2016. 06. 16. EMT megold:

1/ Hengerkondenzátor

$r_1 = 3\,mm$

$r_2 = 6\,mm$

$l = 35\,mm$

$\varepsilon_r = 4$

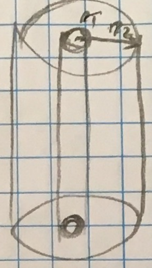
$Q_1 = 0$

$Q_2 = 0$

$\rho = 3\,\frac{\mu C}{m^3}$

$U = ?$

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}\,\frac{As}{Vm}$



Gauss:

$$\oint \vec{E}\,d\vec{A} = \frac{1}{\varepsilon_0 \varepsilon_r} \cdot \int_V \rho\,dV$$

$$E \cdot 2\cancel{\pi} r \cancel{l} = \frac{1}{\varepsilon_0 \varepsilon_r} \cdot \rho \cdot (r^2 - r_1^2)\,\cancel{\pi}\,\cancel{l}$$

$$E(r) = \frac{1}{2\varepsilon_0 \varepsilon_r} \cdot \rho \cdot \frac{(r^2 - r_1^2)}{r}$$

$$E(r) = \frac{1}{2\varepsilon_0 \varepsilon_r} \cdot \rho \cdot \left(r - \frac{r_1^2}{r}\right)$$

$$U = \int_{r_1}^{r_2} \frac{1}{2\varepsilon_0 \varepsilon_r} \cdot \rho \left(r - \frac{r_1^2}{r}\right) dr =$$

$$= \frac{1}{2\varepsilon_0 \varepsilon_r} \rho \int_{r_1}^{r_2} r - \frac{r_1^2}{r}\,dr = \frac{1}{2\varepsilon_0 \varepsilon_r} \rho \left(\int_{r_1}^{r_2} r\,dr - r_1^2 \int_{r_1}^{r_2} \frac{1}{r}\,dr \right.$$

$$= \frac{\rho}{2\varepsilon_0 \varepsilon_r} \cdot \left(\left(\frac{r_2^2 - r_1^2}{2}\right) - r_1^2 \ln \frac{r_2}{r_1} \right) \cong 0{,}3077\,V \approx 0{,}31\,V$$

2/ $\Phi_1 = -20\,V \leftarrow P_1$ $\qquad \Phi(P_2) = \Phi_2 = ?$

$$\int_{P_1}^{P_2} \vec{E}\,d\vec{l} = -30\,V$$

$$-\int_{P_1}^{P_2} \vec{E}\,d\vec{l} = \Phi_2 - \Phi_1$$

$$30 = \Phi_2 - (-20)$$

$$30 = \Phi_2 + 20$$

$$\Phi_2 = 10\,V$$

3/

$\vec{E} = (30; 40; 0)\ \frac{kV}{m}$

$\vec{D} = (0,84; 1,12; 0)\ \mu\frac{As}{m^2}$

$w_e = 0,07 \frac{J}{m^3}$

lineáris $\Leftrightarrow$ $\overline{\overline{\varepsilon}}\vec{E} = \vec{D}$ és $E = 0 \Leftrightarrow D = 0$ minden rendezőre

$D_x = 0$, de $E_x \neq 0$ $\Rightarrow$ nemlineáris

4/

$\left.\begin{array}{l} I_1 = 2\ A \\ I_2 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} \Psi_1 = 2\ Wb \\ \Psi_2 = 0,6\ Wb \end{array}$

$\left.\begin{array}{l} I_1 = 0 \\ I_2 = 0,5\ A \end{array}\right\} \Rightarrow \Psi_1 = ?$

kölcsönös indukciók: $L_{21} = \left.\frac{\Psi_2}{I_1}\right|_{I_2=0}$

$L_{21} = \frac{0,6}{2} = 0,3\ H$

$L_{12} = \left.\frac{\Psi_1}{I_2}\right|_{I_1=0}$

$0,3 = \frac{\Psi_1}{0,5}$

$\Psi_1 = 0,15\ Wb$

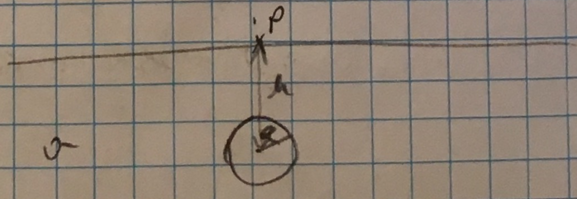
5/ $\sigma = 0,1 \frac{S}{m}$ $\sigma_{lev.} = 0$

$h = 5\ m$

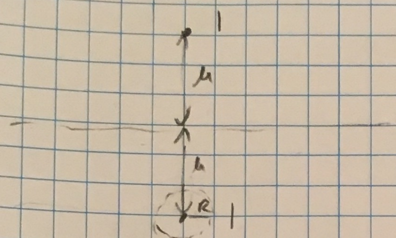
$R = 20\ cm$

$I = 100\ A$

$\varphi(P) = ?$



Áramtüskrótás:



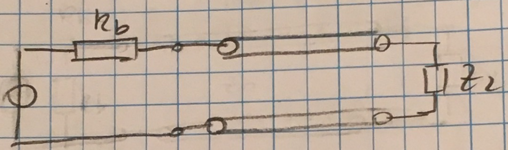
$$\phi(r) = \frac{I}{4\pi\sigma} \cdot \frac{1}{r}$$ ← elektrosztatikai analógiából!

Kis nyomott közelítéssel:

$$\phi(P) = \frac{2I}{4\pi\sigma} \cdot \frac{1}{h} = \frac{2 \cdot 100}{4\pi \cdot 0,1} \cdot \frac{1}{5} \doteq 31,83\ V \approx 31,8\ V$$

6,

$l = 4\ m$
$Z_0 = 75\ \Omega$
$R_b = 10\ \Omega$
szinusz generátor
$\lambda = 2,5\ m$



teljesítményillesztés: $P = max \Leftrightarrow Z_2 = Z_{be2}^*$

$$\beta = \frac{2\pi}{\lambda}$$

$$\beta l = \frac{2\pi}{\lambda} \cdot l = 3,2\pi$$

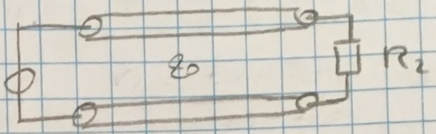
$$Z_{be2} = Z_0 \cdot \frac{R_b + jZ_0 \operatorname{tg}(\beta l)}{Z_0 + jR_b \operatorname{tg}(\beta l)} = \frac{75 \cdot (10 + j75 \operatorname{tg}(3,2\pi))}{75 + j10 \operatorname{tg}(3,2\pi)}$$

$$Z_{be2} = 15,1366 + j53,0244\ \Omega$$

$$Z_2 = 15,1366 - j \cdot 53,0244\ \Omega$$

7.

$Z_0 = 75\,\Omega$
$R_2 = 50\,\Omega$
$U_{max} = 12\,V$



①

$P = P_{beeső} - P_{reflektált} = P^+ - P^-$ ⟸ a forrás által leadott teljesítmény

$$P = \frac{1}{2Z_0}\left(|U^+|^2 - |U^-|^2\right) = \frac{1}{2Z_0}\cdot\left(|U_2^+| - |U_2^-|^2\right)$$

$$\left.\begin{array}{l} U_{max} = (1+|r_2|)U_2^+ \\ r_2 = \dfrac{R_2 - Z_0}{R_2 + Z_0} = -0{,}2 \end{array}\right\} \Rightarrow U_1^+ = 10V \quad P = 0{,}64W$$

$|U_2^-| = |r_2 \cdot U_2^+| = 10 \cdot 0{,}2 = 2V$ ⟸ $r_2 = \dfrac{U_2^-}{U_2^+}$ ← lezárás

② $R_2 = Z_0 \Rightarrow$ illesztett lezárás $\Rightarrow r_2 = 0$

$$P = \frac{1}{2Z_0}\cdot|U_2^+|$$

$$|U_2^+| = \sqrt{2Z_0 \cdot P} = 9{,}8V$$

8.

$E(x,y,z,t) = \hat{e}_y \cdot 20\frac{V}{m}\cdot\cos(2\pi f t + \beta x)$ Levegő

↑ x irányban terjed

$f = 100\,MHz$

$H(2,3,-1,0) = ?$

$$H(x,y,z,t) = -\hat{e}_z \cdot \frac{E_0}{Z_0}\cdot\cos(2\pi f t + \beta x)$$

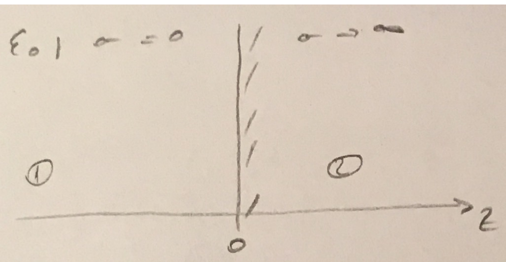
$Z_0 = 377\,\Omega$

$$\lambda = \frac{c}{f} = 3\,m$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{3}$$

$$H(2,3,-1,0) = -\hat{e}_z \cdot 0{,}05305 \cdot \cos\left(\frac{2\pi}{3}\cdot 2\right) \hat{=} 0{,}02653\,\frac{mA}{m}$$

9.



$|E_1^+| = 53 \frac{V}{m}$

$K = ?$

$Z_{02} = \sqrt{\frac{j\omega\mu}{\sigma}} \to 0$

$Z_{01} = 120\pi\,\Omega$

$r_{12} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = \frac{E_1^-}{E_1^+}$

$E_1(z) = E_1^+ e^{-j\beta_1 z} + r_{12} E_1^+ e^{+j\beta_1 z}$

$H_1(z) = \frac{E_1^+}{Z_{01}} \cdot e^{-j\beta_1 z} - r_{12} \frac{E_1^+}{Z_{01}} e^{+j\beta_1 z}$

$$\left.\begin{array}{l} E_2(z) = E_2^+ e^{-j\gamma_2 z} \\ H_2(z) = \frac{E_1^+}{Z_{02}} e^{-j\gamma_2 z} \end{array}\right\}$$ ② : végtelen féltér ⇒ nem található reflektált komponense

$$\left.\begin{array}{l} \gamma_1 = \sqrt{j\omega\mu_1 \cdot j\omega\varepsilon_1} = j\omega\sqrt{\mu_0\varepsilon_0} \\ \gamma_1 = j\beta_1 \\ \gamma_2 = \sqrt{j\omega\mu_2\sigma} \to \infty \end{array}\right\} \Rightarrow \begin{array}{l} E_2(z) \to 0 \\ H_2(z) \to 0 \end{array}$$

$H_{1t} - H_{2t} = K$

H, E térerő tangenciális $\Rightarrow$ $H_1(0) - H_2(0) = K$

$H_2(0) = 0$

$\Downarrow$

$H_1(0) = K$

$H_1(z=0) = \frac{E_1^+}{Z_{01}} + \frac{E_1^+}{Z_{01}}$

$H_1(z=0) = \frac{2E_1^+}{Z_{01}} \Rightarrow |H_1(0)| = \frac{2|E_1^+|}{Z_{01}} = |K|$

$|K| = 0{,}28116 \frac{A}{m}$

10,

$D_{dB} = 3{,}52$ $\quad D_{dB} = 10 \lg(D)$ $\quad 3{,}75 = 10 \lg D$

$P_S = 2\,kW$ $\quad 0{,}375 = \lg D$

$E(r = 500\,m)$ ← maximális sugárzási irány $\quad D = 2{,}25$

$D \triangleq \frac{S_{max}}{S_{átl}} = \frac{S_{max}}{\frac{P_S}{4r^2\pi}} \longrightarrow S_{max} = D \frac{P_S}{4r^2\pi} = 1{,}4324 \frac{mW}{m^2}$

$$\vec{S} = \frac{1}{2}\,\vec{E} \times \vec{H}^* = \frac{1}{2}\left(\overline{E_\vartheta}\cdot\vec{e_\vartheta}\right)\times\left(\overline{H_\varphi}^*\cdot\vec{e_\varphi}\right)$$

$$\vec{S} = \overline{S_r}\cdot\vec{e_r}$$

$$\overline{S_r} = \frac{\overline{E_\vartheta}\cdot\overline{H_\varphi}^*}{2}$$

$$\frac{\overline{E_\vartheta}}{\overline{H_\varphi}} = Z_0 = 377\,\Omega$$

$$\overline{H_\varphi} = \frac{\overline{E_\vartheta}}{Z_0}$$

$$S_r = \frac{\left|\overline{E_\vartheta}\right|^2}{2Z_0}$$

$$\sqrt{2Z_0 S_{max}} = \left|\overline{E_\vartheta}\right| \mathrel{\hat{=}} 1{,}039\,\frac{V}{m} \mathrel{\hat{=}} 1{,}04\,\frac{V}{m}$$