MINTAMEGOLDÁS [-1-] (anal 2, 1. zh, α, 2013. 03. 14.)

1, $\boxed{13}$ $y' + \frac{2}{x} y = \frac{\sin x}{x}$ elsőrendű lineáris

(H): $y' = -\frac{2}{x} y \Rightarrow \underline{\underline{y_{H,ált}(x) = \frac{K}{x^2}}} ;\ K \in \mathbb{R}$ (mint 3/2) 5

(I): Együttható variálása (lásd 1/2) $y_{I,p}(x) = \frac{K(x)}{x^2}$ 2

$$\frac{K'(x)}{x^2} = \frac{\sin x}{x} \Rightarrow K'(x) = x \sin x \Rightarrow K(x) = -x\cos x + \sin x \quad ②\ ②$$

$$\underline{\underline{y_{I,ált}(x) = \frac{K}{x^2} + \frac{-x\cos x + \sin x}{x^2}}} ;\ K \in \mathbb{R} \quad ②$$

2, $\boxed{13}$ $y' = \frac{\operatorname{ctg} y}{\sqrt{x^2-1}}$ $\operatorname{ctg} y \equiv 0$, azaz $y \equiv \frac{\pi}{2} + k\pi ; k \in \mathbb{Z}$ megoldás, tehát

$\underline{\underline{y(x_0) = \frac{\pi}{2} \text{ esetén } y(x) \equiv \frac{\pi}{2}}}$ 3

Ha $\operatorname{ctg} y \neq 0$ szeparálható.

$$\int \frac{\sin y}{\cos y}\, dy = \int \frac{dx}{\sqrt{x^2-1}} \quad ②$$

$$-\ln|\cos y| = \operatorname{arch} x + C \Rightarrow \underline{\underline{y_{ált}(x) = \arccos\left(K \cdot e^{-\operatorname{arch} x}\right)}} \quad K \in \mathbb{R} \quad ③\ ③$$

$$y(0) = -\pi$$

A kezdeti feltételnek megfelelő megoldás NINCS, mert a diff. egy. sem $x=0$-ban, sem $y=-\pi$-ben nincs értelmezve! 2

3, $\boxed{15}$ $y' = \frac{2x+4y}{3x+y} = \frac{4y/x + 2}{y/x + 3}$; $u(x) = \frac{y}{x}$; $y'(x) = u(x) + x u'(x)$ 2

$$x u' + u = \frac{4u+2}{u+3} \Rightarrow u' = \frac{1}{x}\left(\frac{4u+2}{u+3} - u\right) = \frac{1}{x} \frac{-u^2+u+2}{u+3}$$

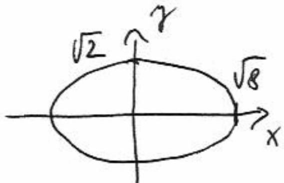
$$\int \frac{u+3}{u^2-u-2}\, du = -\int \frac{dx}{x} \quad ④ \qquad u \neq 2,\ u \neq -1$$

$$\frac{u+3}{u^2-u-2} = \frac{A}{u-2} + \frac{B}{u+1} ; \quad \left.\begin{array}{l} A+B=1 \\ A-2B=3 \end{array}\right\} B = -\frac{2}{3} ; A = \frac{5}{3}$$

$$\frac{5}{3}\int \frac{du}{u-2} - \frac{2}{3}\int \frac{du}{u+1} = \frac{5}{3}\ln|u-2| - \frac{2}{3}\ln|u+1| = -\ln|x| + C \quad ⑥$$

$$\underline{\underline{\frac{5}{3}\ln\left|\frac{y}{x} - 2\right| - \frac{2}{3}\ln\left|\frac{y}{x}+1\right| = \ln|x| + C}} ;\ C \in \mathbb{R} \quad ② \qquad \left.\begin{array}{l} \underline{\underline{y(x) = 2x}} \\ \underline{\underline{y(x) = -x}} \end{array}\right\} ①$$

4, $\boxed{14}$ $y' = \dfrac{1}{x^2+4y^2}$; $(x_0, y_0) = (2, 1)$



a, Az iránymező meredeksége (x_0, y_0)-ban: $\dfrac{1}{2^2+4\cdot 1^2} = \dfrac{1}{8}$ (2)

Izoklína egyenlete: $\dfrac{1}{x^2+4y^2} = \dfrac{1}{8} \Rightarrow x^2+4y^2 = 8 \Rightarrow \dfrac{x^2}{8} + \dfrac{y^2}{2} = 1$ (4)

Ellipszis

b, Legyen $y(x)$ az (x_0, y_0)-on átmenő megoldás.

$y'(x_0) = \dfrac{1}{8}$; $y'' = \dfrac{-(2x+8y\cdot y')}{(x^2+4y^2)^2}$; $y''(x_0) = \dfrac{-(2\cdot 2+8\cdot 1\cdot \frac{1}{8})}{(2^2+4\cdot 1^2)^2} = -\dfrac{5}{64}$ (2)

$y''(x_0) \neq 0 \Rightarrow$ nincs inflexió! (2)

5, $\boxed{14}$ $y''' + 2y'' + y' = 2\cos x + e^{-x}$

$\lambda^3 + 2\lambda^2 + \lambda = \lambda(\lambda+1)^2 = 0 \Rightarrow \lambda_1 = 0$; $\lambda_{2,3} = -1$ ← Belső rezonancia

$y_{H,ált}(x) = C_1 + C_2 e^{-x} + C_3 x e^{-x}$ (6)

$y_{I,p}(x) = A\sin x + B\cos x + Cx^2 e^{-x}$ (3) (külső rez.)

$y'_{I,p}(x) = A\cos x - B\sin x + Ce^{-x}(2x - x^2)$ $/\cdot(1)$

$y''_{I,p}(x) = -A\sin x - B\cos x + Ce^{-x}(x^2 - 4x + 2)$ $/\cdot(2)$

$+$ $y'''_{I,p}(x) = -A\cos x + B\sin x + Ce^{-x}(-x^2 + 6x - 6)$ $/\cdot(1)$

$2\cos x + e^{-x} = \sin x(\cancel{-B} - 2A + \cancel{B}) + \cos x(\cancel{A} - 2B - \cancel{A}) +$
$+ Ce^{-x}\underbrace{(2x - x^2 + 2(x^2 - 4x + 2) - x^2 + 6x - 6)}_{-2}$

$A = 0$; $B = -1$; $C = -\dfrac{1}{2}$ (3)

$y_{I,ált}(x) = C_1 + C_2 e^{-x} + C_3 x e^{-x} - \cos x - \dfrac{x^2}{2} e^{-x}$ (2)

6, $\boxed{11}$ $y = 2x\cdot e^{3x} - 4\cos x$

$$\lambda_{1,2} = 3;\ \lambda_{4,5} = \pm i \quad ④ \implies \underbrace{(\lambda-3)^2}_{\lambda^2-6\lambda+9}\overbrace{(\lambda+i)(\lambda-i)}^{\lambda^2+1} = \lambda^4 - 6\lambda^3 + 10\lambda^2 - 6\lambda + 9 = 0 \quad ③$$

$$y^{(4)} - 6y^{(3)} + 10y'' - 6y' + 9y = 0 \quad ②$$

$$y_{H,ált}(x) = C_1 e^{3x} + C_2 x e^{3x} + C_3 \sin(x) + C_4 \cos(x) \quad ②$$

7, $\boxed{10}$ $f(n) = \frac{13}{2} f(n-1) - 3f(n-2) \implies 2q^2 - 13q + 6 = 0$ ②

$$q_{1,2} = \frac{13 \pm \sqrt{169-48}}{4} = \frac{13 \pm 11}{4} = \left\langle \begin{matrix} 6 \ ② \\ \frac{1}{2} \end{matrix} \right. \implies f_{ált}(n) = A\cdot 6^n + B\left(\tfrac{1}{2}\right)^n \quad ②$$

$A, B \in \mathbb{R}$

$$f(n) \text{ konv.} \iff A = 0 \ ② \iff f(0) = B,\ f(1) = \frac{B}{2} \iff f(0) = 2f(1) \quad ②$$

8, $\boxed{10}$, Lásd B/8. a, ④ b, $\sqrt[n]{a_n} \longrightarrow e^{-5}$ ④ $\implies \sum_n a_n < \infty$ ②

Pót: 9, $\boxed{10}$ $y^{(5)} - 81y' = 0 \implies \lambda^5 - 81\lambda = \lambda\underbrace{(\lambda+3)(\lambda-3)}_{(\lambda^2-9)}\underbrace{(\lambda+3i)(\lambda-3i)}_{(\lambda^2+9)} = 0$ ② ④

$$y_{H,ált}(x) = C_1 + C_2 e^{3x} + C_3 e^{-3x} + C_4 \sin(3x) + C_5 \cos(3x) \quad ④$$

10, $\boxed{10}$ $\sum_{n=1}^{\infty} \frac{(2n)!\,(3n)!}{(5n)!}$; $\quad \frac{a_{n+1}}{a_n} = \frac{(2n+2)!\,(3n+3)!}{(5n+5)!} \cdot \frac{(5n)!}{(2n)!\,(3n)!} =$ ② ②

$$= \frac{(2n+1)(2n+2)\,(3n+1)(3n+2)(3n+3)}{(5n+1)(5n+2)(5n+3)(5n+4)(5n+5)} \longrightarrow \frac{2^2\cdot 3^3}{5^5} < 1 \quad ④$$

$$\implies \sum_n a_n \text{ konvergens} \quad ②$$