Megoldások

1. $L(s)=K\dfrac{1+sT}{s}\dfrac{e^{-T_H s}}{1+sT}=K\dfrac{e^{-T_H s}}{s}$.

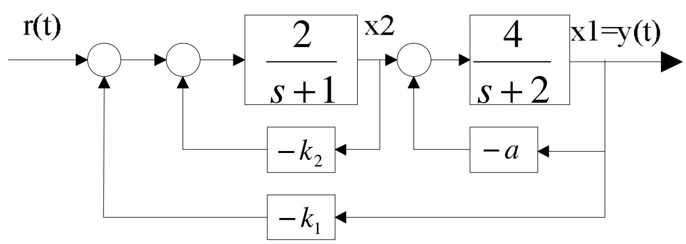
a./ $\omega_c=K$.

Fázisszög romlás a késleltetés miatt: $\omega_c T_H=\dfrac{\pi}{6} \quad\Rightarrow\quad \omega_c=\mathrm{K}=\dfrac{\pi}{6T_H} \quad\Rightarrow\quad \mathrm{C}(\mathrm{s})=\dfrac{\pi(1+Ts)}{6T_H s}$

$$\frac{-\pi}{2}-T_H\omega_\pi=-\pi \quad\Rightarrow\quad \omega_\pi=\frac{\pi}{2T_H} \quad\Rightarrow\quad GM=\frac{\omega_\pi}{K}=\frac{\dfrac{\pi}{2T_H}}{\dfrac{\pi}{6T_H}}=3.$$

b./ $y(0)=0 \quad u(0)=\dfrac{\pi T}{6T_H} \qquad \lim\limits_{t\to\infty} y(t)=1 \quad \lim\limits_{t\to\infty} u(t)=1$.

2.



Az átalakított vázlatot ld. A (*) jelzésű papíron.

$$\mathbf{A}=\begin{bmatrix}-2 & 4\\ 0 & -1\end{bmatrix} \qquad \mathbf{b}=\begin{bmatrix}0\\ 2\end{bmatrix} \qquad \mathbf{bk}^{\mathrm{T}}=\begin{bmatrix}0\\ 2\end{bmatrix}\begin{bmatrix}k_1 & k_2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 2k_1 & 2k_2\end{bmatrix}$$

$$\left|s\mathbf{I}-\mathbf{A}+\mathbf{bk}^{\mathrm{T}}\right|=\begin{vmatrix}s+2 & -4\\ 2k_1 & s+1+2k_2\end{vmatrix}=\mathrm{s}^2+(3+2k_2)\mathrm{s}+2+4k_2+8k_1=$$
$$=(s-p_1)(s-p_2)=\mathrm{s}^2-(p_1+p_2)\mathrm{s}+p_1p_2$$

Innen:
$$k_2=\frac{-3-p_1-p_2}{2}$$
$$k_1=\frac{p_1p_2+4+2(p_1+p_2)}{8}$$

3. $L(z)=\dfrac{z-\alpha}{(z-1)(z-z_1)}$

$$1+L(z)=1+\frac{z-\alpha}{(z-1)(z-z_1)}=0$$

$$z^2-z-z_1z+z_1+z-\alpha=0$$

$$z^2 - z_1 z + z_1 - \alpha = 0 \quad \Rightarrow \quad z_{1,2} = \frac{z_1 \pm \sqrt{z_1^2 - 4z_1 + 4\alpha}}{2}$$

$$z_1 \pm \sqrt{z_1^2 - 4z_1 + 4\alpha} = 2 \quad \Rightarrow \quad z_1^2 - 4z_1 + 4\alpha = 4 - 4z_1 + z_1^2$$

$$4\alpha = 4 \quad \Rightarrow \quad \alpha = 1$$

A stabilitáshoz: $0 < \alpha < 1$

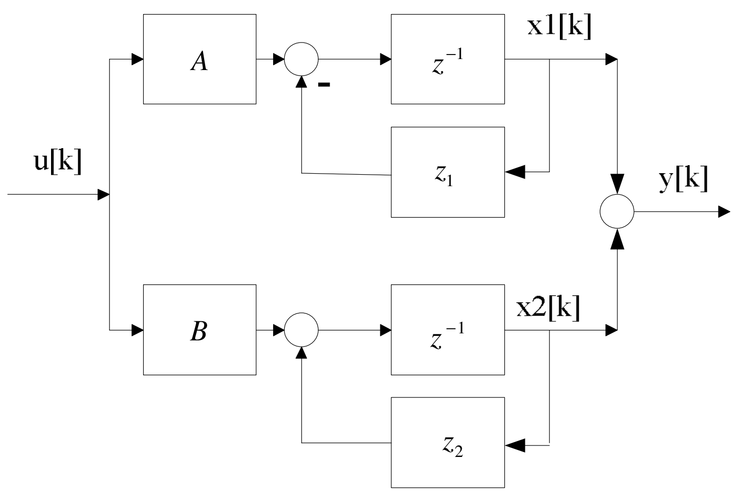
4, $z_1 = e^{-T_s/T_{1st\max}}$ $\quad z_2 = e^{-T_s/T_{2nd\max}}$

5. $G(z) = \dfrac{z + z_o}{(z+z_1)(z+z_2)} = \dfrac{A}{z+z_1} + \dfrac{B}{z+z_2}$

$$A + B = 1 \quad \Rightarrow \quad B = 1 - A$$

$$Az_2 + Bz_1 = z_o \quad \Rightarrow \quad A = \frac{z_o - z_1}{z_2 - z_1}, \quad B = \frac{z_2 - z_o}{z_2 - z_1}$$

$$G(z) = \frac{z + z_o}{(z+z_1)(z+z_2)} = \frac{(z_o - z_1)/(z_2 - z_1)}{z + z_1} + \frac{(z_2 - z_o)/(z_2 - z_1)}{z + z_2}$$



$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} -z_1 & 0 \\ 0 & -z_2 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} \dfrac{z_o - z_1}{z_2 - z_1} \\ \dfrac{z_2 - z_o}{z_2 - z_1} \end{bmatrix} u[k]$$

$$y[k] = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + 0 \cdot u[k]$$

$$x_1[k+1] = -z_1 x_1[k] + \frac{z_0 - z_1}{z_2 - z_1} u[k]$$

$$x_1[0] = 0$$

$$x_1[1] = \frac{z_0 - z_1}{z_2 - z_1}$$

6. $G(z)=\dfrac{5(z+0.8)}{9(z-0.5)(z-0.8)}\cdot z^{-2}=\dfrac{z}{(z-0.5)(z-0.8)}\cdot\dfrac{z+0.8}{1.8z}\cdot z^{-2}=G_+G_-z^{-d}$

$$Q(z)=\frac{R_n}{G_+}=\frac{(z-0.5)(z-0.8)}{z}\cdot\frac{0.8}{z-0.2}=\frac{0.8\left(z^2-1.3z+0.4\right)}{z^2-0.2z}$$

$$y[k]=G_-R_rz^{-2}r[k]=G_-R_rz^{-2}r[k]=\frac{z+0.8}{1.8z}\cdot\frac{0.8}{z-0.2}\cdot z^{-2}r[k]=\frac{z+0.8}{z}\cdot\frac{4/9}{z-0.2}\cdot z^{-2}r[k]$$

$$y[k]=0.2y[k-1]+\frac{4}{9}r[k-3]-\frac{16}{45}r[k-4]$$

$$y[0]=0,\quad y[1]=0,\quad y[2]=0,\quad y[3]=\frac{4}{9}$$

$$u[k]=\frac{R_r}{R_n}Qr[k]=Qr[k]=\frac{0.8\left(z^2-1.3z+0.4\right)}{z^2-0.2z}r[k]$$

$$u[k]=Qr[k]=0.8r[k]-1.04r[k-1]+0.32r[k-2]+0.2u[k-1]$$

$$u[0]=0.8,\quad u[1]=-0.08,\quad u[2]=0.064.$$