy}(f) = H(f)S_{xx}(f), \qquad S_{yy}(f) = |H(f)|^2S_{xx}(f) \tag{17}$$

$$\mathrm{var}\left\{\hat{R}_{xy}(\tau)\right\} = \frac{1}{T}\int_{-T}^{T}\left(1-\frac{|u|}{T}\right)\left(C_x(u)C_y(u) + \mu_x^2C_y(u) + \mu_y^2C_x(u) + \right.$$
$$\left. C_{xy}(\tau+u)C_{xy}(\tau-u) + \mu_x\mu_y\left(C_{xy}(\tau-u) + C_{xy}(\tau+u)\right)\right)du \le \mathrm{var}\left\{\Psi_x^2\right\} \tag{18}$$

$$\mathcal{E}\left\{\hat{S}(f)\right\} = S(f) * W_B(f), \qquad W_B(f) = T\mathrm{sinc}^2(\pi f T), \qquad \mathrm{var}\left\{\hat{S}(f)\right\} \approx S^2(f) \tag{19}$$

$$c(t) = \sum_{i=-\infty}^{\infty} \delta\left(\frac{t - i\Delta t}{\Delta t}\right) \qquad \mathcal{F}\left\{c(t)\right\} = C(f) = \sum_{k=-\infty}^{\infty} \delta\left(\left(f - \frac{k}{\Delta t}\right)\Delta t\right)\Delta t \tag{20}$$

$$X_m(f) = X(f) \star C(f) = \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{\Delta t}\right) \qquad x(t) = \sum_{i=-\infty}^{\infty} x(i\Delta t)\mathrm{sinc}\left(\pi\left(\frac{t}{\Delta t} - i\right)\right) \tag{21}$$

$$\begin{aligned} \mathcal{E}\left\{x\right\} &= \mathcal{E}\left\{x_q\right\} \\ \mathcal{E}\left\{x^2\right\} &= \mathcal{E}\left\{x_q^2\right\} - \frac{q^2}{12} \\ \mathcal{E}\left\{x^3\right\} &= \mathcal{E}\left\{x_q^3\right\} - \mathcal{E}\left\{x_q\right\}\frac{q^2}{4} \end{aligned} \tag{22}$$

$$f_m < 3\frac{x}{q}f_1, \qquad f_m < 9\frac{\sigma}{q}B, \qquad f_m < K\frac{\mathcal{E}\left\{|\dot{x}|\right\}}{q}, \qquad K \approx 0.8\ldots 3 \tag{23}$$

$$X_k = \sum_{i=0}^{N-1} x_i e^{-j2\pi\frac{ki}{N}} = e^{-j\pi\frac{k^2}{N}} \sum_{i=0}^{N-1} x_i e^{-j\pi\frac{i^2}{N}} \cdot e^{+j\pi\frac{(k-i)^2}{N}} \tag{24}$$

$$\mathcal{E}\left\{\hat{R}^c_{xy}(\tau)\right\}\Big|_{\tau=i\Delta t} = \left(1 - \frac{i}{N}\right)R_{xy}(i\Delta t) + \frac{i}{N}R_{xy}\left((i-N)\Delta t\right) \qquad 0 \le i < N \tag{25}$$

$$\Phi_n(u) = \sum_{k=-\infty}^{\infty} \Phi_x(u + k\Psi)\mathrm{sinc}\left(\frac{q(u+k\Psi)}{2}\right) = \sum_{k=-\infty}^{\infty} \Phi_x(u + k\frac{2\pi}{q})\mathrm{sinc}\left(\frac{qu}{2} + k\pi\right), \quad \Psi = \frac{2\pi}{q} \tag{26}$$

$$\hat{x}(n+1) \quad = \quad \frac{1}{n+1}\sum_{k=0}^{n} y(k) = \frac{n}{n+1}\hat{x}(n) + \frac{1}{n+1}y(n) = \hat{x}(n) + \frac{1}{n+1}\left[y(n) - \hat{x}(n)\right] \qquad n = 0, 1, 2, \ldots$$

$$\hat{x}(n+1) = (1 - \frac{1}{Q})\hat{x}(n) + \frac{1}{Q}y(n) = \hat{x}(n+1) = \hat{x}(n) + \frac{1}{Q}\left[y(n) - \hat{x}(n)\right] \qquad \text{ahol } Q > 1$$

$$\hat{x}(n+1) = \frac{1}{N}\sum_{k=n-N+1}^{n} y(k) = \hat{x}(n) + \frac{1}{N}[y(n) - y(n-N)] \tag{27}$$

$$H(z) = \frac{\hat{X}(z)}{Y(z)} = \frac{1}{N}\underbrace{\left(z^{-1} + z^{-2} + \ldots z^{-N}\right)}_{N \text{ tag}} \tag{28}$$

$$H(z)|_{z=e^{j\omega T}} = \frac{1}{N}e^{-j\omega T}\frac{1 - e^{-jN\omega T}}{1 - e^{-j\omega T}} = \frac{1}{N}e^{-j\omega T(1+\frac{N}{2}-\frac{1}{2})}\frac{\sin\frac{N}{2}\omega T}{\sin\frac{1}{2}\omega T} \tag{29}$$