2009.02.18.

Inverz orientáció probléma

1. Inverz Rodrigez-feladat

$$\overline{\overline{Rot}}(\bar{t},\varphi) = C\varphi \bar{\bar{I}} + (1-C\varphi)[\bar{t}\circ\bar{t}] + S\varphi[\bar{t}\times] = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}$$

$\bar{t} = ?$ $(|\bar{t}|=1)$

$\varphi = ?$

Alkalmazás: $\bar{\bar{A}}_a = \overline{\overline{Rot}}(\bar{t},\varphi)\bar{\bar{A}} \Rightarrow \overline{\overline{Rot}}(\bar{t},\varphi) = \bar{\bar{A}}_a \bar{\bar{A}}^T = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix} \xrightarrow[\text{Rodrigez}]{\text{inverz}} \bar{t},\varphi$

$\downarrow$

$\bar{e}_{orient} = (\bar{t},\varphi)$

$$\begin{bmatrix} C\varphi + (1-C\varphi)t_x t_x & (1-C\varphi)t_x t_y - S\varphi t_z & (1-C\varphi)t_x t_z + S\varphi t_y \\ (1-C\varphi)t_y t_x + S\varphi t_z & C\varphi + (1-C\varphi)t_y t_y & (1-C\varphi)t_z t_z - S\varphi t_x \\ (1-C\varphi)t_z t_x - S\varphi t_y & (1-C\varphi)t_z t_y + S\varphi t_x & C\varphi + (1-C\varphi)t_z t_z \end{bmatrix} = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}$$

1.) $3C\varphi + (1-C\varphi)\underbrace{(t_x^2 + t_y^2 + t_z^2)}_{1} = l_x + m_y + n_z \Rightarrow C\varphi = \dfrac{l_x + m_y + n_z - 1}{2}$

arccos helyett célszerűbb arctant használni (meredekebben indul, így pontosabb)

~~arccos~~ $\operatorname{atan}(T\varphi) \rightarrow \varphi$

$(S\varphi, C\varphi) \xrightarrow{\text{atan2}} \varphi$

2.) $\left.\begin{aligned} m_z - n_y &= 2S\varphi t_x \\ n_x - l_z &= 2S\varphi t_y \\ l_y - m_x &= 2S\varphi t_z \end{aligned}\right\} \Rightarrow S\varphi = \dfrac{\sqrt{(m_z-n_y)^2 + (n_x-l_z)^2 + (l_y-m_x)^2}}{2}$

$\Leftrightarrow \varphi \in [0,\pi]$

$(S\varphi, C\varphi) \xrightarrow{\text{atan2}} \varphi \ (1\times)$

$\hookrightarrow$ megoldások száma

$$t_x = \sqrt{\frac{l_x - C\varphi}{1-C\varphi}} \cdot \operatorname{sign}(m_z - n_y)$$

$$t_y = \sqrt{\frac{m_y - C\varphi}{1-C\varphi}} \cdot \operatorname{sign}(\cancel{l_y - m_x}\ n_x - l_z)$$

$$t_z = \sqrt{\frac{n_z - C\varphi}{1-C\varphi}} \cdot \operatorname{sign}(\cancel{n_x - l_z}\ l_y - m_x)$$

Szinguláris konfiguráció: $1 - C\varphi = 0$

nincs forgatás, akármilyen $\bar{t}$ jó

$\bar{e}_{orient} = \bar{t}\cdot 0 = 0$

[2] Inverz Euler feladat

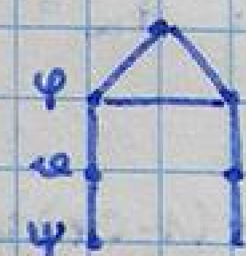
$$\overline{\overline{\text{Euler}}}(\varphi, \vartheta, \psi) = \begin{bmatrix} C\varphi C\vartheta C\psi - S\varphi S\psi & -C\varphi C\vartheta S\psi - S\varphi C\psi & C\varphi S\vartheta \\ S\varphi C\vartheta C\psi + C\varphi S\psi & -S\varphi C\vartheta S\psi + C\varphi C\psi & S\varphi S\vartheta \\ -S\vartheta C\psi & S\vartheta S\psi & C\vartheta \end{bmatrix} = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}$$

Alkalmazás: - nevezetes robotok utolsó 3 csuklója Euler filozófiájú: $q_4 = \varphi$, $q_5 = \vartheta$, $q_6 = \psi$ } Stanford, Puma

- nevezetes robot programozási nyelvek az orientációt Euler-szögben fejezik ki: $o = \varphi$, $a = \vartheta$, $t = \psi$; o, a, t: szögek } VAL, ARPS

1.) $T\varphi = \frac{n_y}{n_x} \xrightarrow{atan} \varphi \quad (2\times)$

2.) $\left.\begin{aligned} C\varphi n_x + S\varphi n_y &= S\vartheta \\ n_z &= C\vartheta \end{aligned}\right\} \rightarrow atan2 \rightarrow \vartheta \quad (1\times)$



3.) $\left.\begin{aligned} -S\varphi l_x + C\varphi l_y &= S\psi \\ -S\varphi m_x + C\varphi m_y &= C\psi \end{aligned}\right\} \rightarrow atan2 \rightarrow \psi \quad (1\times)$

Szinguláris konfig.: $n_x = n_y = 0$

$n_z = +1 \Rightarrow \vartheta = 0 \Rightarrow \left.\begin{aligned} S_{\varphi+\psi} &= l_y \\ C_{\varphi+\psi} &= l_x \end{aligned}\right\} \rightarrow atan2 \rightarrow \varphi + \psi$

$n_z = 1 \Rightarrow \vartheta = \pi \Rightarrow \left.\begin{aligned} S_{\psi-\varphi} &= m_x \\ C_{\psi-\varphi} &= m_y \end{aligned}\right\} \rightarrow atan2 \rightarrow \psi - \varphi$

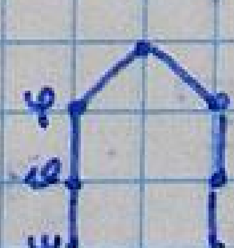
[3] Inverz RPY-szögek

$$\overline{\overline{\text{RPY}}}(\varphi, \vartheta, \psi) = \begin{bmatrix} C\varphi C\vartheta & C\varphi S\vartheta C\psi - S\varphi C\psi & C\varphi S\vartheta C\psi + S\varphi S\psi \\ S\varphi C\vartheta & S\varphi S\vartheta S\psi + C\varphi C\psi & S\varphi S\vartheta C\psi - C\varphi S\psi \\ -S\vartheta & C\vartheta S\psi & C\vartheta C\psi \end{bmatrix} = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}$$

Alkalmazás: - két csukló modellje

1.) $T\varphi = \frac{l_y}{l_x} \xrightarrow{atan} \varphi \quad (2\times)$

2.) $\left.\begin{aligned} S\vartheta &= -l_z \\ C\vartheta &= C\varphi l_x + S\varphi l_y \end{aligned}\right\} \rightarrow atan2 \rightarrow \vartheta \ (1\times)$

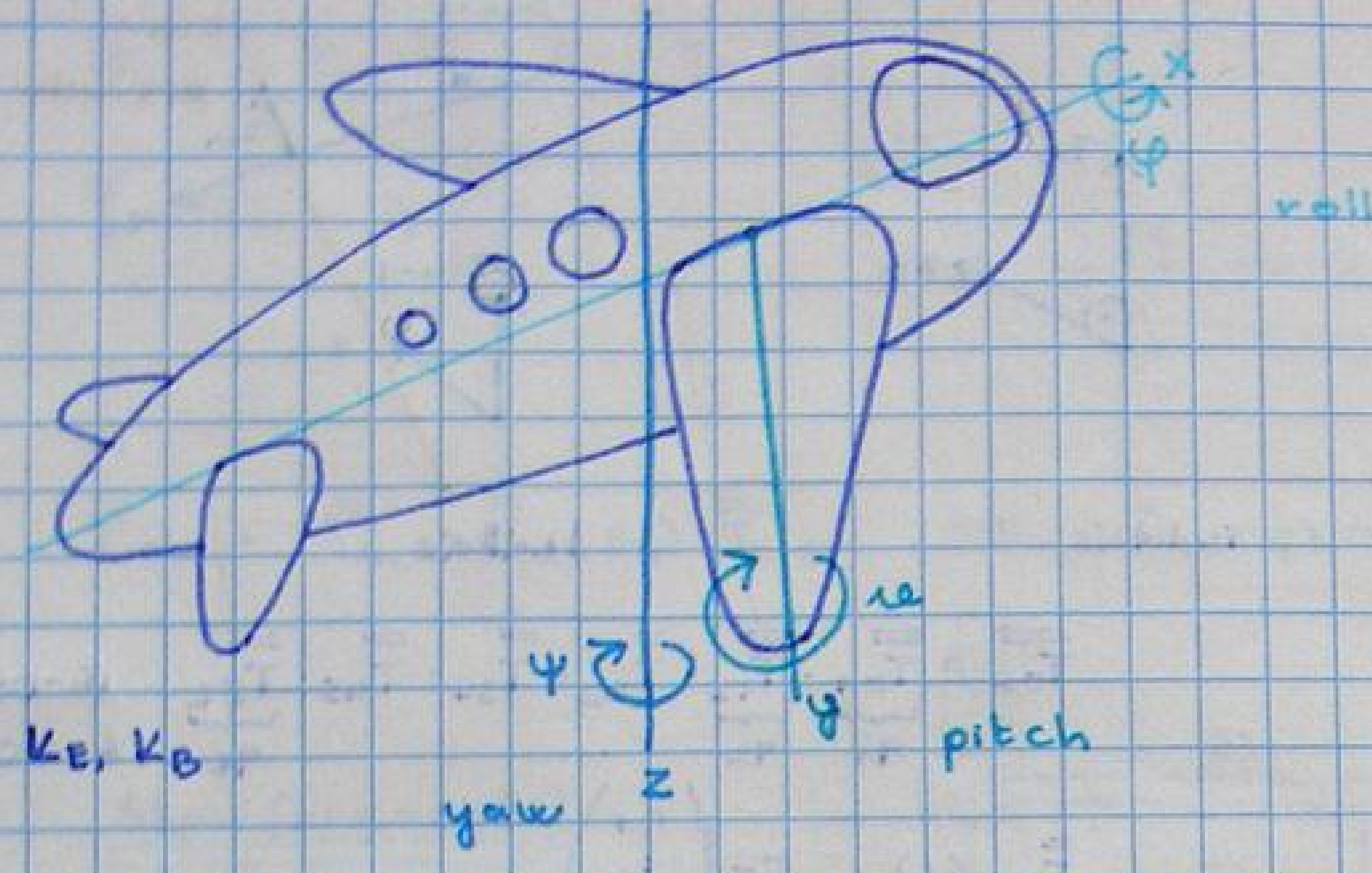


3.) $\left.\begin{aligned} S\psi &= S\varphi n_x - C\varphi n_y \\ C\psi &= -S\varphi m_x + C\varphi m_y \end{aligned}\right\} \rightarrow atan2 \rightarrow \psi \ (1\times)$

Szinguláris konfig.: $l_x = l_y = 0$

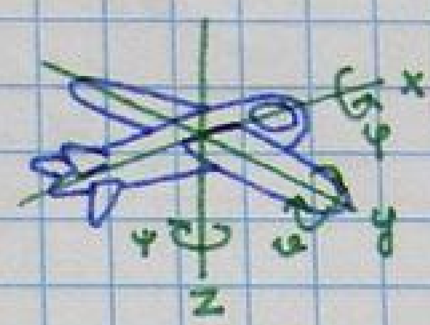
$l_z = +1 \Rightarrow \vartheta = -\frac{\pi}{2} \Rightarrow \left.\begin{aligned} S_{\varphi+\psi} &= -n_y \\ C_{\varphi+\psi} &= -n_x \end{aligned}\right\} \rightarrow atan2 \rightarrow \varphi + \psi$

$l_z = -1 \Rightarrow \vartheta = \frac{\pi}{2} \Rightarrow \left.\begin{aligned} S_{\psi-\varphi} &= m_x \\ C_{\psi-\varphi} &= m_y \end{aligned}\right\} \rightarrow atan2 \rightarrow \psi - \varphi$

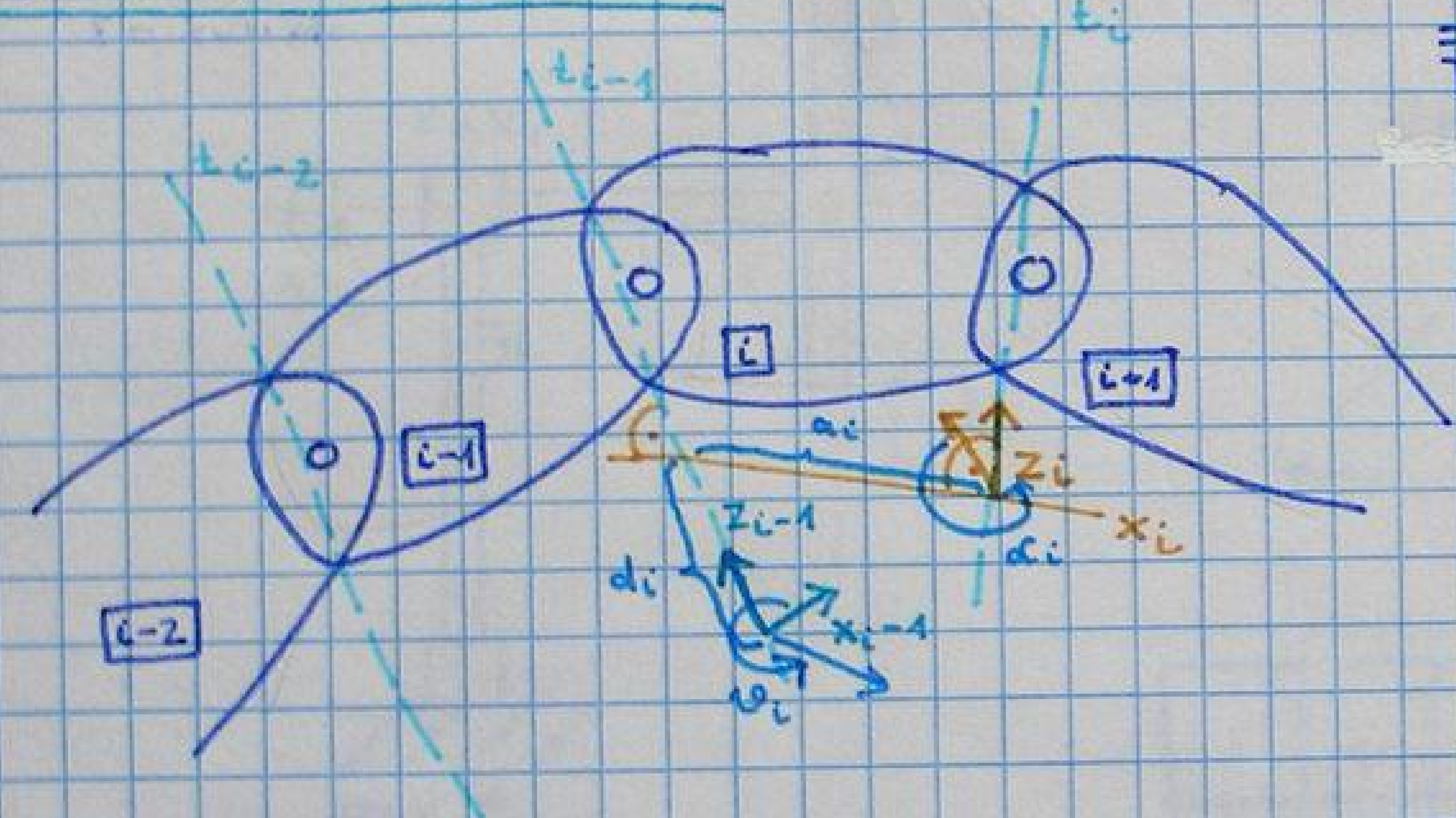


$$\overline{\overline{A}}_{K_E,K_B} = \overline{\overline{Rot}}(z,\psi)\,\overline{\overline{Rot}}(y,\vartheta)\,\overline{\overline{Rot}}(x,\varphi)$$

$RPY(\psi, \vartheta, \varphi)$



Denavit - Hartenberg alak



$$\overline{\overline{T}}_{i-1,i} = \overline{\overline{Rot}}(z_{i-1},\vartheta_i)\,\overline{\overline{Trans}}(z_{i-1},d_i)\cdot\overline{\overline{Trans}}(x_i,a_i)\,\overline{\overline{Rot}}(x_i,\alpha_i)$$

transzláció

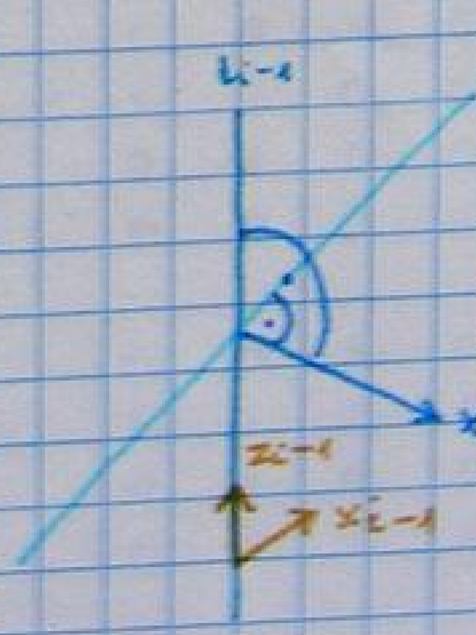
$$T_{i-1,i} = \overline{\overline{Rot}}(z_{i-1},\vartheta_i)\,\overline{\overline{Trans}}(z_{i-1},d_i)\,\overline{\overline{Trans}}(x_i,a_i)\,\overline{\overline{Rot}}(x_i,\alpha_i) =$$

$$= \begin{bmatrix} C_{\vartheta_i} & -S_{\vartheta_i} & 0 \\ S_{\vartheta_i} & C_{\vartheta_i} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{\alpha_i} & -S_{\alpha_i} & 0 \\ 0 & S_{\alpha_i} & C_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{i-1,i} = \begin{bmatrix} C_{\vartheta_i} & -S_{\vartheta_i}C_{\alpha_i} & S_{\vartheta_i}S_{\alpha_i} & a_i C_{\vartheta_i} \\ S_{\vartheta_i} & C_{\vartheta_i}C_{\alpha_i} & -C_{\vartheta_i}S_{\alpha_i} & a_i S_{\vartheta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R \rightarrow q_i = \vartheta_i$ $\vartheta_i, d_i, a_i, \alpha_i$

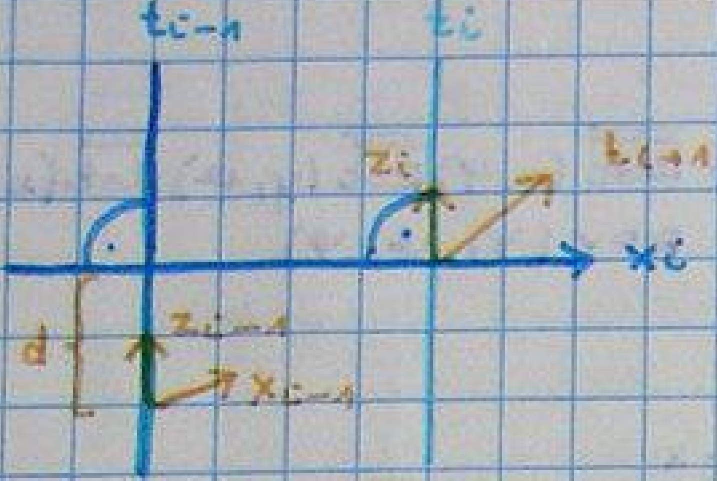
$T \rightarrow q_i = d_i$ $\vartheta_i, d_i, a_i, \alpha_i$



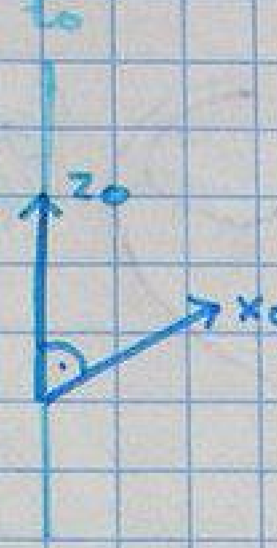
metsző egyenesek

$a_i = 0$

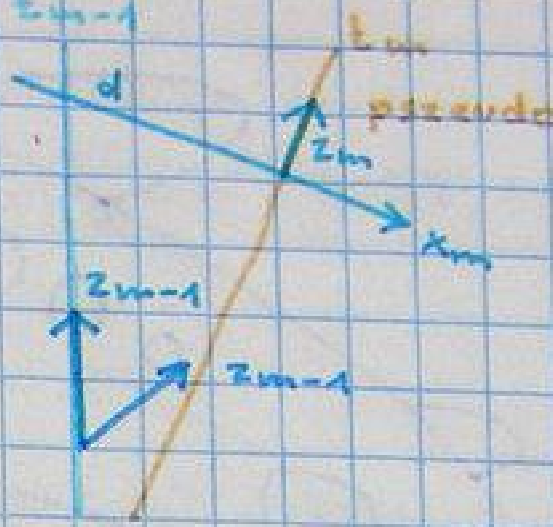
párhuzamos egyenesek



$a_i = 0°$



indulás



leállás

$$\overline{\overline{T}}_{06} = \underbrace{\overline{\overline{T}}_{01}}_{q_1} \cdot \underbrace{\overline{\overline{T}}_{12}}_{q_2} \cdot \overline{\overline{T}}_{23} \cdot \overline{\overline{T}}_{34} \cdot \overline{\overline{T}}_{45} \cdot \underbrace{\overline{\overline{T}}_{56}}_{q_6} \quad \ldots \quad (6\text{-DOF})$$

$$\overline{\overline{T}}_{06}(\overline{q}), \quad \overline{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_6 \end{pmatrix}$$

Direkt geometriai feladat

Inverz = ?