Együttes idő-frekvencia reprezentáció

Wigner-transzformáció

$$W(\omega,t)=\int_{-\infty}^{\infty}\underbrace{x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)}_{\gamma(t,\tau)}e^{-j\omega\tau}\,d\tau=\mathcal{F}^{\tau}\left(\gamma(t,\tau)\right)$$

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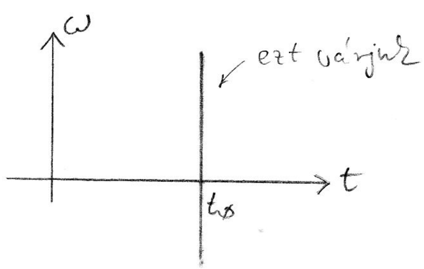
példa

$x(t)=\delta(t-t_\emptyset)$



$$W(\omega,t)=\int_{-\infty}^{\infty}\delta\left(t+\frac{\tau}{2}-t_\emptyset\right)\delta^*\left(t-\frac{\tau}{2}-t_\emptyset\right)e^{-j\omega\tau}\,d\tau$$

$$=\delta(t-\emptyset-t_\emptyset)\,e^{-j\omega\emptyset}=\delta(t-t_\emptyset)$$

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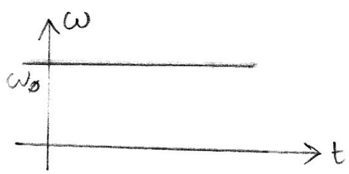
példa

$x(t)=e^{j\omega_\emptyset t}$



$$W(\omega,t)=\int_{-\infty}^{\infty}\cancel{e^{j\omega_\emptyset t}}\cdot e^{j\omega_\emptyset\frac{\tau}{2}}\cdot\cancel{e^{-j\omega_\emptyset t}}\,e^{j\omega_\emptyset\frac{\tau}{2}}\,e^{-j\omega\tau}\,d\tau$$

$$=\mathcal{F}^{\tau}\left\{e^{j\omega_\emptyset\tau}\right\}=2\pi\,\delta(\omega-\omega_\emptyset)$$

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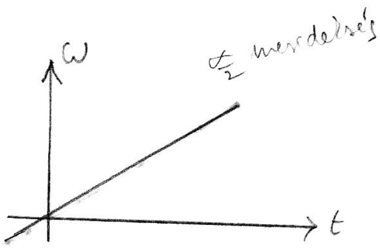
példa

$x(t)=e^{j\frac{\alpha}{2}t^2}$



$$W(\omega,t)=\int_{-\infty}^{\infty}e^{j\frac{\alpha}{2}\left(t+\frac{\tau}{2}\right)^2}e^{-j\frac{\alpha}{2}\left(t-\frac{\tau}{2}\right)^2}e^{-j\omega\tau}\,d\tau$$

$$=\mathcal{F}^{\tau}\left\{e^{j\alpha t\tau}\right\}=2\pi\,\delta(\omega-\alpha t)$$