# Jelek és rendszerek 2

2 házi elfogadva www.hit.bme.hu

3 kis ZH -ból 2 -nek 2.0 -nak kell lennie

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$u(t) = U_0 \cdot \cos(\omega t + \varphi)$

$\bar{u} = U_0 \cdot e^{j\varphi}$

nő a f.

$R \rightarrow R$ $\qquad f \sim \omega \uparrow \qquad = \; (\uparrow)$ nő

$C \rightarrow \frac{1}{j\omega C} \qquad \downarrow$

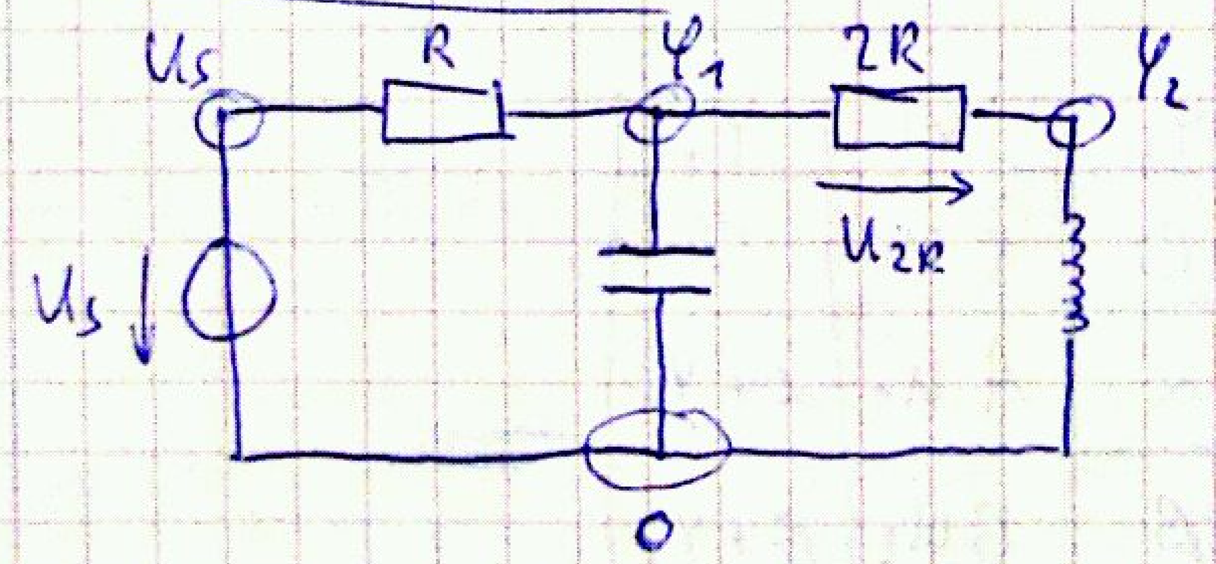
$L \rightarrow j\omega L \qquad \uparrow$

$u(t) = U_0 + U_1 \cdot \cos(\omega_1 t + \varphi_1) + U_2 \cdot \cos(2\omega t + \varphi_2)$

$H(j\omega) = \frac{\bar{Y}(j\omega)}{\bar{U}(j\omega)}$

átviteli karakterisztika

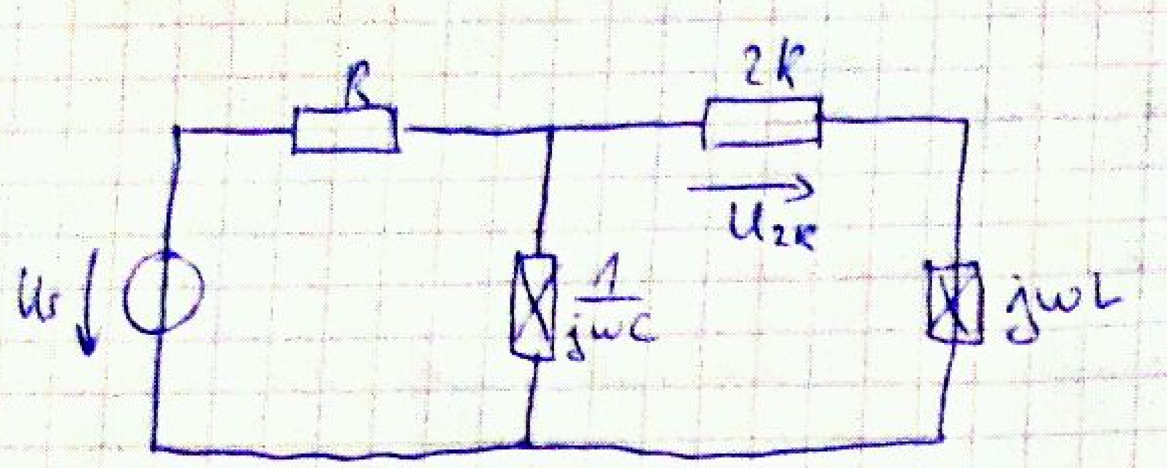
1,




$Y_1, \quad \frac{Y_1 - U_s}{R} + \frac{Y_1}{\frac{1}{j\omega C}} + \frac{Y_1 - Y_2}{2R} = 0$

$Y_2, \quad \frac{Y_2}{j\omega L} + \frac{Y_2 - Y_1}{2R} = 0 \qquad U_{2R} = Y_1 - Y_2$

2,




$$H(j\omega) = \frac{\frac{1}{j\omega C} \otimes (2R + j\omega L)}{\frac{1}{j\omega C} \otimes (2R + j\omega L) + R} \cdot \frac{2R}{2R + j\omega L} = \frac{\frac{2R}{j\omega C}}{\frac{2R}{j\omega C} + \frac{j\omega L}{j\omega C} + \frac{R}{j\omega C} + 2R^2 + Rj\omega L} =$$

$$= \frac{2R}{3R + j\omega(L + 2R^2C) + (j\omega)^2 RLC} = \frac{\frac{2}{LC}}{(j\omega)^2 + j\omega\left(\frac{1}{RC} + \frac{2R}{L}\right) + \frac{3}{LC}}$$

3,

$$\dot{x} = Ax + Bu$$

$$y = c^T x + Du \qquad \frac{d}{dt} \Rightarrow j\omega$$

$$j\omega \bar{x} = \underline{A}\bar{x} + \underline{B}\bar{u}$$

$$\bar{y} = \underbrace{\left[c^T (j\omega \underline{1} - \underline{A})^{-1} \underline{B} + D\right]}_{H(j\omega)} \bar{u}$$

| $\omega$ | $\lvert U \rvert$ | $\varphi_u$ | $\lvert H \rvert$ | $\varphi_H$ | $\lvert Y \rvert$ | $\varphi_Y$ |
|---|---|---|---|---|---|---|
| 0 | $U_0$ | — | $\frac{2}{3}$ | 0 | $\frac{2}{3}U_0$ | 0 |
| $\omega_1$ | $U_1$ | $\varphi_1$ | A | $\alpha$ | $AU_1$ | $\alpha + \varphi_1$ |
| $2\omega$ | $U_2$ | $\varphi_2$ | B | $\beta$ | $BU_2$ | $\beta + \varphi_2$ |

$$H(j\omega)\Big|_{\omega=0} = \frac{2}{3} \qquad H(j\omega)\Big|_{\omega=\omega_1} = A \cdot e^{j\alpha}$$

$$H(j\omega)\Big|_{\omega=\omega_2} = B \cdot e^{j\beta}$$

$$y(t) = \frac{2}{3}U_0 + A U_1 \cos(\omega_1 t + \alpha + \varphi_1) + B U_2 \cos(2\omega_1 t + \beta + \varphi_2)$$

$$p(t) = u(t) \cdot i(t)$$

$$P \quad [W] = \frac{\hat{U}\hat{I}^*}{2} \cdot \cos(\varphi_u - \varphi_i)$$

$$Q \quad [var] = \frac{\hat{U}\hat{I}^*}{2} \cdot \sin(\varphi_u - \varphi_i)$$

$$\bar{S} = P + jQ \qquad [VA]$$

$$S = |\bar{S}| \qquad [VA]$$

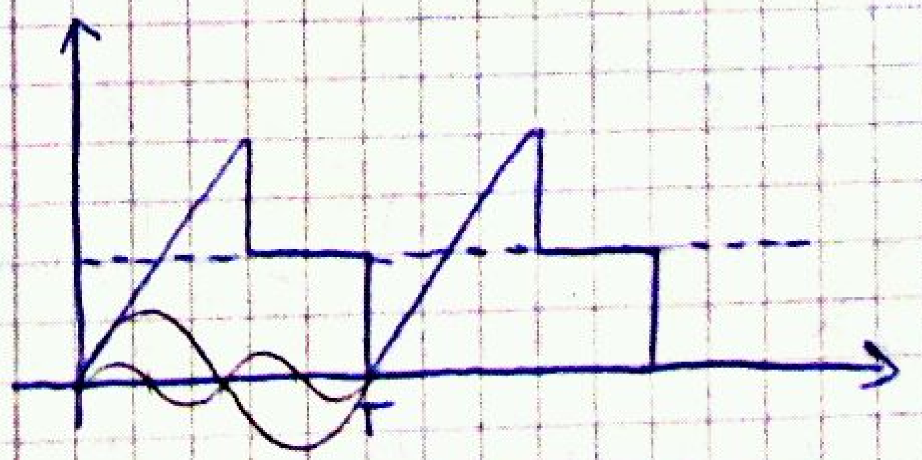
$$U_{eff} = \sqrt{\frac{1}{T}\int_0^T (u(t))^2\, dt}$$

$$U_{eff_2} = \sqrt{U_0^2 + \frac{U_1^2}{2} + \frac{U_2^2}{2} + \frac{U_i^2}{2}}$$

$$u(t) = 10 + 20\cos(2t + 40^\circ) + 30\sin(2t + 15^\circ)$$

$$U_0 = \frac{1}{T}\int_0^T u(t)\, dt \qquad u(t) = u(t + T)$$

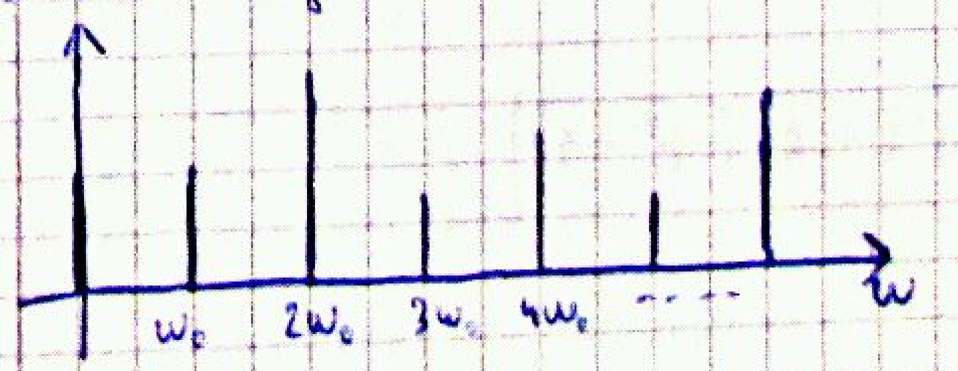
periodicität




$$T = \frac{2\pi}{\omega}$$

$$\omega_0 = \frac{2\pi}{T}$$

Spektrumdiagramm:



Fourier - sorfejtés

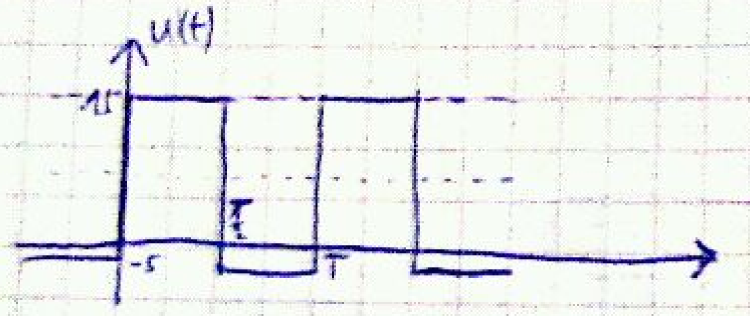
$$u(t) = U_0 + \sum_{i=1}^{\infty} \left( U_i^A \cos i\omega_0 t + U_i^B \sin i\omega_0 t \right)$$
$$U_i^C \cdot \cos(i\omega_0 t + \varphi_i)$$

$$U_i^A = \frac{2}{T} \int_0^T u(t) \cdot \cos i\omega_0 t \, dt \qquad U_i^A + jU_i^B$$

$$U_i^B = \frac{2}{T} \int_0^T u(t) \cdot \sin i\omega_0 t \, dt$$

$$U_i^C = \frac{2}{T} \int_0^T u(t) \cdot e^{j i \omega_0 t} \, dt = A_i e^{j\alpha_i}$$

F/




$$u(t) = \left[ \varepsilon(t) - \varepsilon(t - \tfrac{T}{2}) \right] 15 + \left[ \varepsilon(t - \tfrac{T}{2}) - \varepsilon(t - T) \right] (-5)$$

$$U_0 = \frac{1}{T} \left( \int_0^{T/2} 15 \, dt + \int_{T/2}^{T} -5 \, dt \right) = \frac{1}{T} \left( 15 \frac{T}{2} - 5 \frac{T}{2} \right) = 5$$

$$U_i^A = \frac{2}{T} \left( \int_0^{T/2} 15 \cos i\omega_0 t \, dt + \int_{T/2}^{T} -5 \cos i\omega_0 t \, dt \right) =$$

$$= \frac{2}{T i\omega_0} \left[ \left[ +15 \cdot \sin i\omega_0 t \right]_0^{T/2} - \left[ 5 \sin i\omega_0 t \right]_{T/2}^{T} \right] = 0$$

$$\sin i \frac{2\pi}{T} \cdot \frac{T}{2}$$
$$0 \qquad - \qquad 0$$

$$U_i^B = \frac{2}{T} \left( \int_0^{T/2} 15 \sin i\omega_0 t \, dt + \int_{T/2}^{T} -5 \sin i\omega_0 t \, dt \right) = \frac{2}{T i\omega_0} \left[ 15 \cos i\omega_0 t \right]_{T/2}^{0}$$

$$+ -5 \left[ \cos i\omega_0 t \right]_{T}^{T/2} = \frac{2}{T\omega_0 i} \left[ 15 \left( 1 - (-1)^i \right) - 5 \left( (-1)^i - 1 \right) \right] =$$

$$= \frac{T 2}{T i 2\pi} = \frac{1}{i\pi} \left| 15 (1 - (-1)^i) - 5 ((-1)^i - 1) \right|$$

| $i$ | $U_i^B$ |
|---|---|
| 0 | x |
| 1 | $\frac{40}{\pi}$ |
| 2 | 0 |
| 3 | $\frac{40}{3\pi}$ |
| 4 | 0 |
| 5 | $\frac{40}{5\pi}$ |

$$u(t) = 5 + \frac{40}{\pi} \sin \omega_0 t + \frac{40}{3\pi} \sin 3\omega_0 t + \frac{40}{5\pi} \sin 5\omega_0 t$$

$$U_{eff} = \sqrt{\frac{1}{T} \int_0^T (u(t))^2 \, dt} \qquad = \sqrt{\frac{1}{T} \left[ \int_0^{T/2} 225 \, dt + \int_{T/2}^{T} 25 \, dt \right]} =$$

$$= 225 \frac{T}{2} + 25 \frac{T}{2} = 250 \frac{T}{2}$$

$$U_{eff} = \sqrt{125} = 11,18$$

$$U_{eff_K} = \sqrt{5^2 + \frac{\left(\frac{40}{\pi}\right)^2}{2} + \frac{\left(\frac{40}{3\pi}\right)^2}{2} + \frac{\left(\frac{40}{5\pi}\right)^2}{2}}$$

## Fourier - transzformáció:

jele: $\mathcal{F}$

$$\mathcal{F}\{x(t)\} = X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \, e^{j\omega t} \, d\omega = \mathcal{F}^{-1}(X(j\omega))$$

$$\int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t} \, dt = 1$$

$$u(t) \longrightarrow U(j\omega)$$

$$Y(j\omega) = U(j\omega) \cdot H(j\omega) = H(j\omega)$$

## Fourier-sorfejtés képletei:

$$U_0 = \frac{1}{T}\int_0^T u(t)\,dt$$

$$\begin{matrix} U_i^A \\ U_i^B \end{matrix} = \frac{2}{T}\int_0^T u(t) \begin{matrix}\cos \\ \sin\end{matrix}\, i\omega t\, dt$$

$$U_i^C = \frac{2}{T}\int_0^T u(t)\, e^{j i \omega t}\, dt \qquad \text{komplex}$$

periódikus jeleknél !!!

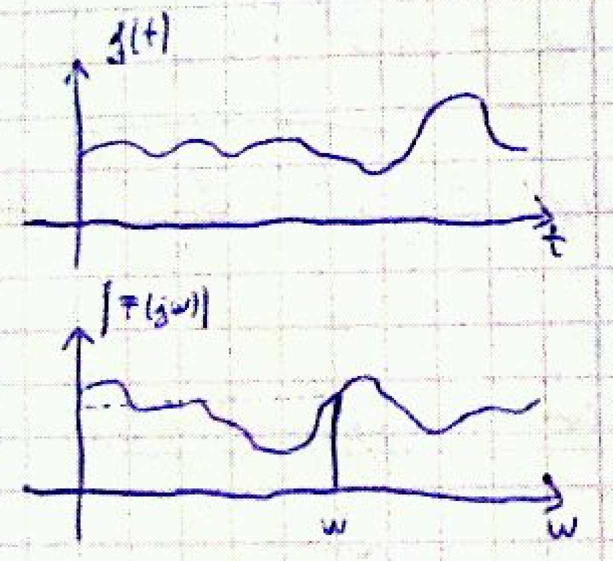
## Fourier-transzformáció

- idő és frekvenciatartomány közti kapcsolat

$$t \xrightarrow{\mathcal{F}} j\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$




nem periódikus jeleknél !!!

I. $C_1 \mathcal{F}\{x(t)\} + C_2 \mathcal{F}\{y(t)\} = \mathcal{F}\{C_1 x(t) + C_2 y(t)\}$

II. csillapítási tétel:

$$\mathcal{F}\{x(t)\, e^{-\alpha t}\}_{\alpha>0} = X(j\omega + \alpha)$$

III. modulációs tétel:

$$\mathcal{F}\{x(t)\, e^{j\omega_0 t}\} = X(j(\omega - \omega_0))$$

IV. konvolúciós tétel:

$$\mathcal{F}\{x(t) * y(t)\} = \mathcal{F}\{x(t)\} \cdot \mathcal{F}\{y(t)\}$$

$\mathcal{F}\{\delta(t)\} = 1$ → gerjesztés → $\delta(t)$

$u(t) = \delta(t) \Rightarrow y(t) = h(t)$ → impulzusválasz

$Y(j\omega) = \mathcal{F}\{\delta(t)\} \cdot H(j\omega) = H(j\omega)$

válasz

(*) $\mathcal{F}\{\delta(t-T)\} = \int_{-\infty}^{\infty} \underset{1}{\delta(t-T)}\, \underset{e^{-j\omega T}}{e^{-j\omega t}}\, \underset{dt}{dt} = e^{-j\omega T}$

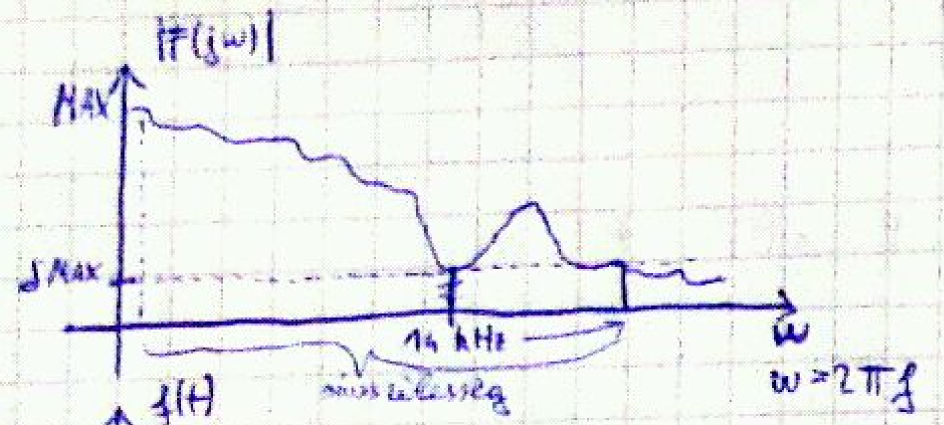
V. eltolás

(*)

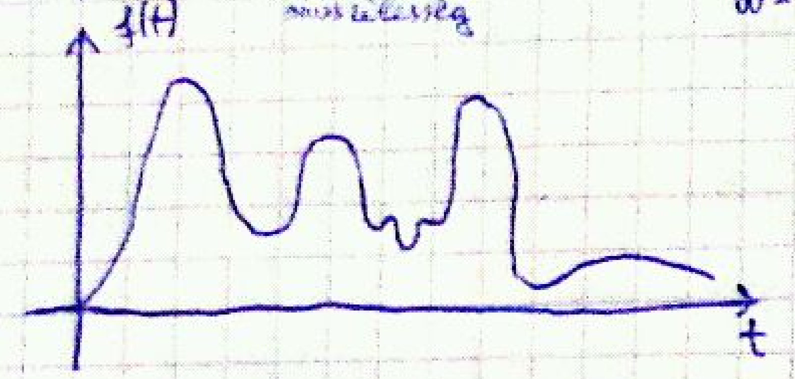
$$\mathcal{F}\{x(t-T)\} = X(j\omega) \cdot e^{-j\omega T}$$

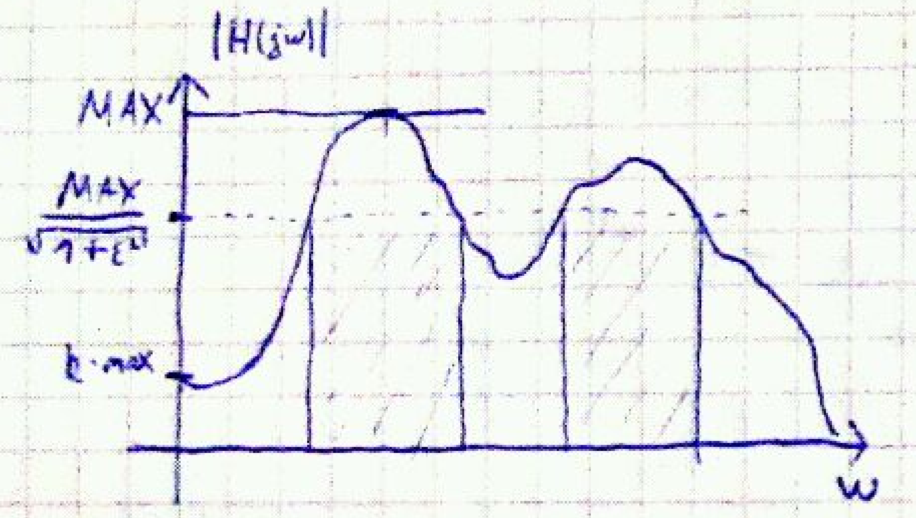
VI.

$$\mathcal{F}\left\{\frac{dx(t)}{dt}\right\} = j\omega \cdot X(j\omega)$$




sávszélesség = $\Delta\omega_j$



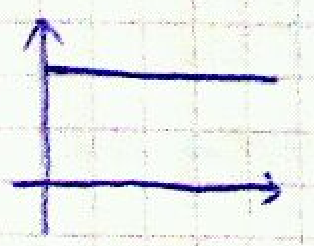




$MAX - \frac{MAX}{\sqrt{1+\varepsilon^2}}$ : vezető

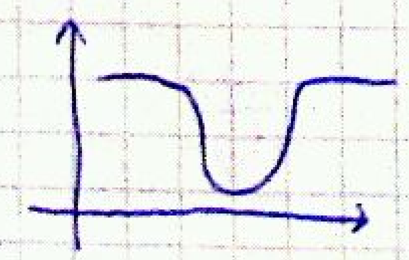
$\frac{MAX}{\sqrt{1+\varepsilon^2}} - k \cdot MAX$ : átmeneti

$k \cdot MAX - 0$ : záró

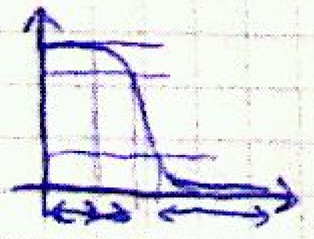
} tartomány



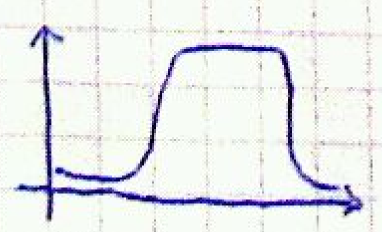
mindent áteresztő



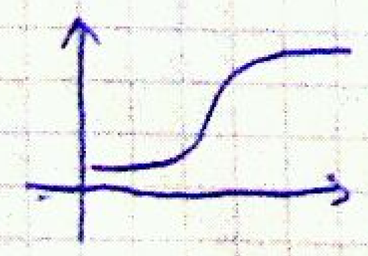
sávzáró



aluláteresztő



sáváteresztő



felüláteresztő

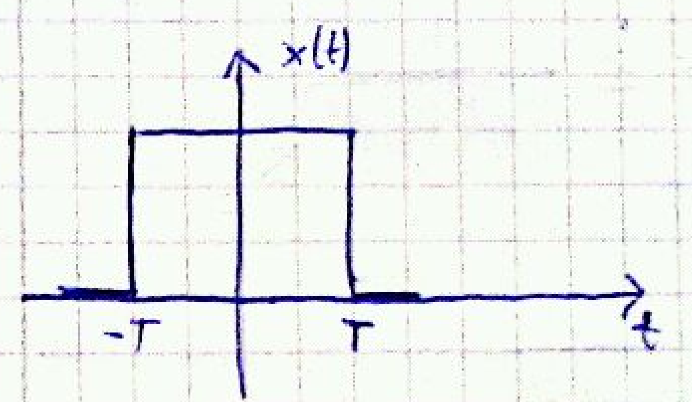
$\mathcal{F}\{\varepsilon(t)\} = \pi\,\delta(\omega) + \frac{1}{j\omega}$

$E = \frac{1}{\pi}\int_0^\infty |x(t)|^2\,dt$

$x(t) \rightarrow X(j\omega)$

$x(t)e^{-\alpha t} \rightarrow X(j\omega+\alpha)$

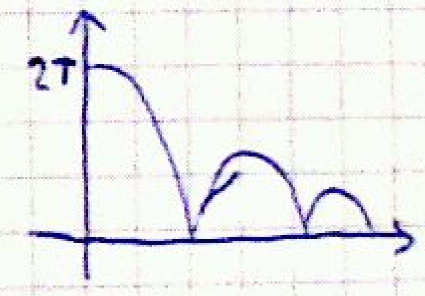
$$\int_{-\infty}^{\infty} \varepsilon(t)\,e^{-\alpha t}\cdot e^{-j\omega t}\,dt = \int_0^\infty e^{-\alpha t}\cdot e^{-j\omega t}\,dt = \left[\frac{e^{-(j\omega+\alpha)t}}{-(j\omega+\alpha)}\right]_0^\infty = \frac{1}{j\omega+\alpha}$$

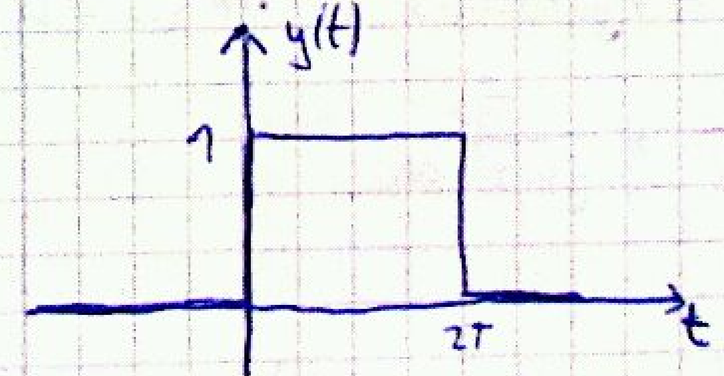



$$\int_{-T}^{T} e^{-j\omega t}\,dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T}^{T} = \frac{e^{j\omega T} - e^{-j\omega T}}{j\omega} =$$

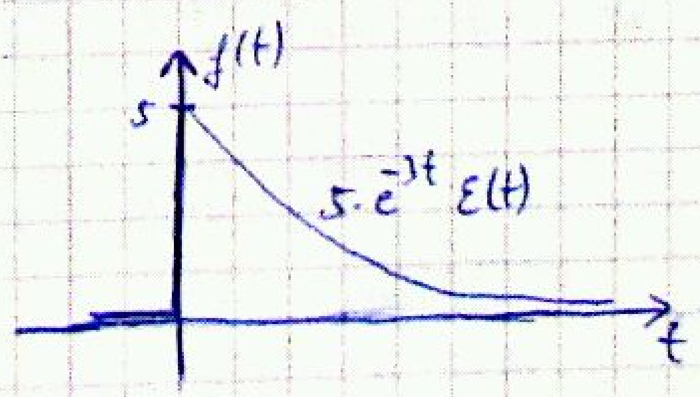
$$= \frac{2\sin\omega T}{\omega} = 2T\cdot\frac{\sin\omega T}{\omega T}$$

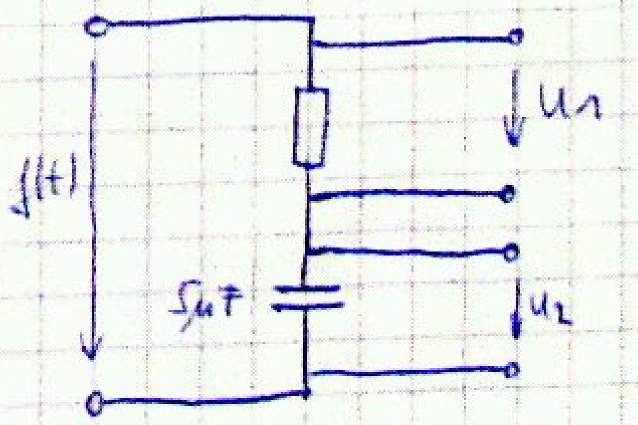
$$\sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$




$$\int_0^{2T} e^{-j\omega t}\,dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^{2T} = \frac{1-e^{-2j\omega T}}{j\omega} = X(j\omega)\cdot e^{-j\omega T}$$




$\delta = 0{,}05$
$\varepsilon = 0{,}1$

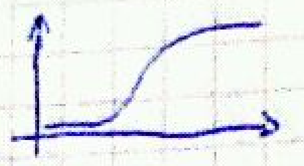
$$F(j\omega) = \mathcal{F}\{5\cdot e^{-3t}\,\varepsilon(t)\} = \frac{5}{j\omega+3}$$

$0 - 60\ \frac{rad}{s}$

$$\frac{5}{\sqrt{3+\omega_1^2}} = \frac{5}{3}\cdot 0{,}05 \qquad T_{max}\Big|_{\omega=0} = \frac{5}{3}$$

$$\omega_1 = \sqrt{\left(\frac{3}{0{,}05}\right)^2 - 9} = 59{,}92$$

$$H_1(j\omega) = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega RC}{1+j\omega RC}$$



$$H_2(j\omega) = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega RC}$$


$$H_{max}\Big|_{\omega=0} = 1 \qquad \frac{1}{\sqrt{1^2+\omega_1^2R^2C^2}} \ge \frac{1}{\sqrt{1^2+0{,}1^2}}$$

($H_{max}$, $\varepsilon$)

$$\frac{1}{1+60^2R^2C^2} \ge \frac{1}{1{,}01}$$

$$\frac{0{,}01}{300^2} \ge R^2$$

$$333\,\Omega \ge R$$

$$T(j\omega) = \frac{5}{j\omega+3} \qquad H(j\omega) = \frac{1}{1+j\omega\, 0{,}0002\cdot 5} = \frac{1}{1+j\omega\cdot 0{,}001}$$

$$U_2(j\omega) = \frac{5}{j\omega+3}\cdot\frac{1}{1+j\omega\cdot 0{,}001} = \frac{\frac{5}{997}}{j\omega+3} + \frac{-\frac{5}{997}}{j\omega+1000}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} U_2(j\omega)e^{j\omega t}\,d\omega \qquad \frac{5}{997}\cdot\left(e^{-3t}-e^{-1000t}\right)\varepsilon(t)$$

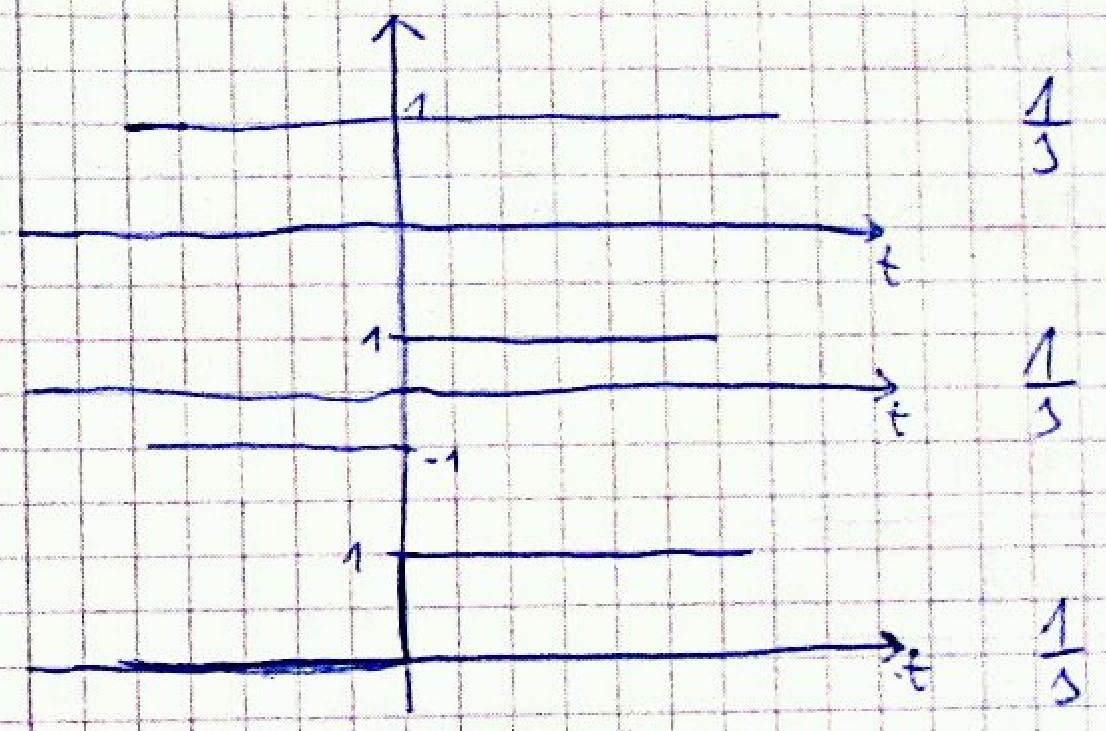
## Laplace-transzformáció:

idő és komplex frekvenciatartomány közti kapcsolat

$$X(t) \longrightarrow X(s)$$

$$s = \delta + j\omega$$

$$\mathcal{L}\{x(t)\} = X(s) = \int_{-0}^{\infty} x(t)\,e^{-st}\,dt$$




$$x(t) = \mathcal{L}^{-1}\{X(s)\} = \frac{1}{2\pi j}\int_{\delta-j\infty}^{\delta+j\infty} X(s)\,e^{st}\,ds$$

1, $C_1\mathcal{L}\{x_1(t)\}\ C_2\mathcal{L}\{x_2(t)\} = \mathcal{L}\{C_1x_1(t)+C_2x_2(t)\}$

2, $\mathcal{L}\{x(t)\,e^{-\alpha t}\varepsilon(t)\} = X(s+\alpha)$ ($\alpha>0$)

$$Y(s) = \int_{-0}^{\infty} x(t)e^{-\alpha t}\cdot e^{-st}\,dt = \int_{-0}^{\infty} x(t)\cdot e^{-(s+\alpha)t}\,dt = X(s+\alpha)$$

2, $\mathcal{L}\{x(t)\,e^{j\omega_0 t}\varepsilon(t)\} = X(s-j\omega_0)$

$$\mathcal{L}\{\varepsilon(t)\cos\omega_0 t\} = \int_{-0}^{\infty}\cos\omega_0 t\, e^{-st}\,dt = \frac{1}{2}\int_{-0}^{\infty}\left(e^{j\omega_0 t}+e^{-j\omega_0 t}\right)e^{-st}\,dt =$$

$$= \frac{1}{2}\left(\frac{1}{s-j\omega_0}+\frac{1}{s+j\omega_0}\right) = \frac{s}{s^2+\omega_0^2}$$

$$\mathcal{L}\{\varepsilon(t)\sin\omega_0 t\} = \frac{\omega_0}{s^2+\omega_0^2}$$

IV. $\mathcal{L}\{x(t) * y(t)\} = \mathcal{L}\{x(t)\}\cdot\mathcal{L}\{y(t)\} = X(s)\cdot Y(s)$

V. $\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = s\cdot X(s) - X(-0)$

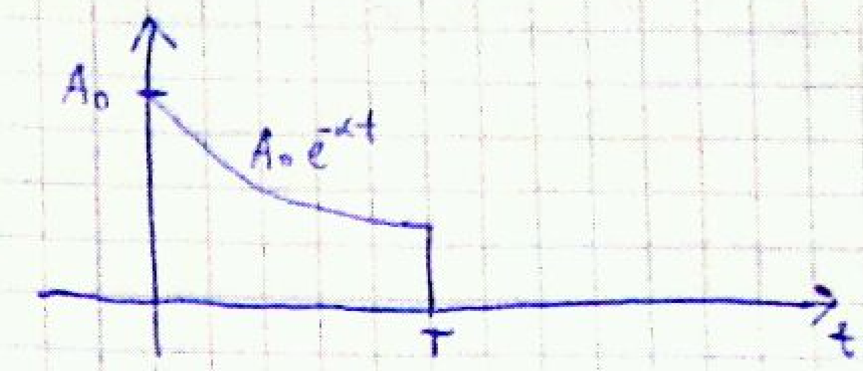
VI. $\mathcal{L}\left\{\int_0^t x(t)\,dt\right\} = \frac{X(s)}{s}$

VII. $x(t\to\infty) = \lim_{s\to 0} s\,X(s)$

VIII. $x(t=+0) = \lim_{s\to\infty} s\,X(s)$

IX. $\mathcal{L}\{\varepsilon(t-T)\,x(t-T)\} = X(s)\,e^{-sT}$

| $f(t)$ | $F(s)$ |
|---|---|
| $\delta(t)$ | $1$ |
| $\varepsilon(t)$ | $\frac{1}{s}$ |
| $t\cdot\varepsilon(t)$ | $\frac{1}{s^2}$ |
| $t^n\,\varepsilon(t)$ | $\cdot\frac{n!}{s^{n+1}}$ |
| $\varepsilon(t)e^{-\alpha t}$ $\alpha>0$ | $\frac{1}{s+\alpha}$ |
| $\varepsilon(t)\cos\omega_0 t$ | $\frac{s}{s^2+\omega^2}$ |
| $\varepsilon(t)\sin\omega_0 t$ | $\frac{\omega_0}{s^2+\omega_0^2}$ |
| $\varepsilon(t-T)x(t-T)$ | $X(s)\cdot e^{-sT}$ |



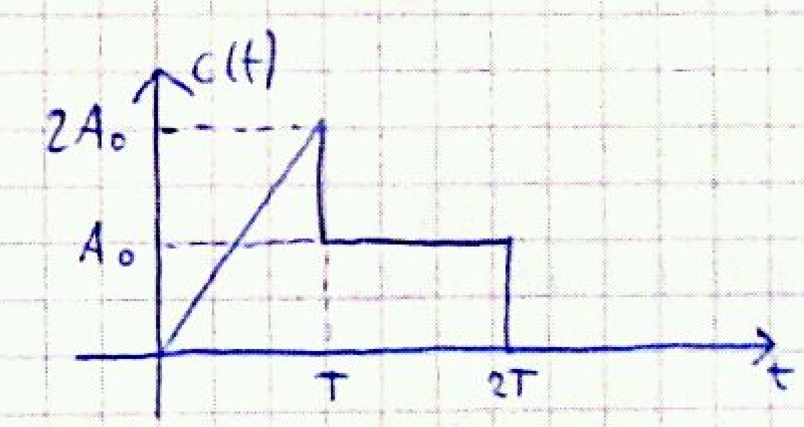

$$[\varepsilon(t)-\varepsilon(t-T)]\,A_0\,e^{-\alpha t} = \varepsilon(t)\,A_0\,e^{-\alpha t} - \varepsilon(t-T)\,A_0 e^{-\alpha(t+T-T)}$$

$$\varepsilon(t)\,A_0 e^{-\alpha t} - \varepsilon(t-T)\,A_0\,e^{-\alpha T}\cdot e^{-\alpha(t-T)}$$

$$\frac{A_0}{s+\alpha}\left(1-e^{-\alpha T}\cdot e^{-sT}\right) = \frac{A_0}{s+\alpha} - \frac{A_0\,e^{-\alpha T}}{s+\alpha}\,e^{-sT}$$

$$b(t) = \varepsilon(t)\cdot 3e^{-4t} + \varepsilon(t)\cdot 5\cdot e^{-6t}$$

$$B(s) = \frac{3}{s+4} + \frac{5}{s+6} = \frac{8s+38}{s^2+10s+24}$$



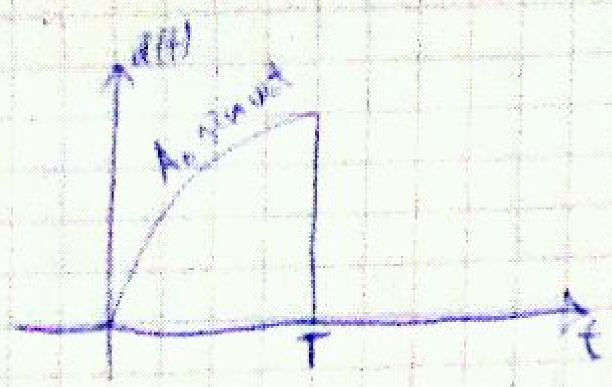

$$c(t) = [\varepsilon(t)-\varepsilon(t-T)]\,\frac{t\cdot 2A_0}{T} + [\varepsilon(t-T)-\varepsilon(t-2T)]\,A_0$$

$$c(t) = \frac{\varepsilon(t)\cdot t\cdot 2A_0}{T} - \varepsilon(t-T)\,\frac{(t-T+T)\,2A_0}{T} + \varepsilon(t-T)A_0 - \varepsilon(t-2T)A_0$$

$$\frac{\varepsilon(t)\cdot t\;2A_0}{T} - \varepsilon(t-T)\,\frac{(t-T)\,2A_0}{T} - \varepsilon(t-T)\,2A_0 + \varepsilon(t-T)A_0 -$$

$$-\,\varepsilon(t-2T)\,A_0$$

$$C(s) = \frac{2A_0}{T}\cdot\frac{1}{s^2}\left(1-e^{-sT}\right) - \frac{A_0}{s}e^{-sT} - \frac{A_0}{s}e^{-2sT}$$




$$d(t)=[\varepsilon(t)-\varepsilon(t-T)]A_0 \sin w_0 t = \varepsilon(t) A_0 \sin w_0 t - \varepsilon(t-T) A_0 \sin w_0 (t+T-T)$$

$$= \varepsilon(t) A_0 \sin w_0 t - \varepsilon(t-T) A_0 \sin w_0 (t-T) \cdot \cos w_0 T - \varepsilon(t-T) A_0 \cos w_0 (t-T) \sin w_0 T$$

$$D(s) = \frac{A_0 w_0}{s^2+w_0^2} - \frac{A_0 \cos w_0 T \cdot w_0}{s^2+w_0^2} e^{-sT} - \frac{A_0 \sin w_0 T \, s}{s^2+w_0^2} e^{-sT}$$

$$\varepsilon(t)\, e^{-5t} \longrightarrow \frac{1}{s+5}$$

$$\lim_{s=0} \frac{s}{s+5} = 0$$

$$\lim_{s\to\infty} \frac{s}{s+5} = 1$$

Inverz Laplace:

$$F(s) = \frac{3s+4}{s^2+5s+6}$$

$$(3s+4) : (s^2+5s+6) = \frac{3}{s} - \frac{11}{s^2} + \frac{37}{s^3} \mp \dots$$

$$-3s+15+\frac{18}{s}$$
$$0-11-\frac{18}{s}$$
$$- -11-\frac{55}{s}-\frac{66}{s^2}$$
$$0+\frac{37}{s}+\frac{66}{s^2}$$

$$f(t) = 3\varepsilon(t) - 11 t \cdot \varepsilon(t) \pm \dots$$

$$T(s) = \frac{3s+4}{s^2+5s+6} = \frac{3s+4}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$$

$$3s+4 = A(s+3)+B(s+2)$$

$$3s = As + Bs$$

$$4 = 3A + 2B \quad \Big\} \quad A=-2, \; B=5$$

$$f(t) = (-2e^{-2t} + 5 \cdot e^{-3t})\,\varepsilon(t)$$

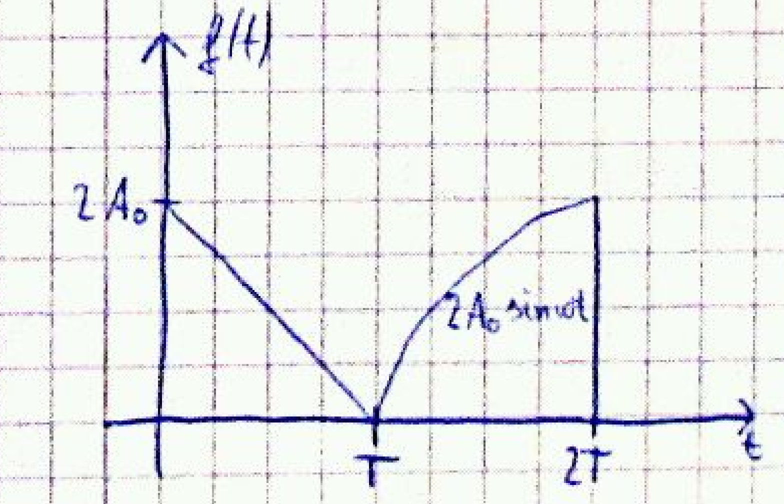
Letakarásos módszer;

$$\frac{3s+4}{(s+2)(s+3)} = \frac{\frac{-2}{1}=-2}{s+2} + \frac{\frac{-5}{-1}=5}{s+3}$$

/ először −2, utána −3 } -t helyettesítünk

$$F(s) = \frac{5s+6}{(s+3)(s+4)^2} = \frac{\frac{-9}{1}}{s+3} + \frac{9}{s+4} + \frac{\frac{-14}{-1}}{(s+4)^2}$$

$$f(t) = [-9e^{-3t} + 9e^{-4t} + 14\, t e^{-4t}]\,\varepsilon(t)$$




$$f(t) = [\varepsilon(t) - \varepsilon(t-T)]\left(\frac{-t\, 2A_0}{T} + 2A_0\right) + [\varepsilon(t-T) - \varepsilon(t-2T)]\, 2A_0 \sin w(t-T)$$

$$f(t) = \varepsilon(t) \frac{-2A_0}{T} t + \varepsilon(t) 2A_0 + \varepsilon(t-T) \frac{2A_0 t}{T} - \varepsilon(t-T) 2A_0 + \varepsilon(t-T) 2A_0 \sin w(t-T)$$
$$- \varepsilon(t-2T) 2A_0 \sin w(t-T)$$

$$F(t) = \frac{-2A_0}{T} \frac{1}{s^2} + 2A_0 \frac{1}{s} + \frac{2A_0}{T} \frac{e^{-sT}}{s^2} + \frac{2A_0}{s} e^{-sT} - \frac{2A_0}{s} e^{-sT} +$$
$$+ \frac{2A_0 w}{s^2+w^2} e^{-sT} - \frac{2A_0 \cos(wT)}{s^2+w^2} e^{-2sT} - \frac{2A_0 \sin(wT) s}{s^2+w^2} e^{-s2T}$$

$$F(s) = \frac{s+2}{s^2+4s+13} = \frac{s+2}{(s+2+3j)(s+2-3j)} = \frac{\frac{-3j}{-6j} = \frac{1}{2}}{s+2+3j} + \frac{\frac{1}{2}}{s+2-3j}$$

$$(s+2) : (s^2+4s+13) = \frac{1}{s} - \frac{2}{s^2}$$

$$-s+4+\frac{13}{s}$$

$$-2 - \frac{13}{s}$$

$$f(t) = \varepsilon(t) - 2t \cdot \varepsilon(t) +$$

$$\left(\frac{1}{2} e^{(-2-3j)t} + \frac{1}{2} e^{(-2+3j)t}\right) \varepsilon(t) =$$

$$= e^{-2t} \cdot \frac{e^{-3jt} + e^{3jt}}{2} \varepsilon(t) = e^{-2t} \cos 3t \, \varepsilon(t)$$

$$G(s) = \frac{3s^2+4s-5}{s^2+6s+8} = 3 + \frac{-14s-29}{s^2+6s+8} = 3 + \frac{-14s-29}{(s+2)(s+4)} =$$

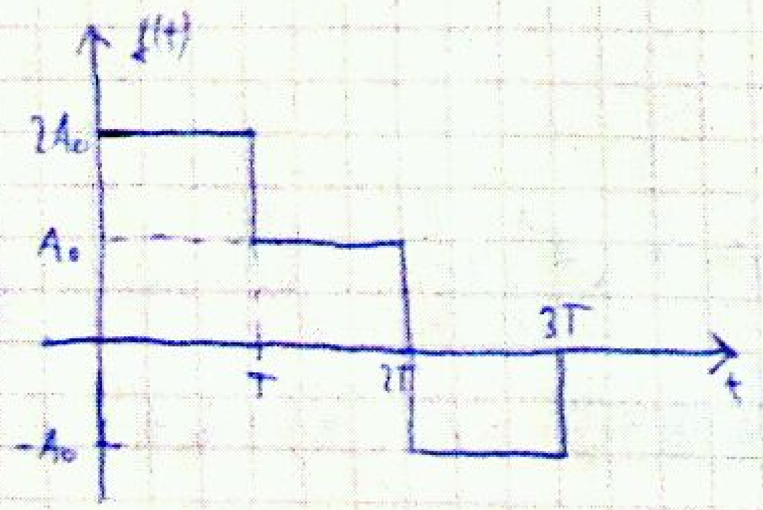
$$(3s^2+4s-5) : (s^2+6s+8) = 3$$

$$-3s^2+18s+24$$

$$-14s-29$$

$$= 3 + \frac{-\frac{1}{2}}{(s+2)} + \frac{\frac{27}{-2}}{(s+4)}$$

$$g(t) = 3\delta(t) + \left[-\frac{1}{2} e^{-2t} - \frac{27}{2} e^{-4t}\right] \varepsilon(t)$$




$$f(t) = [\varepsilon(t) - \varepsilon(t-T)] 2A_0 + [\varepsilon(t-T) - \varepsilon(t-2T)] A_0 + [\varepsilon(t-2T) - \varepsilon(t-3T)](-A_0)$$

$$\varepsilon(t) 2A_0 - \varepsilon(t-T) A_0 - \varepsilon(t-2T) 2A_0 + \varepsilon(t-3T) A_0$$

$$F(s) = \frac{2A_0}{s} - \frac{A_0}{s} e^{-sT} - \frac{2A_0}{s} e^{-2sT} + \frac{A_0}{s} e^{-3sT} = \frac{A_0}{s}\left(2 - e^{-sT} - 2e^{-2sT} + e^{-3sT}\right)$$

$$V(s) = \frac{A_0(s+3)}{s(s+4)}\left(2 - e^{-sT} - 2e^{-2sT} + e^{-3sT}\right)$$

$$V_0(s) = \frac{A_0(s+3)}{s(s+4)} = \frac{\frac{3A_0}{4}}{s} + \frac{\frac{A_0}{4}}{s+4}$$

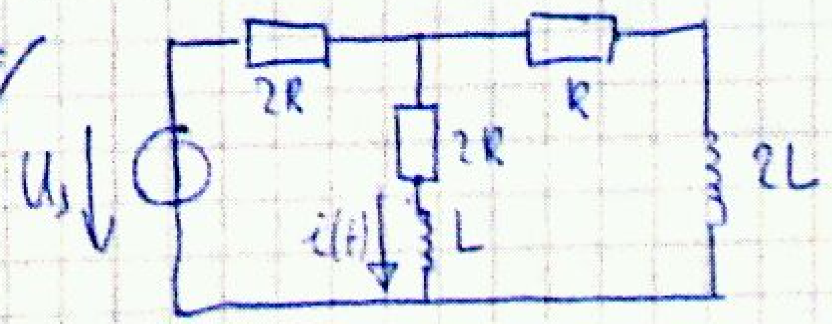
$$y_0(t) = \left(\frac{3A_0}{4} + \frac{1}{4} A_0 e^{-4t}\right) \varepsilon(t)$$

$$y(t) = 2y_0(t) - y_0(t-T) - 2y_0(t-2T) + y_0(t-3T)$$

$$y(t) = \left(\frac{3A_0}{2} + \frac{1}{2} A_0 e^{-4t}\right)\varepsilon(t) - \left(\frac{3A_0}{4} + \frac{1}{4} A_0 e^{-4(t-T)}\right)\varepsilon(t-T) - \left(\frac{3A_0}{2} + \frac{1}{2} A_0 e^{-4(t-2T)}\right)\varepsilon(t-2T) + \left(\frac{3A_0}{4} + \frac{1}{4} A_0 e^{-4(t-3T)}\right)\varepsilon(t-3T)$$

| sima | | frekvencia | | komplex |
|---|---|---|---|---|
| $R$ | $\longrightarrow$ | $R$ | $\longrightarrow$ | $R$ |
| $C$ | $\longrightarrow$ | $\frac{1}{j\omega C}$ | $\longrightarrow$ | $\frac{1}{sC}$ |
| $L$ | $\longrightarrow$ | $j\omega L$ | $\longrightarrow$ | $sL$ |

F/

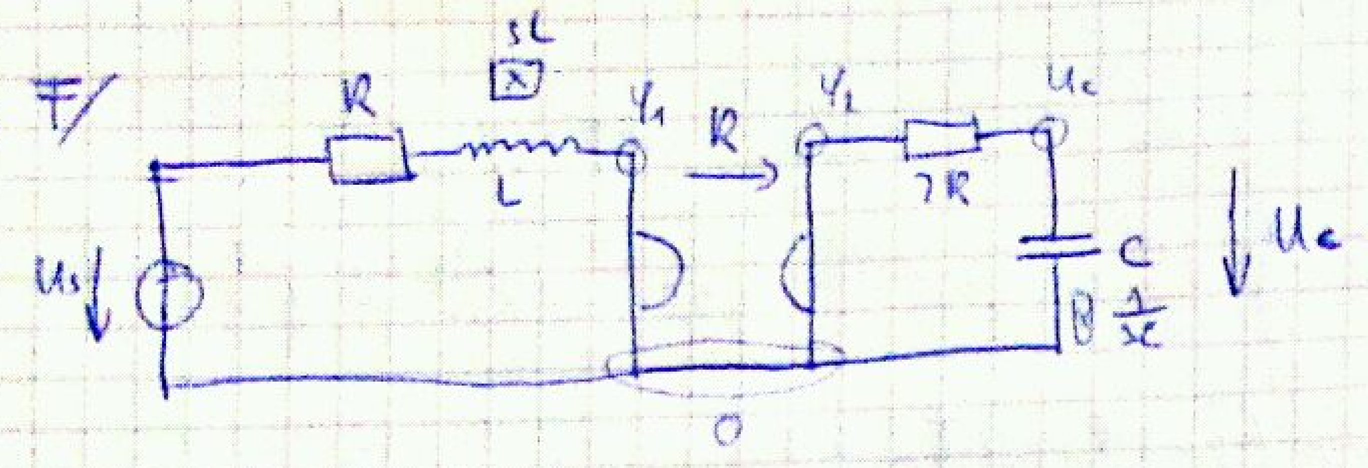



$$\frac{(2R+sL)\times(R+2sL)}{(2R+sL)\times(R+2sL)+2R} \cdot \frac{1}{2R+sL} = \frac{\frac{(2R+sL)(R+2sL)}{3R+3sL}}{\frac{(2R+sL)(R+2sL)}{3R+3sL}+2R} \cdot \frac{1}{2R+sL} =$$

$$= \frac{R+2sL}{2R^2+2s^2L^2+5RsL+6R^2+6RsL} = \frac{R+2sL}{2s^2L^2+11RsL+8R^2} =$$

$$= \frac{s\frac{1}{L} + \frac{R}{2L^2}}{s^2 + \frac{11}{2}\frac{R}{L}s + 4\frac{R^2}{L^2}}$$

– F/



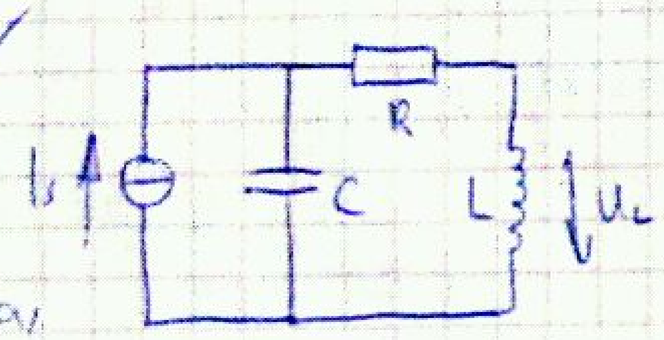

$$Y_2 = R \cdot \frac{U_s - Y_1}{R + sL}$$
$$Y_1 = -R \cdot \frac{U_c - Y_2}{2R}$$
} gírátor

$$\frac{U_c - Y_2}{2R} + \frac{U_c}{\frac{1}{sC}} = 0$$

$U_c = U_s$ ………… (H(s)) ↓ átviteli fv.

(*)

– F/




a/

$$H(s) = \frac{\frac{1}{sC}}{\frac{1}{sC} + R + sL} \cdot sL = \frac{\frac{sL}{sC}}{\frac{1}{sC} + R + sL} = \frac{sL}{1 + RsC + s^2LC} = \frac{s\frac{1}{C}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

(b)

$$sX = Ax + Bu$$
$$y = C^T x + Du$$
$$H(s) = C^T(s\mathbb{1} - A)^{-1}B + D$$

(c)

– (*)

$$Y_2 = R \cdot i_L$$
$$Y_1 = -\frac{U_c - Y_2}{2R} \cdot R$$
$$\frac{U_c - Y_2}{2R} + C\dot{U}_c = 0$$
$$Y_1 + L\dot{i}_L + R \cdot i_L = U_s$$

$$\dot{U}_c = -\frac{1}{2RC} U_c + \frac{1}{2C} i_L$$
$$\dot{i}_L = \frac{1}{2L} U_c - \frac{3R}{2L} i_L + \frac{U_s}{L}$$
$$y = U_c$$

$$sU_c = -\frac{1}{2RC} U_c + \frac{1}{2C} i_L$$
$$s i_L = \frac{1}{2L} U_c - \frac{3R}{2L} i_L + \frac{U_s}{L}$$
$$i_L = \frac{\frac{1}{2L} U_c + \frac{U_s}{L}}{s + \frac{3R}{2L}}$$

$$sU_c = -\frac{1}{2RC} U_c + \frac{1}{2C} \cdot \frac{\frac{1}{2L} U_c + \frac{U_s}{L}}{s + \frac{3R}{2L}}$$

$$\left(s^2 + s\frac{3R}{2L}\right) U_c + \left(s + \frac{3R}{2L}\right)\frac{1}{2RC} U_c - \frac{1}{4LC} U_c = \frac{U_s}{2LC}$$

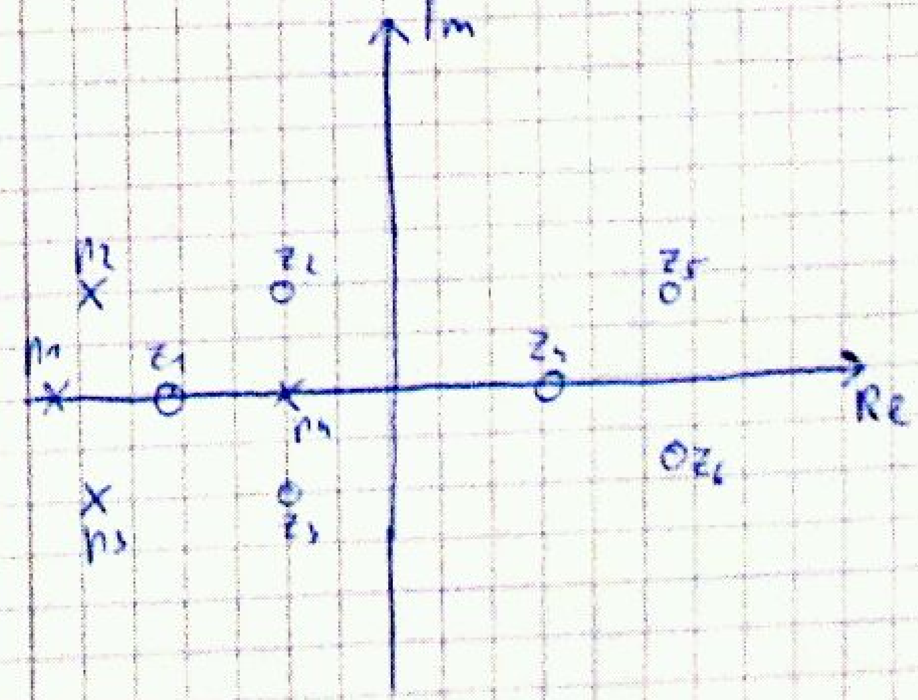
$$U_c = U_s \underbrace{\frac{\frac{1}{2LC}}{s^2 + s\left(\frac{3R}{2L} + \frac{1}{2RC}\right) + \frac{5}{4LC}}}_{H(s)}$$

–

H(s) ⟶ kauzális
H(jω) ⟶ GV stabilis
} jω ⟷ s

– $\mathcal{L}\{h(t)\} = H(s)$

– ábrázolás: pólus-zérus




komplex zérus ⟹ konjugált is

$$H(s) = \frac{(s - z_1)(s - z_2)(s - z_3) \ldots}{(s - p_1)(s - p_2)(s - p_3) \ldots}$$
o zérus
x pólus

pólus csak –Re{ }: negatív

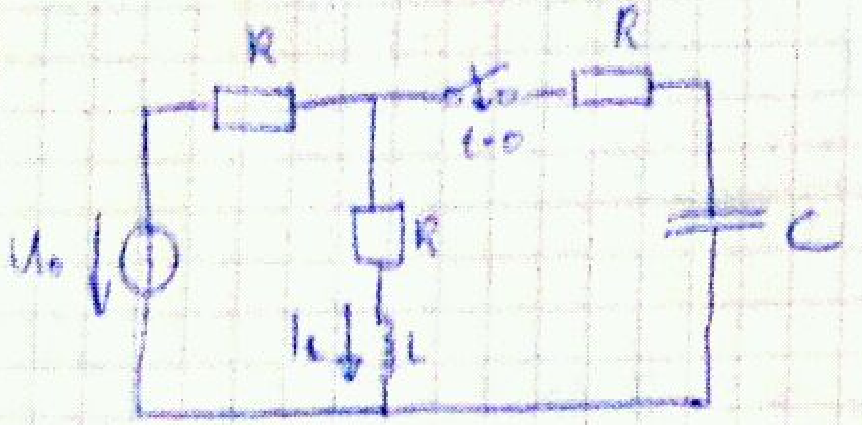
$u(t) \rightarrow U(s)$

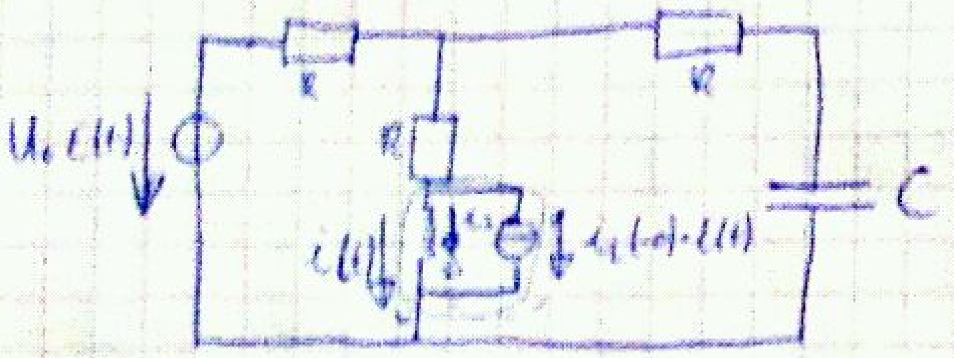
$H(s)$

$Y(s) = U(s) \cdot H(s)$

$Y(s) \rightarrow y(t)$

V/




$i_L(-0) = \frac{U_0}{2R}$

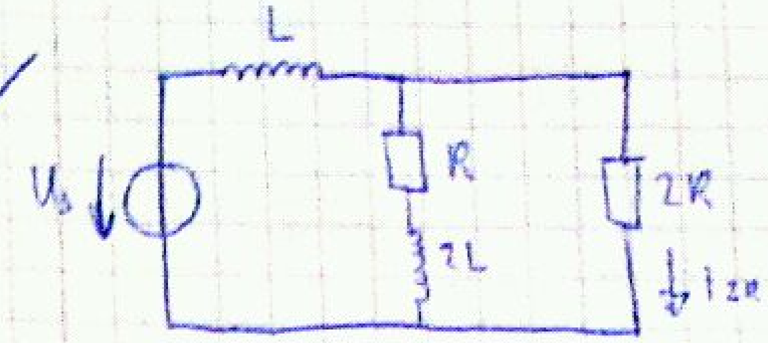
$U(t) = U_0 \, \varepsilon(t) \qquad \mathcal{L}(u(t)) = \frac{U_0}{s}$

$$i(t) = \frac{U_0}{s} \cdot \frac{(R + \frac{1}{sC}) \otimes (R + sL)}{(R + \frac{1}{sC}) \otimes (R+sL) + R} \cdot \frac{1}{R+sL} - \frac{U_0}{2Rs} \cdot \frac{R + R \otimes (R + \frac{1}{sC})}{R + R \otimes (R + \frac{1}{sC}) + sL} + \frac{U_0}{2Rs}$$

egyik, másik, össes

$i_s$

F/

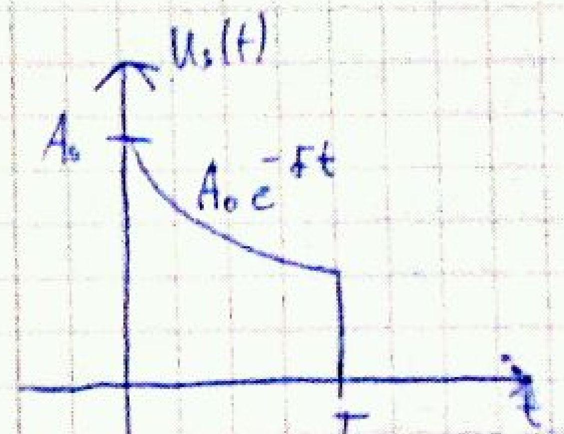



$$H(s) = \frac{(R + s2L) \otimes 2R}{(R + s2L) \otimes 2R + sL} \cdot \frac{1}{2R} = \frac{\frac{(R + 2sL) \cdot 2R}{3R + 2sL}}{\frac{R + 2sL \cdot 2R}{3R + 2sL} + sL} \cdot \frac{1}{2R} =$$

$$= \frac{R + 2sL}{2R^2 + 4RsL + 3RsL + 2s^2L^2} = \frac{s\frac{1}{L} + \frac{R}{2L^2}}{s^2 + s\frac{7R}{2L} + \frac{R^2}{L^2}} = \frac{s+\delta}{(s+\alpha)(s+\beta)} = \frac{A}{s+\alpha} + \frac{B}{s+\beta}$$

$h(t) = (A e^{-\alpha t} + B \cdot e^{-\beta t}) \, \varepsilon(t)$

F/




$[\varepsilon(t) - \varepsilon(t-T)] A_0 e^{-\delta t} = A_0 e^{-\delta t} \varepsilon(t) - A_0 e^{-\delta(t-T)} \varepsilon(t-T) e^{-\delta T} \Rightarrow$

$\Rightarrow \frac{A_0}{s+\delta} (1 - e^{-\delta T} e^{-sT})$

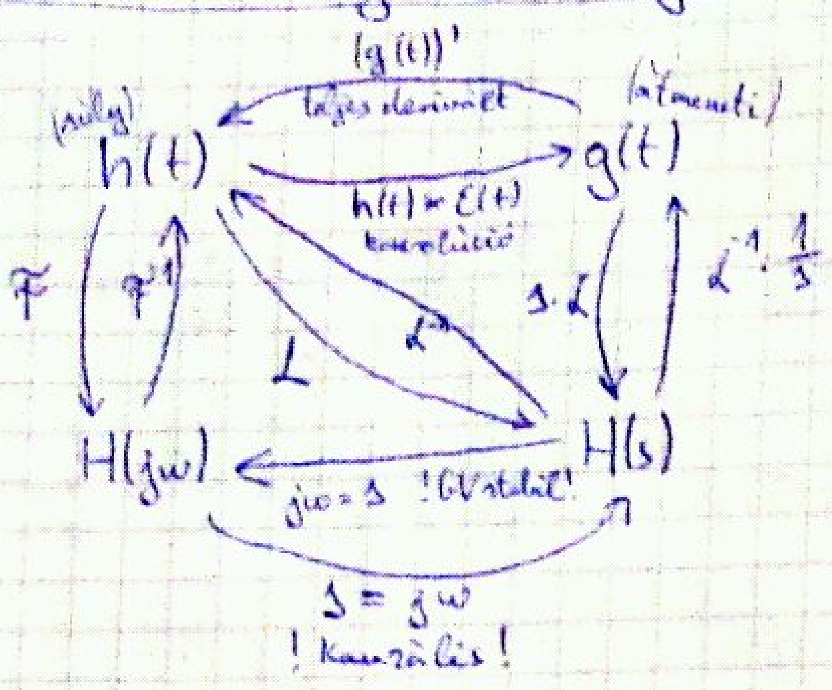
$I_{2R}(s) = \frac{s+\gamma}{(s+\alpha)(s+\beta)} \cdot \frac{A_0}{s+\delta} (1 - e^{-\delta T} e^{-sT})$

$y_0(t) = (C e^{-\alpha t} + D e^{-\beta t} + E e^{-\delta t}) \varepsilon(t)$

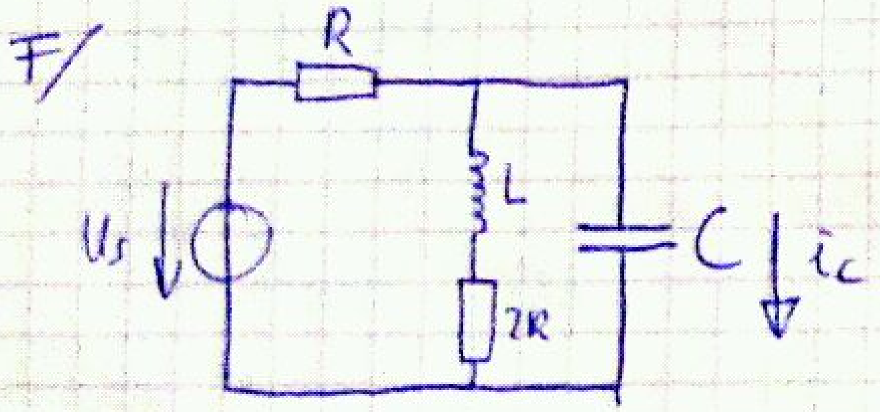
$i_{2R}(t) = y_0(t) - e^{-\delta T} \cdot y_0(t-T)$

$= (C e^{-\alpha t} + D e^{-\beta t} + E e^{-\delta t}) \varepsilon(t) - e^{-\delta T} (C \cdot e^{-\alpha(t-T)} + D e^{-\beta(t-T)} + \dots) \varepsilon(t-T)$

## Rendszert jellemző fv-ek




F/




$$H(s)=\frac{(2R+sL)\times\frac{1}{sC}}{(2R+sL)\times\frac{1}{sC}+R}\cdot\frac{1}{\frac{1}{sC}}=\frac{\frac{(2R+sL)\cdot\frac{1}{sC}}{2R+sL+\frac{1}{sC}}}{\frac{(2R+sL)\cdot\frac{1}{sC}}{(2R+sL)+\frac{1}{sC}}+R}\cdot sC=$$

$$=\frac{2R+sL}{\frac{2R}{sC}+\frac{L}{C}+2R^2+RsL+\frac{R}{sC}}=\frac{2RsC+s^2LC}{3R+2R^2sC+sL+Rs^2LC}=$$

$$=\frac{\frac{1}{R}\left(s^2+s\frac{2R}{L}\right)}{s^2+s\left(\frac{2R}{L}+\frac{1}{RC}\right)+\frac{3}{LC}}$$

F/ $\frac{s^2+2s}{s^2+7s+12}=1+\frac{-5s-12}{s^2+7s+12}=1+\frac{-5s-12}{(s+3)(s+4)}=1+\frac{3}{s+3}+\frac{-8}{s+4}$

$\Rightarrow h(t)=\delta(t)+\left[3e^{-3t}-8e^{-4t}\right]\cdot\varepsilon(t)$

F/ $H(s)=\frac{s^2+2s}{s^2+7s+12}\cdot\frac{1}{s}=\frac{s+2}{s^2+7s+12}=\frac{-1}{s+3}+\frac{2}{s+4}\Rightarrow g(t)=\left(2\cdot e^{-4t}-e^{-3t}\right)\varepsilon(t)$

$g(t)=\{H(s)\cdot\frac{1}{s}\}\mathcal{L}^{-1}$

F/ $u(t)=5\cdot e^{-4t}\,\varepsilon(t)$ $\quad y(t)=?$

$U(s)=\frac{5}{s+4}$

$Y(s)=U(s)\cdot H(s)=\frac{5}{s+4}\cdot\frac{s^2+2s}{(s+3)(s+4)}=\frac{\frac{15}{1}}{s+3}+\frac{-10}{s+4}+\frac{\frac{40}{-1}}{(s+4)^2}\Rightarrow$

$\Rightarrow y(t)=\left[15e^{-3t}-10e^{-4t}-40te^{-4t}\right]\varepsilon(t)$

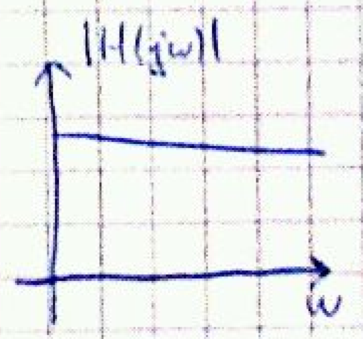
$u(t)=5\cdot\cos 3t$ $\quad y(t)=?$ $\quad j\omega=s$

$H(j\omega)\big|_{\omega=3}=\frac{-9+6j}{-9+21j+12}=\frac{-9+6j}{3+21j}=\frac{-3+2j}{1+7j}=\frac{(-3+2j)(1-7j)}{50}=$

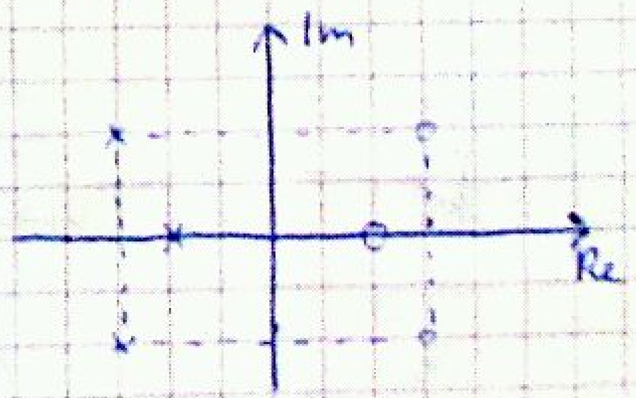
$=\frac{11+23j}{50}=A\cdot e^{j\alpha}$

$y(t)=5\cdot A\cos(3t+\alpha)$

### Mindent áteresztő rendszer

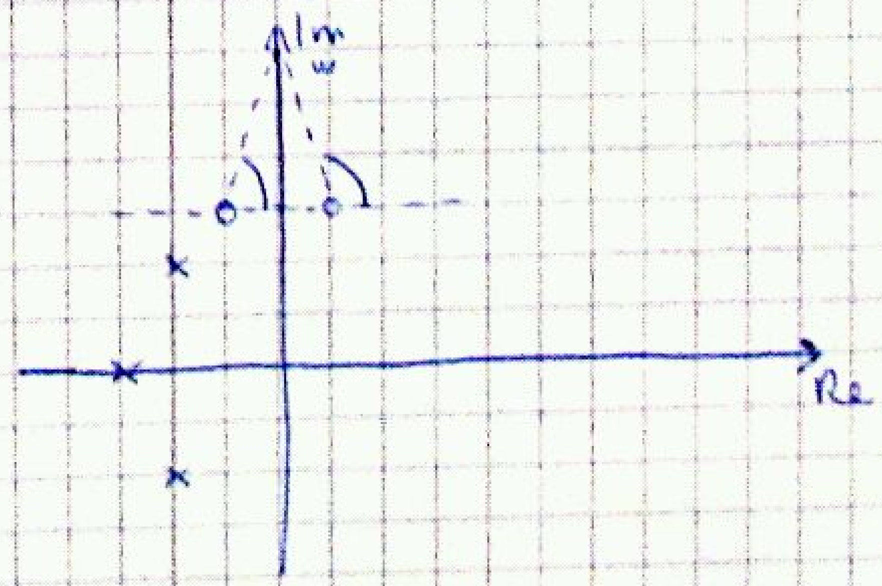



$H(s)=\frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$




$\bar{p}_i=-\bar{z}_i^*$

minimál fázis



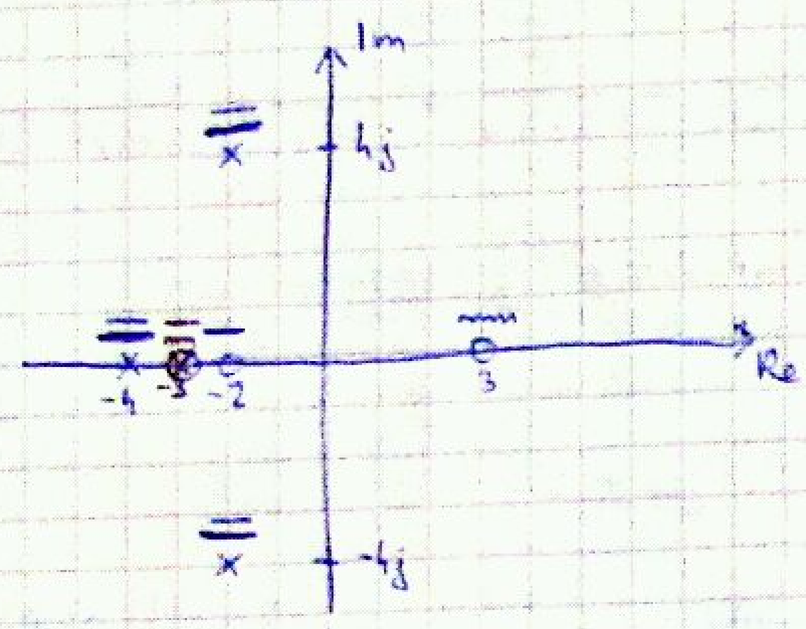

$Re\{p_i\}<0$

$Re\{z_i\}\le 0$

7/

$$H(s) = \frac{(s+2)(s-3)}{(s+2+4j)(s+2-4j)(s+4)} \cdot 7$$

— mindent áteresztőben lehet
— minimál fázisúban lehet

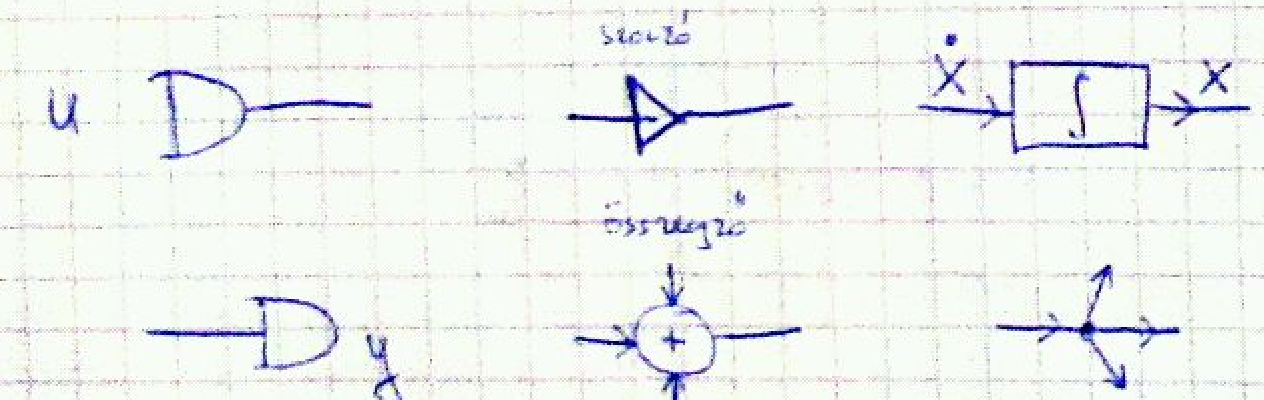



$$H_{\text{mindent áteresztő}} = 7 \cdot \frac{s-3}{s+3}$$ → kell, mert $p_i = -\bar{z}_i^*$

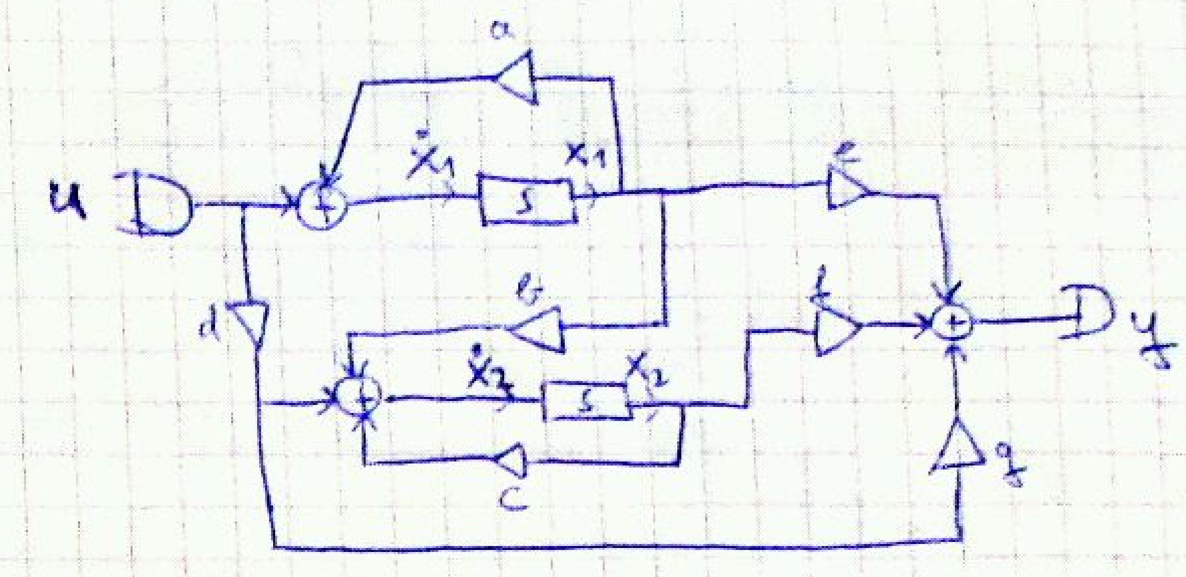
$$H_{\text{minimál fázis}}(s) = \frac{(s+2)(s+3)}{(s+2+4j)(s+2-4j)(s+4)}$$

~~Diszkrét idejű hálózatok~~

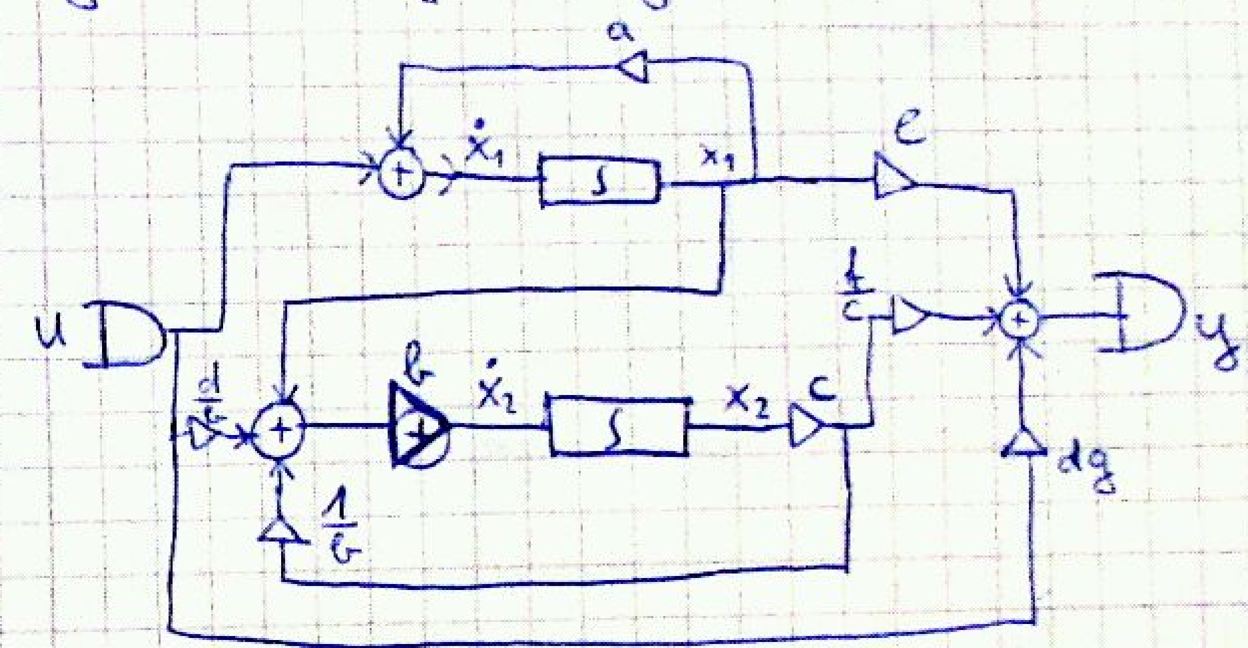
$$\dot{x} = Ax + Bu$$
$$y = c^T x + Du$$



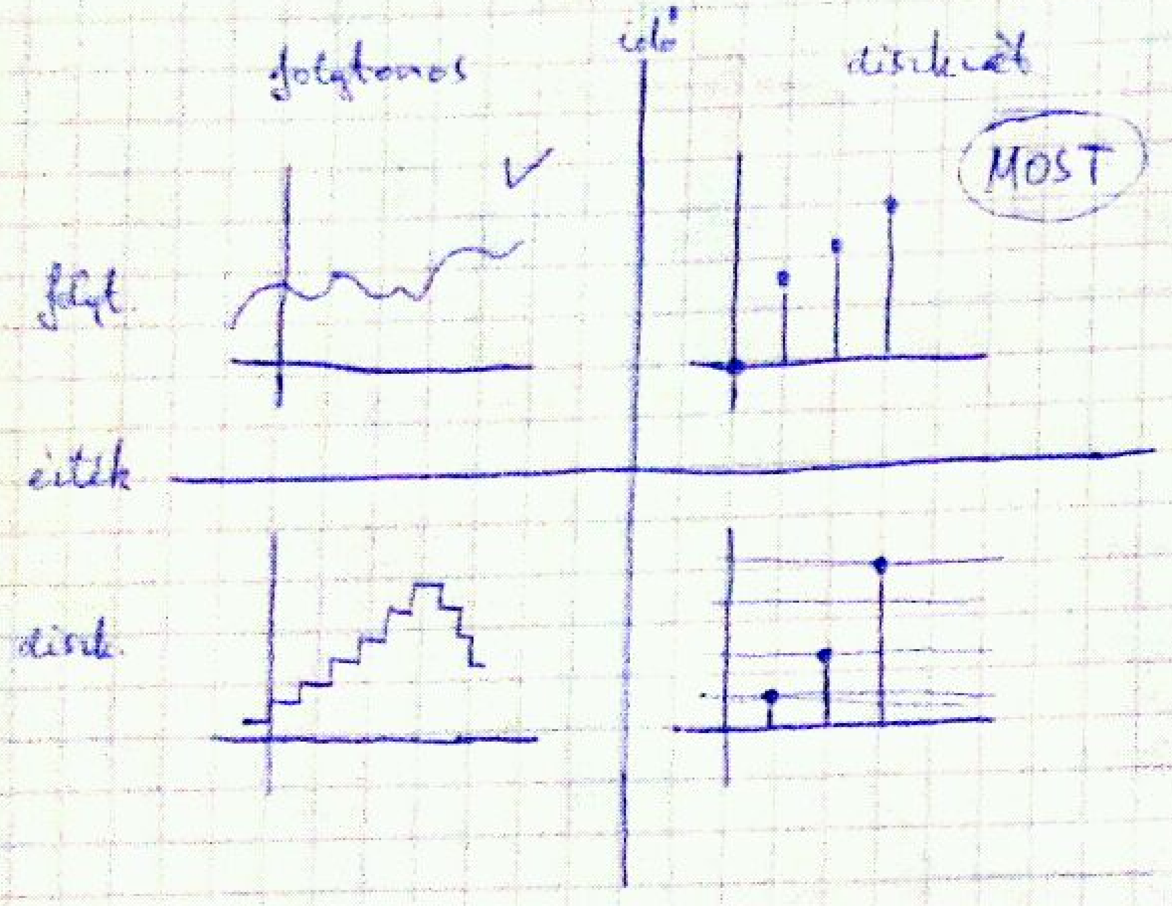

7/

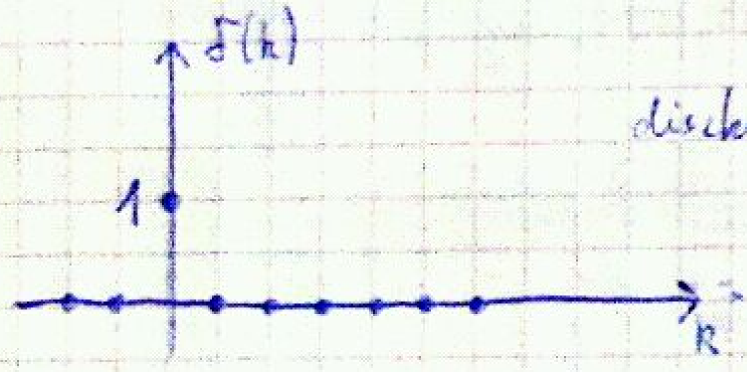



$$\dot{x}_1 = a x_1 + u$$
$$\dot{x}_2 = b x_1 + c x_2 + du$$
$$y = e x_1 + f x_2 + dg\, u$$



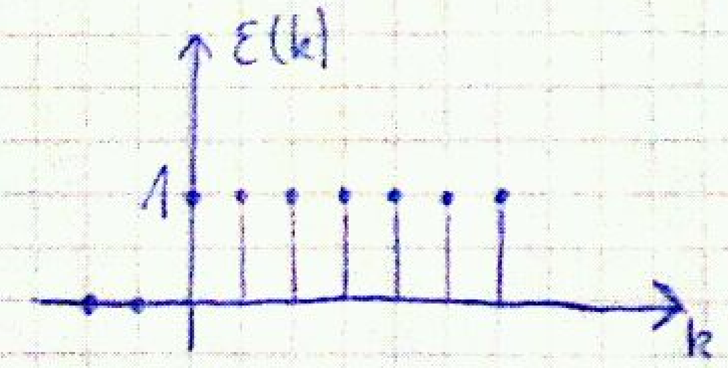
# Diszkrét idejű hálózatok




diszkrét idő, folytonos érték

k: lépésszám, diszkrét idő változója




$$\varepsilon(k) = \sum_{i=0}^{\infty} \delta[k-i]$$

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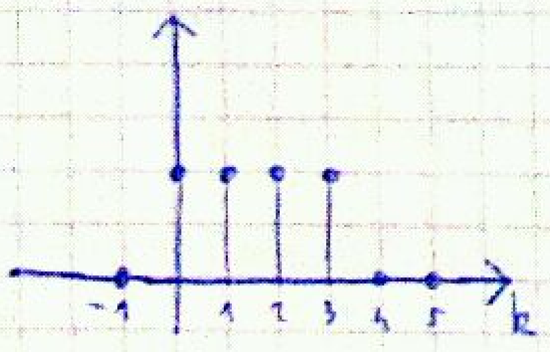
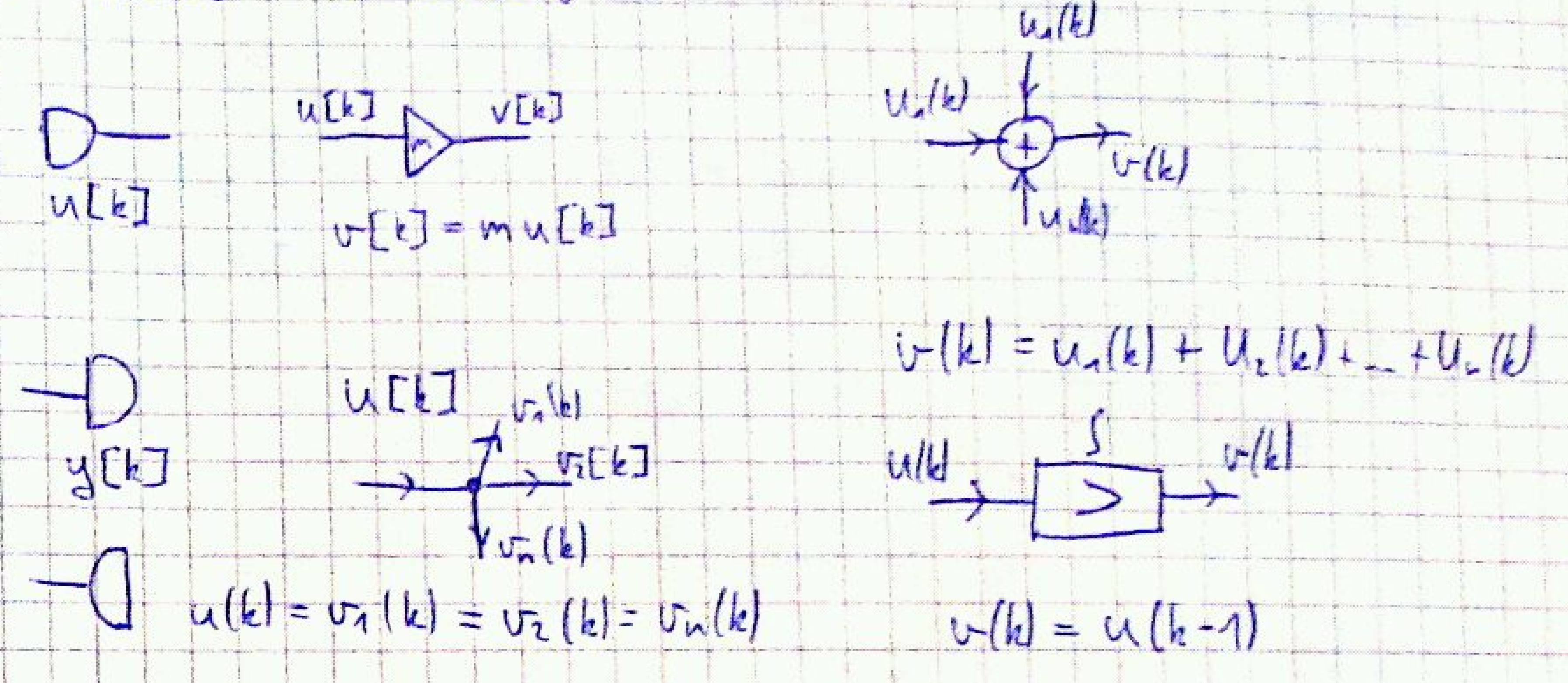



$\varepsilon(k) - \varepsilon(k-4)$

mint – , mert

folytonosban $X(t) = x(t-T)$
diszkrétben $X(k) = X(k+K)$ } periodikus

V: kimenet U: bemenet

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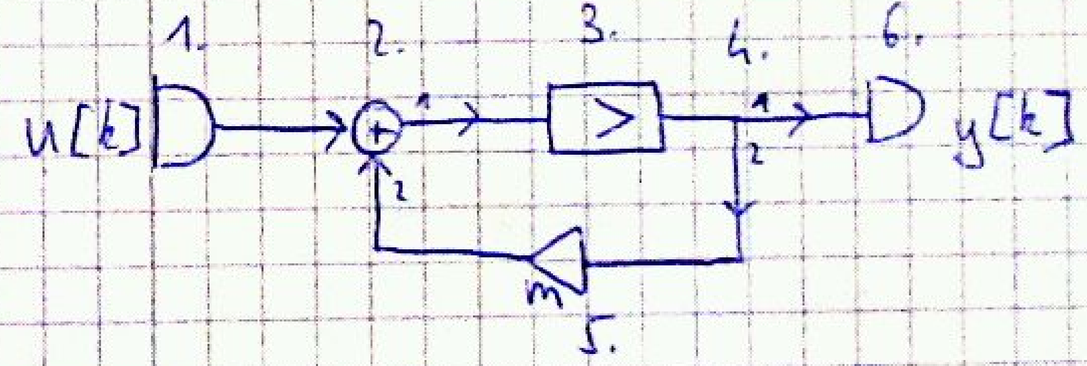


| | |
|---|---|
| $v_1[k] = u[k]$ | $u_{21}(k) = v_1(k) = u(k)$ |
| $my(k)+u(k) = v_2[k] = u_{21}[k] + u_{22}[k]$ | $u_{22}(k) = v_5(k) = my(k)$ |
| $v_3[k] = u_3[k-1]$ | $u_3(k) = v_2(k) = my(k) + u(k)$ |
| $y(k) = v_{41}(k) = v_{42}(k) = u_4(k)$ | $u_4(k) = v_3(k) = y(k)$ |
| $my(k) = v_5(k) = m\, u_5(k)$ | $u_5(k) = v_{42}(k) = y(k)$ |
| $y(k) = u_6(k)$ | $u_6(k) = v_{41}(k) = y(k)$ |

$y(k) = my(k-1) + u(k-1)$ → Rendszeregyenlet

$$y(k) + \sum_{i=1}^{n} a_i\, y(k-i) = \sum_{\ell=0}^{m} b_\ell \cdot u(k-\ell)$$

$$y(k) - 0{,}8\,y(k-1) + 0{,}15\,y(k-2) = 3u(k) + 4u(k-1)$$

| k | u | y=h |
|---|---|---|
| -2 | 0 | 0 |
| -1 | 0 | 0 |
| 0 | 1 | 3 |
| 1 | 0 | 6,4 |
| 2 | 0 | 4,67 |

$\delta(k)$

$$y(0) = 0{,}8\,y(-1) - 0{,}15\,y(-2) + 3u(0) + 4u(-1)$$
(0, 0, 1, 0)

$$y(1) = 0{,}8\,y(0) - 0{,}15\,y(-1) + 3u(1) + 4u(0)$$
(3, 0, 0, 1)

$$y(2) = 0{,}8\,y(1) - 0{,}15\,y(0) + 3u(2) + 4u(1)$$
$0{,}8\cdot 6{,}4$ $\quad -0{,}15\cdot 3$ $\quad 0 \quad 0$

$5{,}12 \quad -0{,}45$

$$y(k) = \sum_{i=1}^{n} a_i\, y(k-i) = \sum_{l=0}^{m} b_l \cdot u(k-l) \Rightarrow h(k) = \sum_{l=0}^{m} b_l\, h_0(k-l)$$

Differenciaegyenlet

$$x(k+1) = A\,x(k) + B\,u(k)$$
$$x = x_f + x_g$$
$$x(k+1) = A\,x(k)$$

homogén $\rightarrow$ $x_f = C\lambda^k$

$$\lambda C\lambda^k = C\lambda^{k+1} = A\,C\lambda^k$$
$$\lambda x_f = A x_f$$

$$y(k) + \sum_{i=1}^{n} a_i\, y(k-i) = u(k) \Rightarrow h_0(k)$$

$$y(k) + \sum_{i=1}^{n} a_i\, y(k-i) = 0$$

$$C\lambda^k + \sum_{i=1}^{n} a_i\, C\cdot\lambda^{k-i} = 0$$

$$C\lambda^{k-n}\left(\lambda^n + \sum_{i=1}^{n} a_i^n\,\lambda^{n-i}\right) = 0$$

$$h_0(k) = C_1\lambda_1^k + C_2\lambda_2^k + C_n\lambda_n^k$$

$h(0) = 1$

$h_0(-1) = 0$

$h_0(-2) = 0$

$h_0(-n+1) = 0$

$$\lambda^n + \sum_{i=1}^{n} a_i\,\lambda^{n-i} = 0$$

$\lambda_1, \lambda_2 \ldots \lambda_n$

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$$y(k) - 0{,}8\,y(k-1) + 0{,}15\,y(k-2) = 3u(k) + 4u(k-1)$$
$$-\|- \qquad = u(k)$$
$$y(k) - 0{,}8\,y(k-1) + 0{,}15\,y(k-2) = 0$$
$$C\lambda^k - 0{,}8\,C\lambda^{k-1} + 0{,}15\,C\lambda^{k-2} = 0$$
$$C\lambda^{k-2}(\lambda^2 - 0{,}8\lambda + 0{,}15) = 0$$
$$\lambda^2 - 0{,}8\lambda + 0{,}15 = 0$$
$$\lambda_{1,2} = \begin{cases} 0{,}3 \\ 0{,}5 \end{cases}$$

$$h_0(k) = C_1\cdot 0{,}3^k + C_2\cdot 0{,}5^k$$

$$h_0(0) = 1 = C_1 + C_2$$
$$h_0(-1) = 0 = \frac{C_1}{0{,}3} + \frac{C_2}{0{,}5}$$

$\Rightarrow$ $C_1 = 1 - C_2$

$0{,}5(1 - C_2) + 0{,}3\,C_2 = 0$

$0{,}5 = 0{,}2\,C_2$

$C_2 = 2{,}5 \Rightarrow C_1 = -1{,}5$

$$h_0(k) = -1{,}5\cdot 0{,}3^k + 2{,}5\cdot 0{,}5^k$$

$-4{,}5\cdot 0{,}3\cdot 0{,}3^{k-1} + 7{,}5\cdot 0{,}5\cdot 0{,}5^{k-1}$

$$h(k) = 3h_0(k) + 4h_0(k-1) =$$
$$= \varepsilon(k)(-4{,}5\cdot 0{,}3^k + 7{,}5\cdot 0{,}5^k) + \varepsilon(k-1)(-6\cdot 0{,}3^{k-1} + 10\cdot 0{,}5^{k-1})$$
$$= 3\delta(k) + \varepsilon(k)(-7{,}35\cdot 0{,}3^{k-1} + 13{,}75\cdot 0{,}5^{k-1})$$

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$$y[k] + 0{,}6\,y[k-1] + 0{,}08\,y[k-2] = 4u(k) - 3u(k-2)$$
$$= u(k)$$

$$C\lambda^k + 0{,}6\,C\lambda^{k-1} + 0{,}08\,C\lambda^{k-2} = 0$$

$$\lambda^2 + 0{,}6\lambda + 0{,}08 = 0$$

$$\lambda_{1,2} = \begin{cases} -0{,}2 \\ -0{,}4 \end{cases}$$

$$h_0(k) = C_1(-0{,}2)^k + C_2(-0{,}4)^k$$

$$h_0(0) = 1 = C_1 + C_2$$
$$h_0(-1) = 0 = \frac{C_1}{-0{,}2} + \frac{C_2}{-0{,}4}$$
$$\Rightarrow \quad C_1 = 1 - C_2$$
$$-0{,}4(1-C_2) - 0{,}2C_2 = 0$$
$$C_2 = 2 \qquad C_1 = -1$$

$$h_0(k) = -1\cdot(-0{,}2)^k + 2(-0{,}4)^k$$

$$h(k) = 4\cdot h_0(k) - 3h_0(k-2) = \left(-4\cdot(0{,}2)^k + 8(-0{,}4)^k\right)\varepsilon(k) - \left(-3(0{,}2)^{k-2} + 6(-0{,}4)^{k-2}\right)\varepsilon(k-2)$$

$$h(k) = 4\delta(k) - 2{,}4\,\delta(k-1) + \varepsilon(k-2)\left(\underbrace{[3-0{,}16]}_{2{,}84}(-0{,}2)^{k-2} + \underbrace{(-6+1{,}44)}_{-4{,}56}(-0{,}4)^{k-2}\right)$$

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$$y(k) + y(k-1) + 0{,}5\,y(k-2) = 8u(k-2)$$

$$\lambda^2 + \lambda + 0{,}5 = 0 \qquad \lambda_{1,2} = -0{,}5 \pm j0{,}5$$

$$h_0(k) = C_1(-0{,}5+j0{,}5)^k + C_2(-0{,}5-j0{,}5)^k$$

$$h_0(0) = 1 = C_1 + C_2$$
$$h_0(-1) = 0 = \frac{C_1}{-0{,}5+j0{,}5} + \frac{C_2}{-0{,}5-j0{,}5}$$
$$\Rightarrow \quad C_1 = 1 - C_2$$
$$(-0{,}5-j0{,}5)(1-C_2) + (0{,}5+j0{,}5)C_2 = 0$$

$$C_2 = 0{,}5 - j0{,}5 \qquad C_1 = 0{,}5 + j0{,}5$$

$$h_0(k) = \underbrace{(0{,}5+j0{,}5)}_{\frac{\sqrt{2}}{2}e^{j\frac{\pi}{4}}}(-0{,}5+j0{,}5)^k + (0{,}5-j0{,}5)(-0{,}5-j0{,}5)^k$$

$$h_0(k) = \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}\right)^k \frac{e^{j\frac{3\pi k}{4}}e^{j\frac{\pi}{4}} + e^{-j\frac{3\pi k}{4}}e^{-j\frac{\pi}{4}}}{2} = \sqrt{2}\left(\frac{\sqrt{2}}{2}\right)^k \cos\left(\frac{3\pi}{4}k + \frac{\pi}{4}\right)$$

$$h(k) = 8\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)^{k-2}\cdot\cos\left(\frac{3\pi}{4}(k-2) + \frac{\pi}{4}\right)\varepsilon(k-2)$$

Konvolúció:

$$y[k] = \sum_{i=0}^{k} h[k-i]\cdot u[i]$$

$$h(k) = A\cdot a^k\,\varepsilon(k)$$
$$u(k) = B\cdot b^k\,\varepsilon(k)$$

$$\sum_{i=0}^{k} A a^{k-i}\cdot B b^i = ABa^k\sum_{i=0}^{k}\left(\frac{b}{a}\right)^i = ABa^k\cdot\frac{\left(\frac{b}{a}\right)^{k+1} - 1}{\frac{b}{a} - 1} = AB\frac{b^{k+1} - a^{k+1}}{b-a} =$$

$$= \Big[\underbrace{AB\frac{b}{b-a}b^k}_{X_{gerj.}} + \underbrace{AB\frac{a}{a-b}a^k}_{X_{függ.}}\Big]\varepsilon(k)$$

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$$h(k) = 4\cdot 0{,}3^k\,\varepsilon(k)$$
$$u_1(k) = 5\,\varepsilon(k)$$
$$u_2(k) = 5$$

$$y(0) = h(0)\,u(0) = 4\cdot 5 = 20$$
$$y(1) = h(1)\,u(0) + h(0)\,u(1) = 26$$
$$y(2) = h(2)\,u(0) + h(1)\,u(1) + h(0)\,u(2) = 27{,}8$$

$$\sum_{i=0}^{k} 5\cdot 4\cdot 0{,}3^i = 20\cdot\frac{0{,}3^{k+1} - 1}{0{,}3 - 1} = \left(-\frac{20\cdot 0{,}3}{0{,}7}\,0{,}3^k + \frac{20\cdot 1}{0{,}7}\right)\varepsilon(k)$$

$$\sum_{i=-\infty}^{k} = 20\cdot 0{,}3^i = \frac{20}{0{,}7}$$

F/

$$y(k) = y_{s}(k) + y_{g}(k)$$

$$y_{s}(k) = C_1\lambda_1^k \ldots + C_n\lambda_n^k$$

$y(k) + 0{,}6y(k-1) + 0{,}08y(k-2) = 4u(k) - 3u(k-2)$

| k | u | h |
|---|---|---|
| -1 | 0 | 0 |
| 0 | 1 | 4 |
| 1 | 0 | -2,4 |
| 2 | 0 | -1,88 |

$y_f(k) = C_1(-0{,}2)^k + C_2(-0{,}4)^k$

$h(1) = -2{,}4 = C_1(-0{,}2)^1 + C_2(-0{,}4)^1$

$h(2) = -1{,}88 = C_1(-0{,}2)^2 + C_2(-0{,}4)^2$

$h(k) = 4\delta(k) + \varepsilon(k-1)\left(C_{11}(-0{,}2)^k + C_{12}(-0{,}4)^k\right)$

| u(k) | $y_g(k)$ |
|---|---|
| $\delta(k)$ | — |
| $A\cdot\varepsilon(k)$ | $B$ |
| $A a^k$ | $B a^k$ |
| $A\cos(\vartheta k)$ | $B\cos(\vartheta k+\alpha)$ |

7/

$y(k) + 0{,}6y(k-1) + 0{,}08y(k-2) = 4u(k) - 3u(k-2)$

$u(k) = 5\cdot 0{,}9^k\,\varepsilon(k)$

$y_f(k) = C_1(-0{,}2)^k + C_2(-0{,}4)^k$

$B\cdot 0{,}9^k + 0{,}6\,B\,0{,}9^{k-1} + 0{,}08\cdot B\cdot 0{,}9^{k-2} = 4\cdot 5\cdot 0{,}9^k - 3\cdot 5\cdot 0{,}9^{k-2}$

$B\cdot(0{,}9^2 + 0{,}6\cdot 0{,}9 + 0{,}08) = 20\cdot 0{,}9^2 - 15$

$B = \frac{12}{0{,}8+0{,}54+0{,}08} = 0{,}84$

$y_g(k) = 0{,}84\cdot 0{,}9^k$

| k | u | y |
|---|---|---|
| -2 | 0 | 0 |
| -1 | 0 | 0 |
| 0 | 5 | 20 |
| 1 | 4,5 | 6 |
| 2 | 4,05 | -4 |

$y(0) = 20$

$y(1) = 18 - 12 = 6$

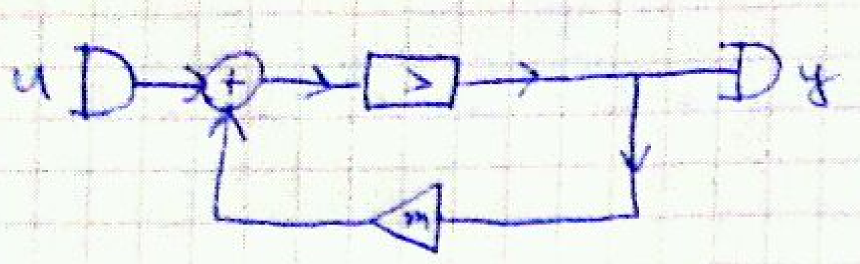
$y(2) = 16{,}2 - 15 - 3{,}6 - 1{,}6$

$y(1) = 6 = C_1\cdot(0{,}2) + C_2(-0{,}4) + 0{,}84\cdot 0{,}9$

$y(2) = -4 = C_1(-0{,}2)^2 + C_2(-0{,}4)^2 + 0{,}84\cdot 0{,}9^2$

$\Rightarrow C_1 \ldots,\ C_2 \ldots$

$y(k) = 20\,\delta(k) + \varepsilon(k-1)\left(C_1(-0{,}2)^k + C_2(-0{,}4)^k + 0{,}84\cdot 0{,}9^k\right)$




$$x(k+1) = A\,x(k) + B\,u(k)$$
$$y(k) = c^T x(k) + D\,u(k)$$

| k | u | x | y=h |
|---|---|---|---|
| 0 | 1 | 0 | D |
| 1 | 0 | B | $C^T B$ |
| 2 | 0 | AB | $C^T A B$ |
| 3 | 0 | $A^2 B$ | $C^T A^2 B$ |

$$y(k) = D\,\delta(k) + \varepsilon(k-1)\, C^T A^{k-1} B$$

Lagrange matrix:

$$A^k = \sum_{i=1}^{n} L_i \cdot \lambda_i^k \qquad L_i = \prod_{\substack{n=1\\ n\neq i}}^{n} \frac{A - \lambda_n \mathbb{1}}{\lambda_i - \lambda_n}$$

$$y(k) = C^T A^k X(0) + \sum_{i=0}^{k-1} C^T \cdot A^{k-1-i} B \cdot u(i) + D\,u(k)$$

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$$A = \begin{bmatrix} -0,5 & 4 \\ 0 & 0,3 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \lambda_1 = -0,5 \quad \lambda_2 = 0,3$$

$$C^T = [-3 \;\; 4] \qquad D = 5 \qquad \lambda^k \to |\lambda| < 1$$

$$L_1 = \prod_{n=2}^{2} \frac{A - 0,3\,\mathbb{1}}{-0,5-(0,3)} = \begin{bmatrix} 1 & -5 \\ 0 & 0 \end{bmatrix}$$

$$L_2 = \frac{A-(-0,5)\mathbb{1}}{0,3-(-0,5)} = \begin{bmatrix} 0 & 5 \\ 0 & 1 \end{bmatrix}$$

$$K_1 = [-3 \;\; 4] \begin{bmatrix} 1 & -5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 9$$

$$K_2 = [-3 \;\; 4] \begin{bmatrix} 0 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = -11$$

$$h(k) = 5\,\delta(k) + \varepsilon(k-1)\left(K_1 \cdot \overset{\lambda_1}{(-0,5)}^{k-1} + \overset{K_2}{11}(0,3)^{k-1}\right)$$

$$x[k+1] = A\,x(k) + B\,u(k)$$
$$y(k) = c^T x(k) + D\,u(k)$$

$$h(k) = D\,\delta(k) + \varepsilon(k-1)\, C^T \cdot A^{k-1} \cdot B$$
$$y(k) = C^T A^k x(0) + \sum_{i=1}^{k} C^T A^{k-1-i} \cdot B\,u(i) + D\,u(k)$$

$$A^k = \sum_{i=1}^{N} L_i \lambda_i^k \qquad L_i = \prod_{\substack{n=1\\ n\neq i}}^{N} \frac{A - \lambda_n \mathbb{1}}{\lambda_i - \lambda_n} \qquad \sum_{i=1}^{N} L_i = \mathbb{1}$$

F/

$$A = \begin{bmatrix} -0,3 & 0,06 \\ 1 & -0,2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad c^T = [3 \;\; 2] \quad D = 0$$

$$(-0,3-\lambda)(-0,2-\lambda) - 0,06 = 0$$
$$\lambda^2 + 0,5\,\lambda = 0$$
$$\lambda_1 = -0,5 \qquad \lambda_2 = 0$$

$$L_1 = \frac{\begin{bmatrix} -0,3 & 0,06 \\ 1 & -0,2 \end{bmatrix}}{-0,5} = \begin{bmatrix} 0,6 & -0,12 \\ -2 & 0,4 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0,4 & 0,12 \\ 2 & 0,6 \end{bmatrix}$$

$$h(k) = \varepsilon(k-1)\left(\underset{-2,64}{c^T L_1 B}(-0,5)^{k-1} + \underset{16,64}{c^T L_2 B}\, 0^{k-1}\right)$$

$$h(k) = \varepsilon(k-1)\,(-2,64(-0,5)^{k-1} + 16,64\,\delta(k-1))$$

| k | $x_1$ | $x_2$ | y | u |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 2 | 4 | 14 | 0 |
| 2 | -0,16 | 1,2 | 1,32 | 0 |

$$x_1:\ \underset{B_1}{2}\cdot \underset{A_{11}^{k-1}}{-0,3} + \underset{B_2}{4}\cdot \underset{A_{12}^{k-1}}{0,06}$$

$$x_2:\ \underset{B_1}{2}\cdot \underset{A_{21}^{k-1}}{1} + ((-0,2)\cdot 4)$$

$$y:\ x_1\cdot c_1^T + x_2\cdot c_2^T$$

F/

$$A = \begin{bmatrix} -0,6 & 1 \\ 0,26 & -0,4 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad c^T = [2\ 0] \quad D = 3$$

$$\lambda^2 + \lambda + 0,5 = 0$$

$$\lambda_{1,2} = -0,5 \pm j0,5$$

$$L_1 = \begin{bmatrix} 0,5+0,1j & j \\ -0,26j & 0,5-j0,1 \end{bmatrix}$$

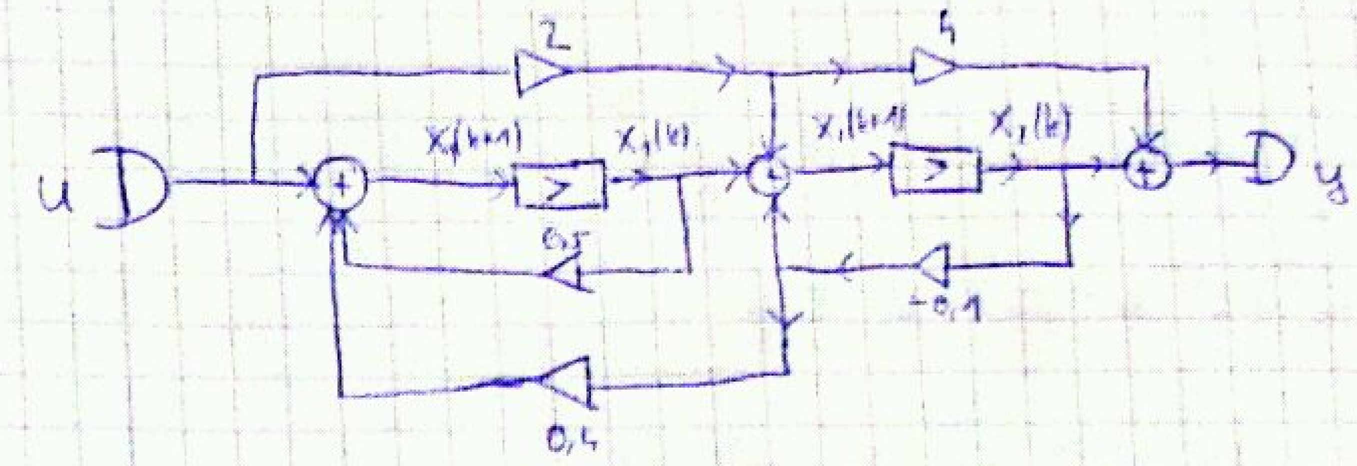
$$L_2 = \begin{bmatrix} 0,5-0,1j & -j \\ +0,26j & 0,5+0,1j \end{bmatrix}$$

$$c^T\cdot L_1\cdot B = [2\ 0]\begin{bmatrix} 0,5+0,1j & j \\ -0,26j & 0,5-0,1j \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 2+2,2j = K_1$$

$$2-2,2j = K_2$$

$$h(k) = 3\delta(k) + \varepsilon(k-1)\left[\underset{2,97e^{j47,7°}}{(2+2,2j)}\underset{\frac{\sqrt{2}}{2}e^{j\frac{3\pi}{4}}}{(-0,5+j0,5)^{k-1}} + (2-2,2j)(-0,5-j0,5)^{k-1}\right] =$$

$$= 3\delta(k) + \varepsilon(k-1)\left(2\cdot 2,97\cdot\left(\frac{\sqrt{2}}{2}\right)^{k-1}\cdot\cos\left(\frac{3\pi}{4}(k-1) + 47,7°\right)\right)$$




$$X_1(k+1) = u(k) + (0,4)(-0,1)X_2(k) + 0,5X_1(k)$$

$$X_2(k+1) = 2u(k) + X_1(k) + (-0,1)X_2(k)$$

$$y(k) = 8u(k) + X_2(k)$$

folytonos

$$u(t) = U_0 \cos\omega t + \varphi$$

$$\bar{u} = U_0\cdot e^{j\omega t}e^{j\varphi}$$

$$\hat{U} = U_0 e^{j\varphi}$$

$$U_L = L\frac{di_L}{dt} \qquad \frac{d}{dt} \Rightarrow j\omega$$

diszkrét

$$u(k) = U_0\cdot\cos(\vartheta k + \varphi) \qquad \vartheta = \frac{\pi\cdot 2n}{K}$$

$$\bar{u} = U_0\cdot e^{j\vartheta k}e^{j\varphi}$$

$$\hat{U} = U_0 e^{j\varphi}$$

Állapotváltozós leírás normál alakja:

$$H(e^{j\vartheta}) = \underline{c}^T (e^{j\vartheta}\underline{1} - \underline{A})^{-1} B + D$$

$$y(k) + \sum_{i=1}^{n} a_i\, y(k-i) = \sum_{\ell=0}^{m} b_\ell \cdot u(k-\ell)$$

$$H(e^{j\vartheta}) = \frac{\sum_{\ell=0}^{m} b_\ell\, e^{-j\ell\vartheta}}{1+\sum_{i=1}^{n} a_i\, e^{-ji\vartheta}} = \frac{\overline{Y}}{\overline{U}}$$

$$H = \frac{Y}{U}$$

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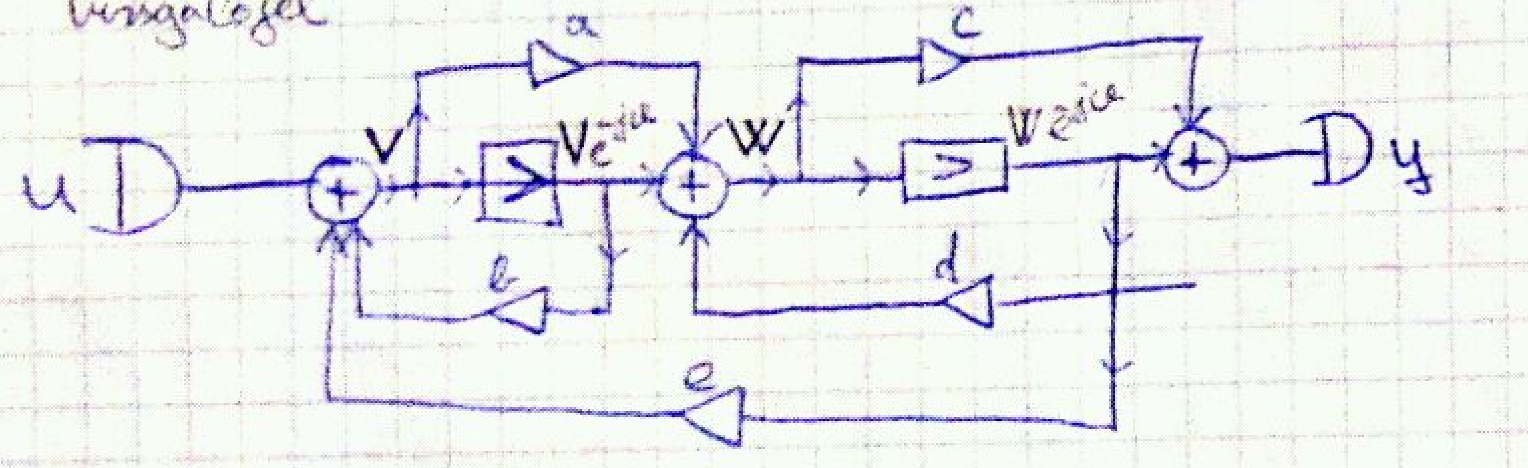
vizsgálójel




$V = bVe^{-j\vartheta} + eWe^{-j\vartheta} + U$ ⇒ $W = \dots U$

$W = aV + Ve^{-j\vartheta} + dWe^{-j\vartheta}$ ⇒ $V = \frac{(1-de^{-j\vartheta})W}{a+e^{-j\vartheta}}$

$Y = cW + We^{-j\vartheta}$ $\qquad \frac{Y}{U} = H(e^{j\vartheta})$

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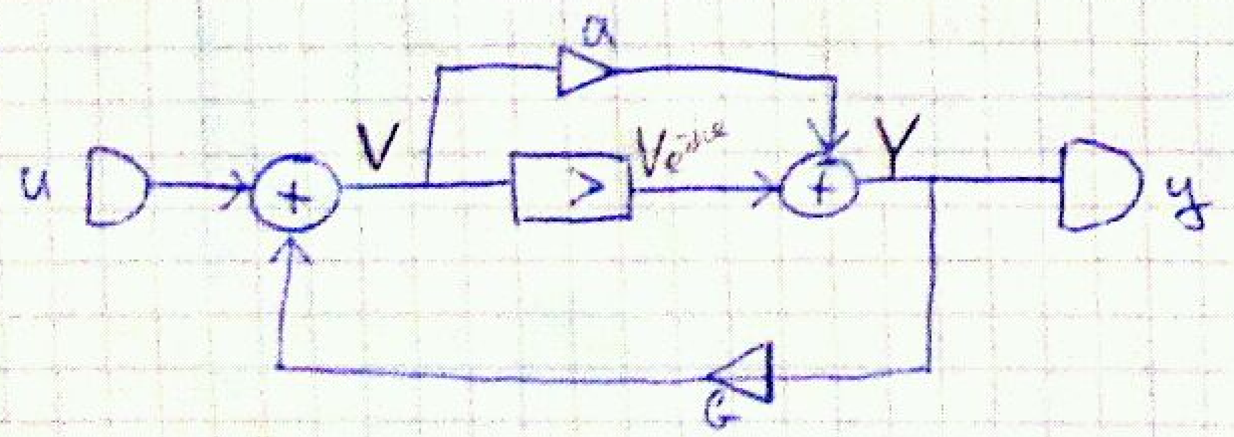


$V = U + G \cdot Y$

$Y = aV + Ve^{-j\vartheta}$ ⇒ $V = \frac{Y}{a+e^{-j\vartheta}}$

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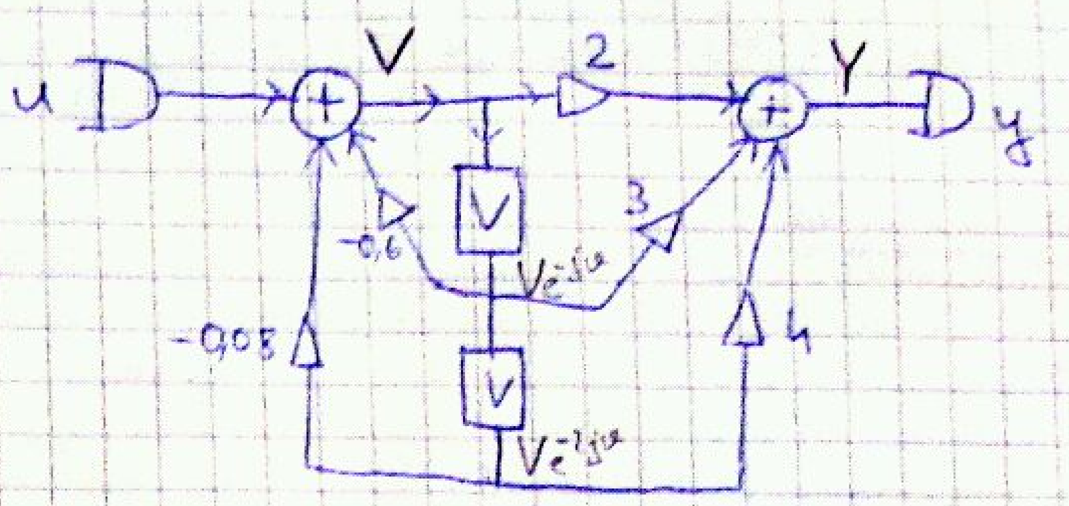



$$V = U - 0{,}6Ve^{-j\vartheta} - 0{,}08Ve^{-2j\vartheta} \Rightarrow V = \frac{U}{1+0{,}6e^{-j\vartheta}+0{,}08e^{-2j\vartheta}}$$

$$Y = 2V + 3Ve^{-j\vartheta} + 4Ve^{-2j\vartheta}$$

$$Y = \underbrace{\frac{2+3e^{-j\vartheta}+4e^{-2j\vartheta}}{1+0{,}6e^{-j\vartheta}+0{,}08e^{-2j\vartheta}}}_{H(e^{j\vartheta})} U$$

Rendszeregyenlet:

$$y(k) + 0{,}6\,y(k-1) + 0{,}08(k-2) = 2u(k) + 3u(k-1) + 4u(k-2)$$

$$u(k) = 10 + 5\cos k\frac{\pi}{2} + 4\sin\left(k\frac{3\pi}{4} + \frac{\pi}{2}\right)$$

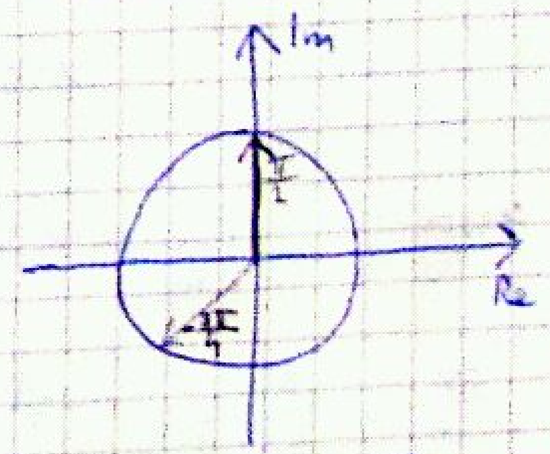
| $\vartheta$ | $\lvert U\rvert$ | $\varphi_U$ | $\lvert H\rvert$ | $\varphi_H$ | $\lvert Y\rvert$ | $\varphi_Y$ |
|---|---|---|---|---|---|---|
| 0 | 10 | — | $\frac{9}{1,68}$ | — | $\frac{9}{1,68}$ | — |
| $\frac{\pi}{2}$ c | 5 | — | A | $\alpha$ | 5A | $\alpha$ |
| $\frac{3\pi}{4}$ s | 4 | $\frac{\pi}{2}$ | B | $\beta$ | 4B | $\beta+\frac{\pi}{2}$ |

$e^{j\vartheta} = \cos\vartheta + j\sin\vartheta$

$e^{-j\vartheta} = \cos\vartheta - j\sin\vartheta$

$$H(e^{j\vartheta})\Big|_{\vartheta=0} = \frac{2+3+4}{1+0{,}6+0{,}08}$$

$$H(e^{j\vartheta})\Big|_{\vartheta=\frac{\pi}{2}} = \frac{2-3j-4}{1-0{,}6j-0{,}08} = \frac{-2-3j}{0{,}92-0{,}6j} = Ae^{j\alpha}$$




$$H(e^{j\vartheta})\Big|_{\vartheta=\frac{3\pi}{4}} = \frac{2+3\cos\frac{3\pi}{4}-3j\sin\frac{3\pi}{4}+4j}{1+0{,}6\cos\frac{3\pi}{4}-3j\sin\frac{3\pi}{4}+0{,}08j} = Be^{j\beta}$$

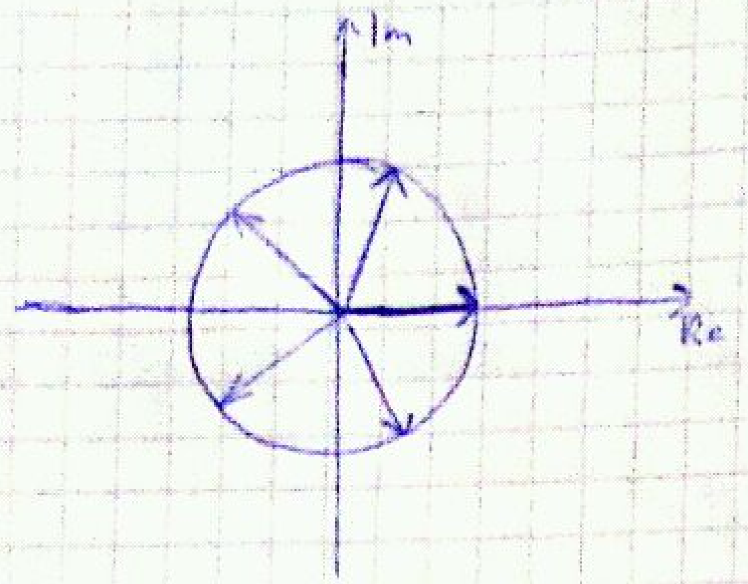
$e^{j\frac{\pi}{2}} = j$ $\qquad e^{j\frac{3\pi}{2}} = -j$

$e^{j\pi} = -1$

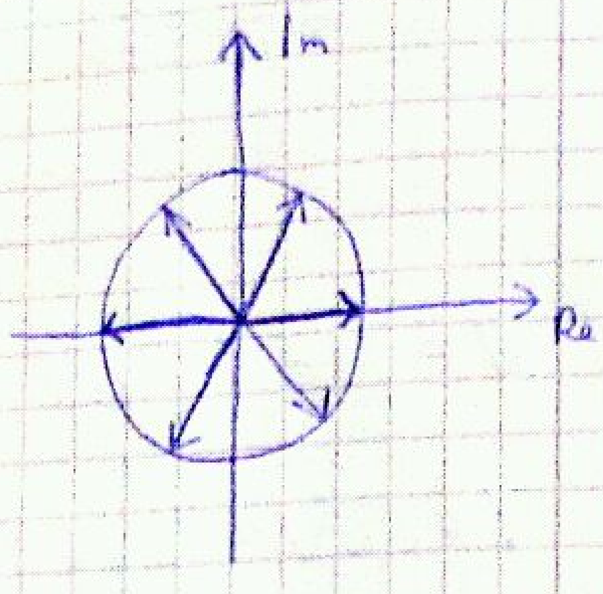
$y(k) = \frac{90}{1{,}68} + 5A\cos(k\frac{\pi}{2}+\alpha) + 4B\sin(k\frac{2\pi}{4}+\beta+\frac{\pi}{2})$

Fourier - sorfejtés

Páratlan




páros:




$\overline{F}_i = \frac{1}{K}\sum_{k=0}^{K-1} f[k]\cdot e^{-jik\vartheta_0}$ $\quad \vartheta_0 = \frac{2\pi}{K} \quad 0 \ldots \frac{K}{2}$

$\overline{F}_i = \overline{F}_{K-i}^{*}$

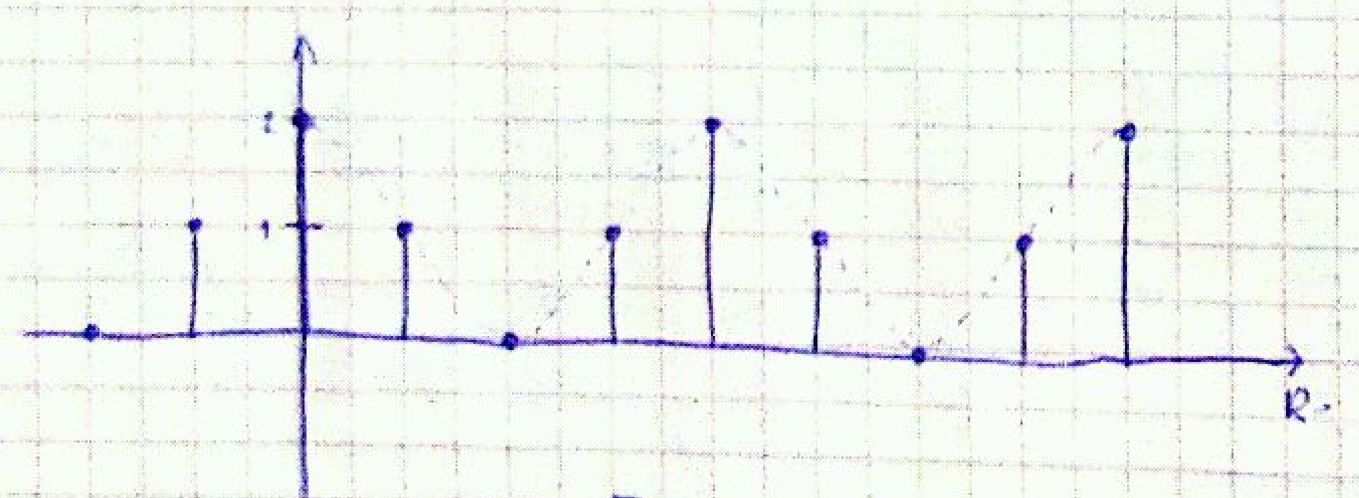
$F_0 = \overline{F}_0$

$F_i^A = 2\,\mathrm{Re}\{\overline{F}_i\}$

$F_i^B = -2\,\mathrm{Im}\{\overline{F}_i\}$ $\quad$ ha $0 < i < \frac{K}{2}$

$F_{\frac{K}{2}}^A = F_{\frac{K}{2}}^B = \overline{F}_{\frac{K}{2}} = 0$

F/




$u(k) = [2,1,0,1] \quad u(k+K) = u(k)$

$\overline{F}_0 = \frac{1}{4}\left[2\,e^{-j0\cdot0\cdot\frac{2\pi}{4}} + 1\cdot e^{-j0\cdot1\cdot\frac{2\pi}{4}} + 0\cdot e^{-j0\cdot2\cdot\frac{2\pi}{4}} + 1\cdot e^{-j0\cdot3\cdot\frac{2\pi}{4}}\right] = 1$

$\overline{F}_1 = \frac{1}{4}\left[2\,\underbrace{e^{-j1\cdot0\frac{2\pi}{4}}}_{1} + 1\cdot\underbrace{e^{-j1\cdot1\cdot\frac{2\pi}{4}}}_{-j} + 0\cdot\underbrace{e^{-j1\cdot2\cdot\frac{2\pi}{4}}}_{-1} + 1\cdot\underbrace{e^{-j1\cdot3\frac{2\pi}{4}}}_{j}\right] = 0{,}5$

$\overline{F}_2 = \frac{1}{4}\left[2\cdot\underbrace{e^{-j2\cdot0\frac{2\pi}{4}}}_{1} + 1\cdot\underbrace{e^{-j2\cdot1\frac{2\pi}{4}}}_{-1} + 0\cdot\underbrace{e^{-j2\cdot2\frac{2\pi}{4}}}_{1} + 1\underbrace{e^{-j2\cdot3\cdot\frac{2\pi}{4}}}_{-1}\right] = 0$

$\overline{F}_3 = \frac{1}{4}\left[\underbrace{\ldots}_{1} + \underbrace{\ldots}_{j} + \underbrace{\ldots}_{-1} + \underbrace{\ldots}_{-j}\right] = 0{,}5$

$u(k) = F_0 + \sum_{i=1}^{K}(F_i^A\cos i\vartheta_0 k + F_i^B\sin i\vartheta_0 k)$

$F_0 = \overline{F}_0 = 1$

$F_1^A = 2\cdot\mathrm{Re}\{\overline{F}_1\} = 1$

$F_1^B = -2\,\mathrm{Im}\{\overline{F}_1\} = 0$

$F_2 = \overline{F}_2 = 0$

$u(k) = 1 + 1\cos\frac{\pi}{2}k$

| k | u(k) |
|---|---|
| 0 | 2 |
| 1 | 1 |
| 2 | 0 |
| 3 | 1 |

„Pont visszakaptuk!"

F/ $u(k) = [5,-1,0,3] \rightarrow K = 4 \quad \vartheta = \frac{\pi}{2}$

$\overline{F}_0 = \frac{1}{4}[5 + (-1) + 0 + 3] = \frac{7}{4}$

$\overline{F}_1 = \frac{1}{4}\left[5 + (-1)\underbrace{e^{-j1\cdot1\frac{2\pi}{4}}}_{+j} + 0 + 3\underbrace{e^{-j3\cdot1\cdot\frac{2\pi}{4}}}_{3j}\right] = \frac{5+4j}{4}$

$\overline{F}_2 = \frac{1}{4}\left[5 + (-1)\underbrace{e^{-j2\cdot1\frac{2\pi}{4}}}_{-1} + 0 + 3\underbrace{e^{-j2\cdot3\frac{2\pi}{4}}}_{-1}\right] = \frac{3}{4}$

$F_0 = \frac{7}{4}$

$F_1^A = 2\cdot\mathrm{Re}\{\overline{F}_1\} = \frac{5}{2}$

$F_1^B = -2\cdot\mathrm{Im}\{\overline{F}_1\} = -2$

$F_2^A = \overline{F}_2 = \frac{3}{4}$

$u(k) = \frac{7}{4} + \frac{5}{2}\cos\frac{\pi}{2}k - 2\sin\frac{\pi}{2}k + \frac{3}{4}\cos\pi k$

## Fourier-transzformáció:

$$X(e^{j\vartheta}) = \sum_{i=-\infty}^{\infty} x[i]\, e^{-j\vartheta i}$$

| $x[k]$ | $X(e^{j\vartheta})$ |
|---|---|
| $\delta(k)$ | $1$ |
| $\varepsilon(k)\, a^k$ $\;|a|<1$ | $\frac{1}{1-ae^{-j\vartheta}} = \frac{e^{j\vartheta}}{e^{j\vartheta}-a}$ |
| $x[k-\tau]$ | $X(e^{j\vartheta})\, e^{-\tau j\vartheta}$ |

$$x(k) = a^{|k|} = \varepsilon(-k)\, a^{-k} + \varepsilon(k)\, a^k - \delta(k)$$

$$X(e^{j\vartheta}) = \frac{1}{1-ae^{j\vartheta}} + \frac{1}{1-ae^{-j\vartheta}} - 1 = \frac{2 - ae^{-j\vartheta} - ae^{j\vartheta} - 1 - a^2 + 2\cos\vartheta}{1 + a^2 - 2a\cos\vartheta} =$$

$$= \frac{1-a^2}{1+a^2-2a\cos\vartheta}$$

$$x[k] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\vartheta})\, e^{j\vartheta k}\, d\vartheta$$

GV-stabilitás: $\sum_{-\infty}^{\infty} |h[k]| < \infty$ vagy $|\lambda| < 1$

(szükséges is elégséges) (elégséges)

Pl.: $y(k) - 2y(k-1) = 3u(k)$

$$C\lambda^k - 2C\lambda^{k-1} = 0$$

$$\lambda - 2 = 0 \quad \lambda = 2 \;\lightning \Rightarrow H(e^{j\vartheta}) \not\exists \;\frac{3}{1-2e^{-j\vartheta}}$$

$$h_0(k) = 2^k$$

$$h(k) = 3\cdot 2^k\, \varepsilon(k)$$

## Laplace-transzformáció (Z-transzformáció):

$$X(z) = \sum_{k=0}^{\infty} x(k)\cdot z^{-k}$$ – csak belépő gerjesztéseknél

| $x(k)$ | $X(z)$ |
|---|---|
| $\delta(k)$ | $1$ |
| $\varepsilon(k)$ | $\frac{1}{1-z^{-1}} = \frac{z}{z-1}$ |
| $A\cdot\varepsilon(k)\, a^k$ $\;|a|<1$ | $\frac{A\cdot z}{z-a}$ |
| $A\cdot k\cdot a^k\cdot\varepsilon(k)$ | $\frac{A\cdot a\cdot z}{(z-a)^2}$ |
| $x[k-\tau]\,\varepsilon[k-\tau]$ | $X(z)\, z^{-\tau}$ |

Számlálóban mindig kell egy $z$!!!

F/ $a(k) = 2\cdot 0{,}4^k\cdot\varepsilon(k)$

$$\sum_0^{\infty} 2\cdot 0{,}4^k\cdot z^{-k} = 2\sum_0^{\infty} \cdot \frac{1}{1-0{,}4z^{-1}} = \frac{2z}{z-0{,}4}$$

F/ $b(k) = 3\cdot(-0{,}5)^k\,(\varepsilon(k) - \varepsilon(k-2)) = 3\cdot(-0{,}5)^k\,\varepsilon(k) - 3(-0{,}5)^2(-0{,}5)^{k-2}\varepsilon(k-2) =$

$$\Rightarrow \frac{3z}{z+0{,}5} - \frac{3(-0{,}5)^2 z}{z+0{,}5}\cdot z^{-2} = \frac{z}{z+0{,}5}\left(3 - 3(-0{,}5)^2 z^{-2}\right)$$

F/ $\frac{2z^2 + 4z + 3}{z^2 + 0{,}8z + 0{,}15}$

$$(2z^2 + 4z + 3) : (z^2 + 0{,}8z + 0{,}15) = 2 + \frac{2{,}4}{z} + \frac{0{,}78}{z^2} \pm \dots$$

$2{,}4\cdot z^{-1}$

$-\;2z^2 + 1{,}6z + 0{,}3$

$2{,}4z + 2{,}7$

$-\;2{,}4z + 1{,}92 + 0{,}36/z$

$0{,}78 - \frac{0{,}36}{z}$

$\vdots$

$$2\delta(k) + 2{,}4\,\delta(k-1) + 0{,}78\,\delta(k-2) \dots$$

Jó, de nem a legjobb megoldás

2. megoldás

I $A(z)\ \dfrac{2z^2+4z+3}{z^2+0{,}8z+0{,}15} = 2 + \underbrace{\dfrac{2{,}4z+2{,}7}{z^2+0{,}8z+0{,}15}}_{B(z)}$

$a(k) = 2\delta(k) + b(k)$

II $\dfrac{2{,}4z+2{,}7}{z^2+0{,}8z+0{,}15}\, z\cdot z^{-1} = Z\left(\dfrac{-7{,}5}{z+0{,}5} + \dfrac{9{,}9}{z+0{,}3}\right) z^{-1} = \left(\dfrac{-7{,}5z}{z+0{,}5} + \dfrac{9{,}9z}{z+0{,}3}\right) z^{-1} =$

$b[k] = \varepsilon(k-1)\left(-7{,}5\cdot(-0{,}5)^{k-1} + 9{,}9\cdot(-0{,}3)^{k-1}\right)$

F/II $C(z) = \dfrac{4z^2+5z}{z^2+0{,}6z+0{,}08} = Z\,\dfrac{4z+5}{(z+0{,}4)(z+0{,}2)} = \dfrac{-17z}{z+0{,}4} + \dfrac{21z}{z+0{,}2}$

$C(k) = \varepsilon(k)\cdot\left(-17\cdot(-0{,}4)^k + 21(-0{,}2)^k\right)$

F/II $D(z) = \dfrac{4z}{z^2-0{,}5z-0{,}24} = Z\left(\dfrac{\frac{4}{1{,}1}}{z-0{,}8} + \dfrac{-\frac{4}{1{,}1}}{z+0{,}3}\right)$

$D(k) = \varepsilon(k)\left(\frac{4}{1{,}1}\cdot 0{,}8^k - \frac{4}{1{,}1}\cdot(-0{,}3)^k\right)$

F/ $K(z) = \dfrac{3z^2+5z}{z^2+z+0{,}5} = Z\left(\dfrac{\frac{3{,}5-1{,}5j}{-j}}{z+0{,}5+j0{,}5} + \dfrac{\frac{3{,}5+1{,}5j}{j}}{z+0{,}5-j0{,}5}\right)$

$k(k) = \varepsilon(k)\cdot\left((1{,}5+3{,}5j)(-0{,}5-0{,}5j)^k + (1{,}5-3{,}5j)(-0{,}5+0{,}5j)^k\right)$

$3{,}8\cdot e^{j66^\circ}\quad \left(\frac{\sqrt{2}}{2}\cdot e^{-j\frac{3\pi}{4}}\right)^k \quad 3{,}8\cdot e^{-j66^\circ}\quad \frac{\sqrt{2}}{2}e^{j\frac{3\pi}{4}}$

$= 2\cdot 3{,}8\left(\frac{\sqrt{2}}{2}\right)^k \cos\left(\frac{3\pi}{4}k - 66^\circ\right)\varepsilon(k)$

F/ $F(z) = \dfrac{4z}{(z-0{,}5)^2}$ $\quad 0{,}5\cdot A = 4 \to A = 8$

$f(k) = \varepsilon(k)\, k\cdot 0{,}5^k\cdot 8$

F/ $G(z) = \dfrac{3z}{z^3+0{,}5z^2-0{,}14z} = Z\left(\dfrac{3}{z^2+0{,}5z-0{,}14}\right)\cdot z^{-1} = \ldots$

$g(k) = \varepsilon(k-1)$

F/ $H(z) = \dfrac{4z^2+2z}{(z-0{,}5)(z-0{,}6)^2} = Z\cdot\dfrac{4z+2}{(z-0{,}5)(z-0{,}6)^2} = Z\left(\dfrac{400}{z-0{,}5} + \dfrac{-400}{z-0{,}6} + \dfrac{44}{(z-0{,}6)^2}\right)$

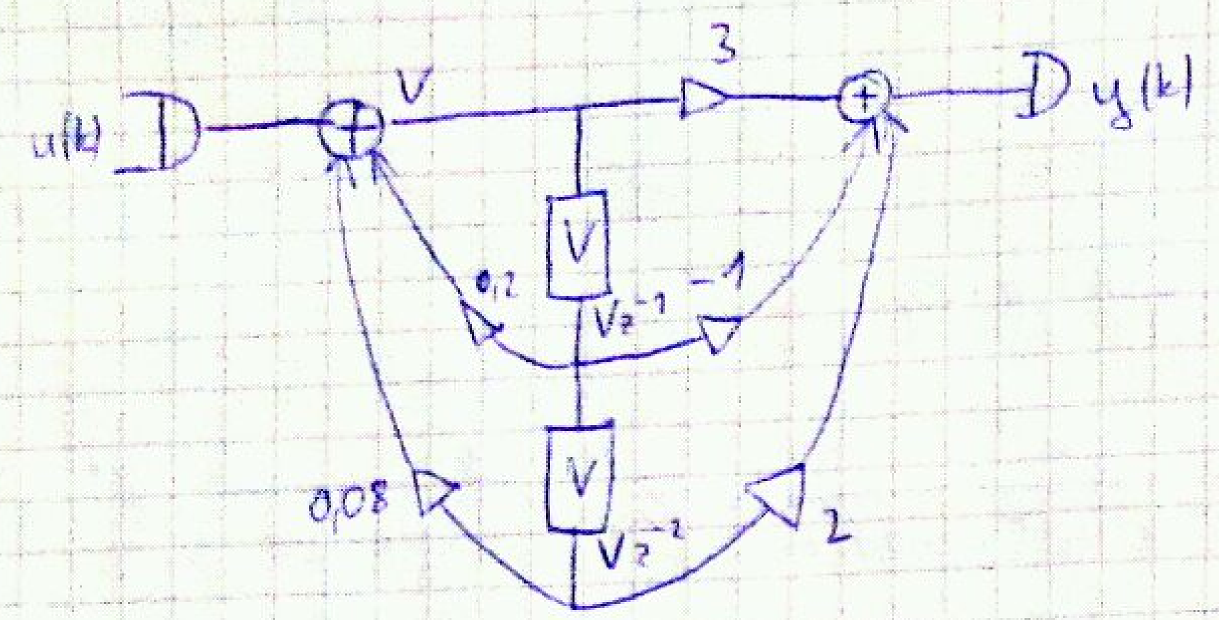
$h(k) = \left(400\cdot 0{,}5^k - 400\cdot 0{,}6^k + \frac{44}{0{,}6}\cdot k\cdot 0{,}6^k\right)\varepsilon(k)$

F/ $u(k) = \left(\varepsilon(k) - \varepsilon(k-5)\right) k\cdot 0{,}4^k = \varepsilon(k)\,k\cdot 0{,}4^k - \varepsilon(k-5)(k-5+5)\cdot$

$\cdot 0{,}4^{(k-5+5)} = \varepsilon(k)\,k\cdot 0{,}4^k - \varepsilon(k-5)(k-5)\,0{,}4^5\, 0{,}4^{(k-5)} - \varepsilon(k-5)\cdot 5\cdot 0{,}4^5\cdot 0{,}4^{k-5}$

$U(z) = \dfrac{0{,}4z}{(z-0{,}4)^2} - \dfrac{0{,}4z}{(z-0{,}4)^2}\cdot 0{,}4^5\cdot z^{-5} - \dfrac{5z}{z-0{,}4}\cdot 0{,}4^5 z^{-5}$

F/




$y(k)$? ha $u(k) = 4 \cos \frac{\pi}{2} k$

$y(k)$, ha $u(k) = 4 \cdot 0{,}8^k (\varepsilon(k) - \varepsilon(k-6))$

$V = U + 0{,}2\, V z^{-1} + 0{,}08\, V z^{-2}$ $\qquad V = \frac{U}{1 - 0{,}2 z^{-1} - 0{,}08 z^{-2}}$

$Y = 3V + 2Vz^{-2} - Vz^{-1}$

$$Y = \underbrace{\frac{3 - z^{-1} + 2z^{-2}}{1 - 0{,}2 z^{-1} - 0{,}08 z^{-2}}}_{H(z)} U$$

$$H(z) = \frac{3z^2 - z + 2}{z^2 - 0{,}2z - 0{,}08}$$

rendszeregyenlet:

$$y(k) - 0{,}2y(k-1) - 0{,}08y(k-2) = 3u(k) - u(k-1) + 2u(k-2)$$

$(3z^2 - z + 2) : (z^2 - 0{,}2z - 0{,}08) = 3$

$-3z^2 - 0{,}6z - 0{,}24$

$-0{,}4z + 2{,}24$

$$H(z) = 3 + \frac{-0{,}4z + 2{,}24}{z^2 - 0{,}2z - 0{,}08} = \left(\frac{A}{z - 0{,}4} + \frac{B}{z + 0{,}2}\right) z \cdot z^{-1} + 3$$

$$h(k) = 3\delta(k) + \varepsilon(k-1)\left(A \cdot (0{,}4)^{k-1} + B(-0{,}2)^{k-1}\right)$$

$u(k)\ 4 \cdot 0{,}8^k (\varepsilon(k) - \varepsilon(k-6))$

$u(k)\ 4\ 0{,}8^k \varepsilon(k) - 4 \cdot 0{,}8^6 \cdot 0{,}8^{k-6} \varepsilon(k-6)$

$U(z) = \frac{4z}{z - 0{,}8}(1 - 0{,}8^6 \cdot z^{-6})$ $\qquad Y(z) = Y_1(z) \cdot (1 - 0{,}8^6 \cdot z^{-6})$

$$Y_1(z) = U(z) \cdot H(z) = z \cdot \frac{12z^2 - 4z + 8}{(z-0{,}8)(z-0{,}4)(z+0{,}2)} = z\left(\frac{31{,}2}{z - 0{,}8} + \frac{-34{,}7}{z - 0{,}4} + \frac{15{,}5}{z + 0{,}2}\right)$$

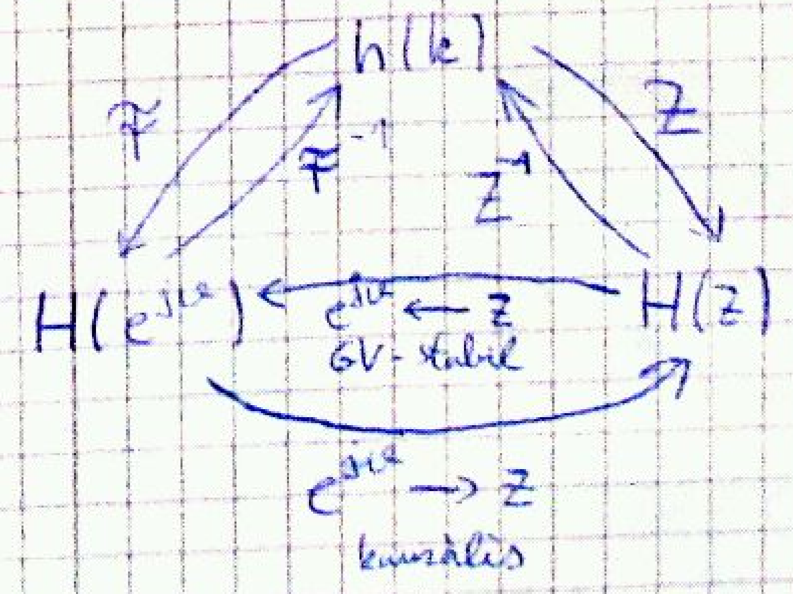
$$y_1(k) = \varepsilon(k)\left(31{,}2\,(0{,}8)^k - 34{,}7 \cdot (0{,}4)^k + 15{,}5 \cdot (-0{,}2)^k\right)$$

$$y(k) = \text{—}\|\text{—} - 0{,}8^6 \cdot \varepsilon(k-6)\left(31{,}2 \cdot 0{,}8^{k-6} - 34{,}7\ 0{,}4^{k-6} + 15{,}5\,(-0{,}2)^{k-6}\right.$$

$$\left.\frac{3 - e^{-j\vartheta} + 2e^{-2j\vartheta}}{1 - 0{,}2 e^{-j\vartheta} - 0{,}08 e^{-2j\vartheta}}\right|_{\vartheta = \frac{\pi}{2}} = \frac{3 + j - 2}{1 + 0{,}2j + 0{,}08} = 1{,}28 \cdot e^{j0{,}729}$$

$$y(k) = 4 \cdot 1{,}28 \cdot \cos\left(k\frac{\pi}{2} + 0{,}729\right)$$

Rendszert jellemző fv-ek



# Stabilitás:

$$X^N + a_1 X^{N-1} + a_2 X^{N-2} + \dots + a_{N-1} X + a_N = 0$$

(1) $a_i$ : $a_0 = 1$ $\quad i = 1 \dots N$

(2) $b_i = a_0 \cdot a_i - a_{N-i} \cdot a_N$ $\quad i = 0 \dots N-1$

(3) $c_i = b_0 \cdot b_i - b_{N-1-i} \cdot b_{N-1}$ $\quad i = 0 \dots N-2$

⋮

(N) $g_i = f_0 \cdot f_i - f_{2-i} f_2$ $\quad i = 0, 1$

$|a_N| < |a_0| = 1$

$|b_{N-1}| < |b_0|$

$|c_{N-2}| < |c_0|$

⋮

$|g_1| < |g_0|$

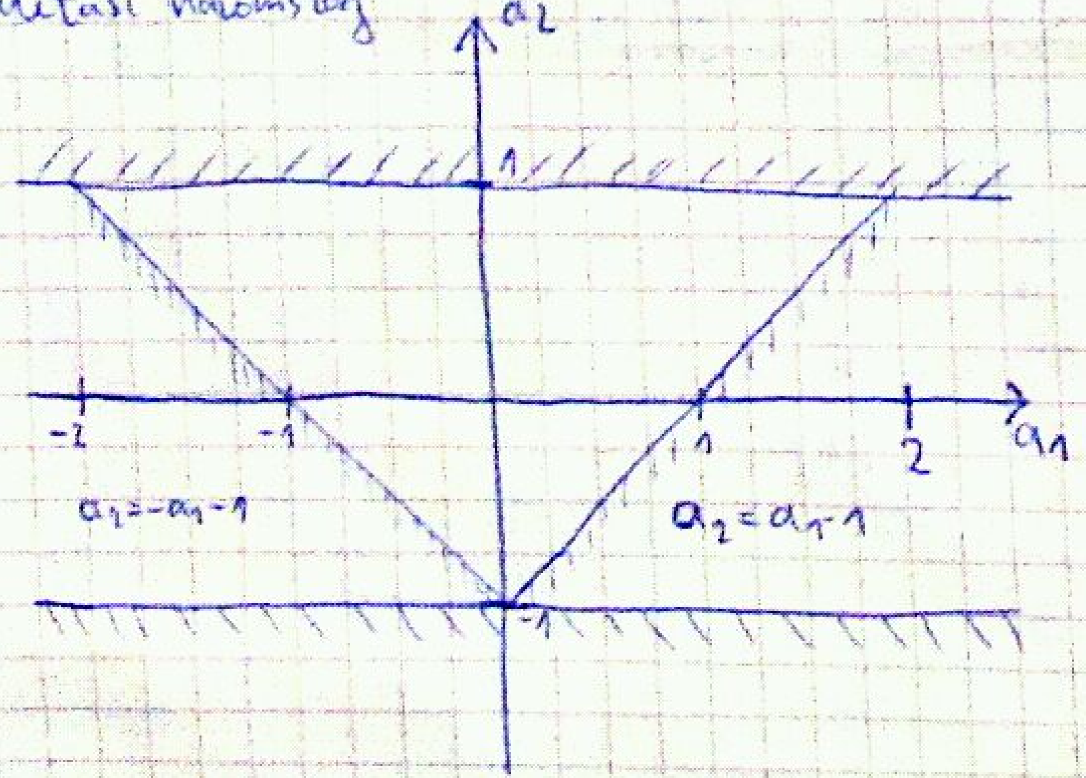
$N = 2$ $\quad b_0 = 1 - a_2^2 = (1-a_2)(1+a_2)$

$b_1 = a_1 - a_1 \cdot a_2 = a_1(1-a_2)$

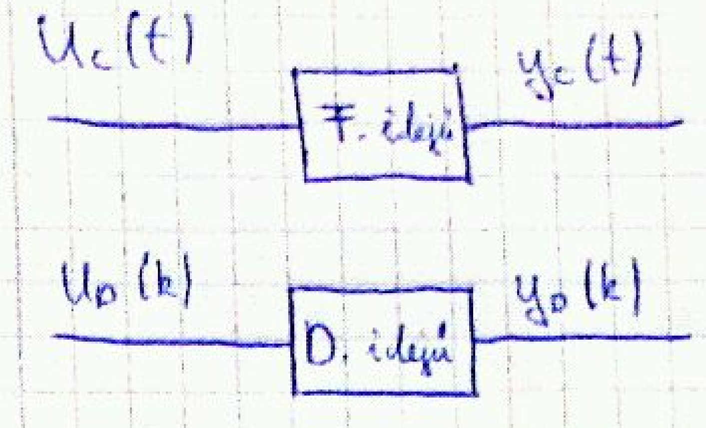
$|a_2| < 1$

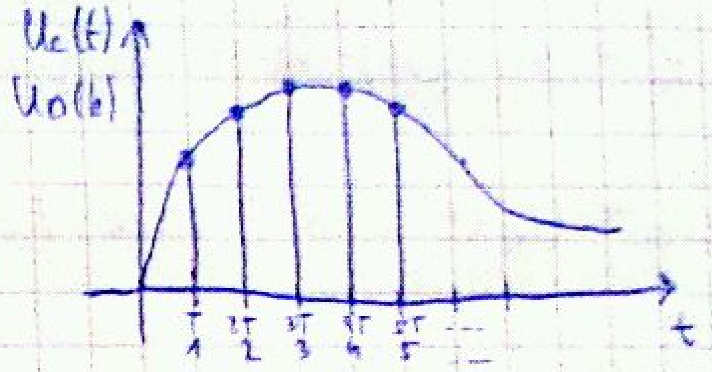
$|a_1| < 1 + a_2$

stabilitási háromszög




# Szimuláció:




$$u_D(k) = u_c(kT+0) \Rightarrow y_D(k) = y_c(kT+0)$$

$$y_c(t) = \int_{-\infty}^{t} h_c(t-\tau)\, u(\tau)\, d\tau$$

$$h_c(t) = D\delta(t) + \varepsilon(t) \cdot f(t)$$

$$y_c(kT+0) = \int_{-0}^{kT+0} D\delta(t)\, u_c(kT+0)\, dt + \int_{0}^{kT+0} f(t)\, u_c(kT+0-t)\, dt$$

$$y_D(k) = \sum_{i=0}^{k} h_D(i)\, u_D(k-i)$$

$$y_c(kT+0) \cong \begin{cases} D \cdot u_c(+0) \\ D \cdot u_c(kT+0) + T \sum_{i=1}^{k} f(iT)\, u_c(kT-iT+0) \end{cases}$$

$$y_c(t) = \int_0^t u_c(t)\, dt$$

$h_c(t) = \varepsilon(t)$ $\quad H_c(s) = \frac{1}{s}$

$h_D(k) = T \cdot \varepsilon(k-1)$ $\quad H_D(z) = T \cdot \frac{1}{z-1}$

$H_c(s)$ $\qquad\qquad$ $H_D(z)$

$u_c(t) = e^{j\omega t}$ $\qquad$ $u_D(k) = e^{j\omega kT} = e^{j\vartheta k}\Big|_{\vartheta = \omega T}$

$\bar{Y}_c = H_c(j\omega)\cdot \bar{u} = H(j\omega)\, e^{j\omega t}$

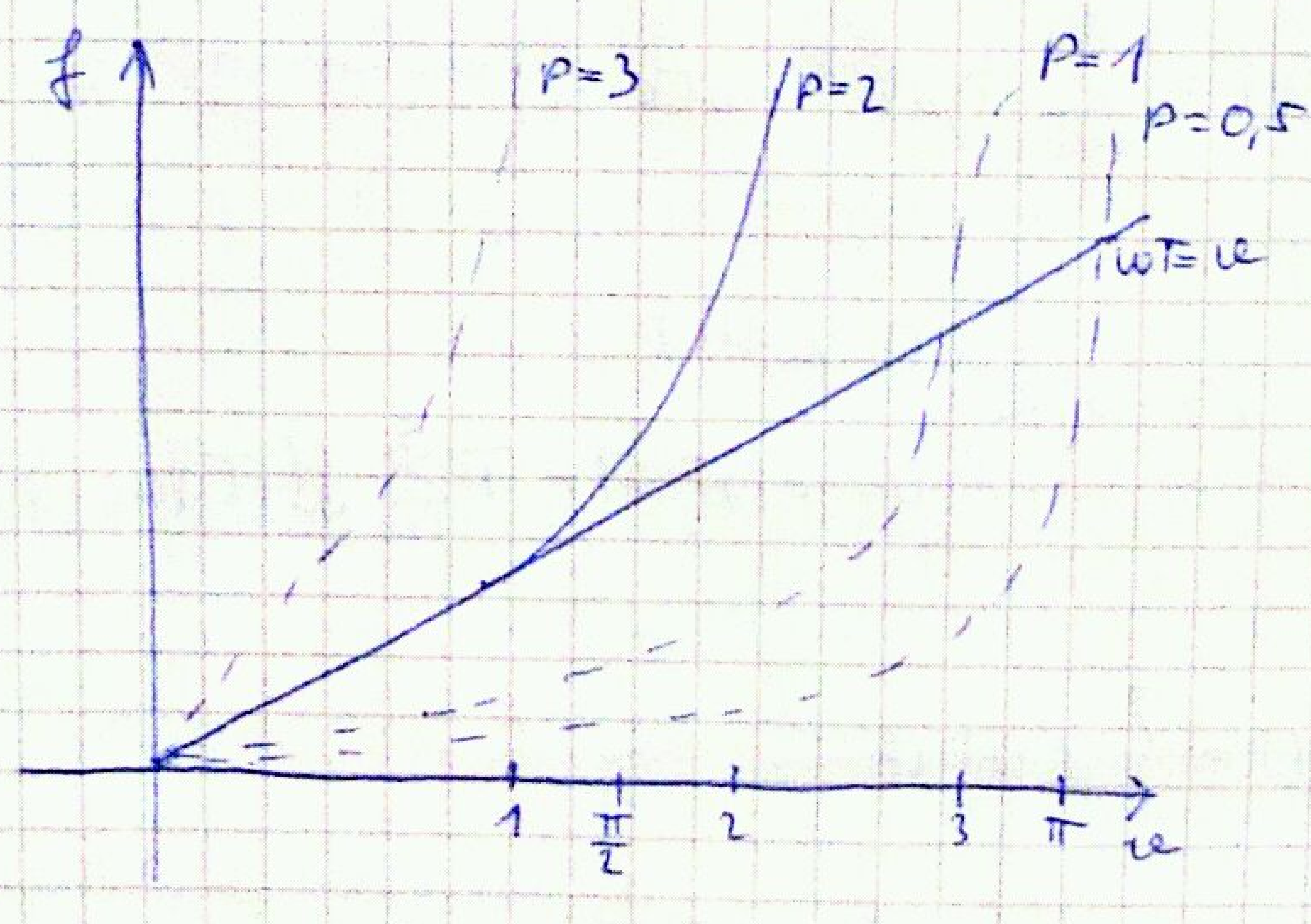
$\bar{Y}_D = H_D(e^{j\vartheta})\, \bar{u} = H(e^{j\vartheta})\cdot e^{j\vartheta k}$

$y_D(k) = y_c(kT)$

$H_D(e^{j\vartheta}) = H_c(j\omega)\Big|_{\omega = \frac{\vartheta}{T}} \Longrightarrow H_D(z) = H_c(s)\Big|_{s = \frac{\ln z}{T}}$

$s = \frac{p}{T}\cdot\frac{z-1}{z+1}$ $\qquad$ $z = \frac{1 + \left(\frac{sT}{p}\right)}{1 - \left(\frac{sT}{p}\right)}$

$H_{D_2}(z) = H_c(s)\Big|_{s = f(z)}$ $\qquad$ $f(z) = \frac{p}{T}\cdot\frac{z-1}{z+1}$




$H_c(s) = \frac{1}{s}$ $\qquad$ $H_D(z) = \frac{T}{2}\cdot\frac{(z+1)}{(z-1)}$ $\qquad$ $h_D(k) = \frac{T}{2}\varepsilon(k) + \frac{T}{2}\varepsilon(k-1) = T\left\{\frac{1}{2}\delta(k) + \varepsilon(k-1)\right\}$