(α)

1. a) $\forall \varepsilon>0,\ \exists x_\varepsilon<0,\ \forall x<x_\varepsilon : |f(x)|<\varepsilon$ $\boxed{5}$

b) $\sqrt{x^2+1}+x=\frac{1}{\sqrt{x^2+1}-x}$ (5) $=\frac{1}{\sqrt{x^2+1}+|x|}$ (2) $<\varepsilon$

elegendő: $\frac{1}{|x|}<\varepsilon$ (2) $\Rightarrow x_\varepsilon=\frac{1}{\varepsilon}$ $\boxed{9}$

2. a) $xe^{x^3}=0$ ha $x=0$, $ax^3+b=b$ ha $x=0$ (3)

$\boxed{5}$ $\Rightarrow b=0$ és $\forall a$ (2)

b) $b=0$ (1) $(xe^{x^3})'=e^{x^3}+3x^3e^{x^3}\big|_{x=0}=1$

$\boxed{5}$ $(ax^3+b)'=3ax^2\big|_{x=0}=0$ (2)

nem diff-ható bármi is az a! (2)

3. a) ÉT: $-1-\sqrt{2}<x<-1+\sqrt{2}$ (5), ÉK: $(-\infty, \ln 2)$ (4)

b) $f'=\frac{-2-2x}{1-2x-x^2}=\frac{2x+2}{x^2+2x-1}$ (5)

c) $f'(-1)=0$ (2), +/− előjelváltás (3) $\Rightarrow$ lok. max

4. a) $\lim\limits_{x\to\frac{\pi}{2}}\frac{tg(4x)}{\sin 2x}=\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin 4x}{\sin 2x}$ (2) $=\lim\limits_{\dots}\frac{4\cos 4x}{2\cos 2x}$ (2) $=-2$ (3)

b) $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{1}{ctg x}-\frac{1}{\cos x}\right)=\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin x-1}{\cos x}$ (2) $=\lim\limits_{x\to\frac{\pi}{2}}\frac{\cos x}{-\sin x}=0$ (2) (3)

c) $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x}}=\lim\limits_{x\to 0}e^{\frac{\ln\cos x}{x}}$ (2)

$\lim\limits_{x\to 0}\frac{\ln\cos x}{x}=\lim\limits_{x\to 0}\frac{-\sin x}{\cos x}$ (2) $=0$, tehát 1 (3)

d) $\lim\limits_{x\to\infty}\frac{e^{-x}(e^{5x}-e^{-5x})}{e^{6x}+e^{-6x}}$ (2) $=\lim\limits_{x\to-\infty}\frac{e^{4x}-e^{-6x}}{e^{6x}+e^{-6x}}$ (2) $=-1$ (2)

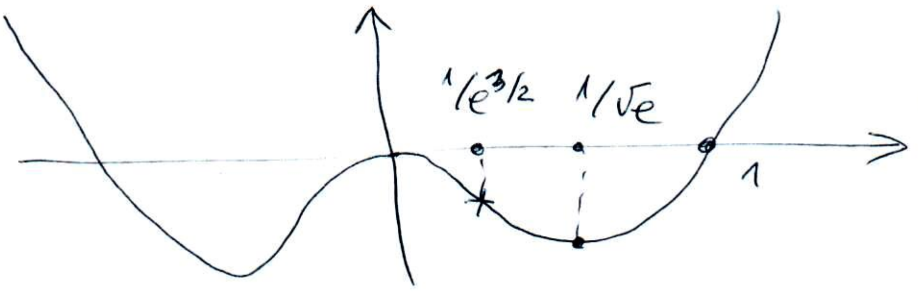
5. a) $f'(x)=2x(1+\ln x^2)$ (2), $f\uparrow$, $(-1/\sqrt{e},0)\cup(1/\sqrt{e},\infty)$ (2)

$f\downarrow$ ......, 0 − lok max (2)

$\pm 1/\sqrt{e}$ − lok. min

b) $f''=6+2\ln x^2$ (3), $(-e^{-3/2}, e^{3/2})$ − konkáv (3) $\pm e^{-3/2}$ infl. (3)

c) $\boxed{6}$



6

d) ÉT: $x \neq 0$ 1 ÉK: $[-3e^{-3}, \infty)$ 2

$\boxed{9}$ folytonos $\forall x \neq 0$-ra, $x = 0$ javítható!
3 3

β

1. a) $\forall \varepsilon > 0, \exists x_\varepsilon < 0, \forall x < x_\varepsilon \; |f(x) - \frac{1}{2}| < \varepsilon$ $\boxed{5}$

$\boxed{9}$ b) $\sqrt{x^2 \cancel{+x}} + x - \frac{1}{2} = \frac{-x}{\sqrt{x^2 - x} - x} - \frac{1}{2} =$ 3

$= \frac{|x|}{2(\sqrt{x^2 - x} - x)^2} < \varepsilon$ ha $\frac{|x|}{2|x|^2} < \varepsilon \Longrightarrow x_\varepsilon = \frac{1}{2\varepsilon}$
3 3

2. a) $\boxed{5}$ $x e^{x^4}|_{x=0} = 0$, $ax^3 + b|_{x=0} = b \Longrightarrow b = 0$, $\forall a$
3 2

$\boxed{5}$ b) $(x e^{x^4})' = e^{x^4} + 4x^4 e^{x^4}|_{x=0} = 1$, $(ax^3 + b)' = $ 2
$= 3ax^2|_{x=0} = 0$
$b = 0$ 1

nem diff-ható! 2

3. $\boxed{9}$ a) ÉT: $-2 < x < 0$ 5 ÉK: $[0, -\infty)$ 4

$\boxed{5}$ b) $f' = \frac{-2-2x}{-2x - x^2} = \frac{2x+2}{x^2+2x}$ 3

$\boxed{5}$ c) $f'(-1) = 0$, $+/-$ előjelváltás lok. max
2 3

4.

7 a) $\lim\limits_{x\to\frac{\pi}{2}} \frac{tg\,6x}{\sin 2x} \overset{2}{=} -\lim\limits_{x\to\frac{\pi}{2}} \frac{\sin 6x}{\sin 2x} \overset{2}{=} -\lim\limits_{x\to\frac{\pi}{2}} \frac{6\cos 6x}{2\cos 2x}$

$= -3$ 3

7 b) $\lim\limits_{x\to\pi} \left(\frac{1}{ctg\frac{x}{2}} - \frac{1}{\cos\frac{x}{2}}\right) \overset{2}{=} \lim\limits_{x\to\pi} \frac{\sin\frac{x}{2}-1}{\cos\frac{x}{2}} =$

$\overset{2}{=} \lim\limits_{x\to\pi} \frac{\frac{1}{2}\cos\frac{x}{2}}{-\frac{1}{2}\sin\frac{x}{2}} = 0$ 3

7 c) $\lim\limits_{x\to 0} (\cos 2x)^{\frac{1}{x}} = \lim\limits_{x\to 0} e^{\frac{\ln\cos 2x}{x}}$ 2

$\lim\limits_{x\to 0} \frac{\ln\cos 2x}{x} = \lim\limits_{x\to 0} \frac{-2\sin 2x}{\cos 2x} = 0$ 2 3

d) u a

5. u a mint a z 2