1. $y''(x^2+1) = 2xy'$ $\quad y' = z$ (1P)

$z'(x^2+1) = 2x\cdot z$ SZEPARÁLHATÓ

$\frac{f'}{f}$ (1P)

$z'\cdot\frac{1}{z} = \frac{2x}{x^2+1}$ $\quad z \equiv 0 \Rightarrow y = C$ MEGOLDÁS (1P)

$\ln|z| = \ln(x^2+1) + c$ (1P)

$|z| = (x^2+1)\cdot K_1$ (1P)

$z = (x^2+1)\cdot K_2$ $\quad K_2 \neq 0$ (1P)

VAGY-VAGY

$y' = (x^2+1)\cdot K_2 \Rightarrow \boxed{y = \left(\frac{x^3}{3} + x\right)\cdot K_2 + K_3}$ (1P) (1P)

7PONT

2. $\sin^7 x \cdot y' - \frac{\sin^8 x}{\cos x} y = 1$

$\sin^7 x \neq 0$
LEOSZTUNK (1P)

$$y' - \frac{\sin x}{\cos x} y = \frac{1}{\sin^7 x}$$

$N = e^{\int p(x)dx} = e^{\int \frac{-\sin x}{\cos x}dx} = e^{\ln|\cos x|} =$ (1P) (1P)

$= \cos x$ (1P)

$(y\cdot N)' = \frac{1}{\sin^7 x}\cdot \cos x$ (1P)

$y\cdot N = \int \cos x \cdot \sin^{-7} x \, dx$ (1P)

$y\cdot N = \frac{\sin^{-6} x}{-6} + C$ (1P) (1P)

HOMOGÉN EGYENLET

$y' = \frac{\sin x}{\cos x} y$ $\quad y \equiv 0$ MO

$\int \frac{1}{y} dy = \int \frac{\sin x}{\cos x} dx$ (1P)

$\ln|y| = -\ln|\cos x| + c$ (1P)

$y_H = \frac{K}{\cos x}$ $\quad K \neq 0$ (1P)

DE INNEN $K=0$ IS.

INHOMOGÉN MO:

$y = K(x)\cdot\frac{1}{\cos x}$ (1P)

$$y=\frac{\sin^{-6}x}{-6\cdot\cos x}+\frac{C}{\cos x} \quad (1P)$$

$$K'(x)\frac{1}{\cos x}+\cancel{K(x)\left(\frac{1}{\cos x}\right)'-\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}K(x)}=\frac{1}{\sin^7 x} \quad (1P)$$

$$K'(x)=\frac{\cos x}{\sin^7 x}$$

$$K(x)=\int\cos x\cdot\sin^{-7}x\,dx \quad (1P)$$

$$K(x)=\frac{\sin^{-6}x}{-6} \quad (1P)$$

$$y=K(x)\cdot\frac{1}{\cos x}+\frac{K}{\cos x}=\frac{\sin^{-6}x}{-6\cos x}+\frac{K}{\cos x} \quad (1P)$$

KEZDETIÉRTÉK-PROBLÉMA MO.

$$y\left(\frac{\pi}{4}\right)=1736$$

$$1736=\frac{\sin^{-6}\left(\frac{\pi}{4}\right)}{-6\cdot\cos\left(\frac{\pi}{4}\right)}+\frac{C}{\cos\left(\frac{\pi}{4}\right)} \quad (1P)$$

$$1736=\frac{\sqrt{2}^{6}}{-6\cdot\frac{1}{\sqrt{2}}}+\frac{C}{\frac{1}{\sqrt{2}}}$$

$$1736=\frac{\sqrt{2}^{7}}{-6}+C\cdot\sqrt{2} \quad (1P)$$

$$C=\frac{1}{\sqrt{2}}\left(1736+\frac{4\sqrt{2}}{3}\right) \quad (1P)$$

$\boxed{12\ \text{PONT}}$

$\boxed{3.}$

$$y''-6y'+9y=2\operatorname{ch}3x$$

$$\lambda^2-6\lambda+9=0 \quad (1P)$$

$$\lambda_1=\lambda_2=3 \quad (1P)$$

$$y_H=C_1\cdot e^{3x}+C_2\cdot x\cdot e^{3x}$$

(1P) (1P)

PARTIKULÁRIS MEGOLDÁSNÁL KÜLSŐ REZONANCIA MIVEL

$$2\operatorname{ch}3x=2\,\frac{e^{3x}+e^{-3x}}{2}=e^{3x}+e^{-3x} \quad (1P)$$

$$y_{IP}=x^2e^{3x}\cdot A+e^{-3x}\cdot B$$

(1P) (1P)

$$y' = 2Ax \cdot e^{3x} + Ax^2 \cdot 3e^{3x} + B(-3)e^{-3x}$$ (1P)

$$y'' = 2Ae^{3x} + 6Ax \cdot e^{3x} + 6Ax \cdot e^{3x} + 9Ax^2 e^{3x} + 9Be^{-3x}$$ (2P)

$$y'' - 6y' + 9y = e^{3x} + e^{-3x}$$

$2A$	1	e^{3x}	(1P)
$9B + 18B + 9B$	1	e^{-3x}	(1P)
0	0	$x \cdot e^{3x}$	(1P)
0	0	$x^2 e^{3x}$	

$2A = 1 \Rightarrow A = \frac{1}{2}$ (1P)

$36B = 1 \Rightarrow B = \frac{1}{36}$ (1P)

$$y = C_1 \cdot e^{3x} + C_2 \cdot x \cdot e^{3x} + \frac{1}{2}x^2 e^{3x} + \frac{1}{36}e^{-3x}$$ (1P)

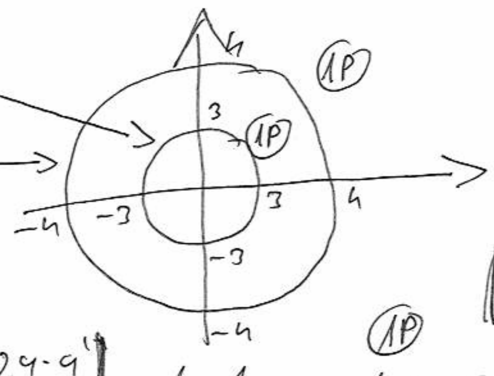
$\boxed{\text{16 PONT}}$

$\boxed{4.}$ $y' = \sqrt{x^2 + y^2} - 3$

(1P)

HA $k=0$ AKKOR $x^2 + y^2 = 9$

HA $k=1$ AKKOR $x^2 + y^2 = 16$ (1P)



$\boxed{\text{8 PONT}}$

(2P) (1P)

$P(3;0)$: $y'' = \frac{1}{2}(\underset{9}{x^2} + \underset{0}{y^2})^{-\frac{1}{2}} \cdot (\underset{6}{2x} + \underset{0}{2y \cdot y'}) = \frac{1}{2} \cdot \frac{1}{3} \cdot 6 = 1 > 0$

LOK MIN ← (1P)

$\boxed{5.}$

a) $\sum_{n=1}^{\infty} \sqrt{\frac{n+2}{9n+7}}$

$\lim_{n\to\infty} \sqrt{\frac{n+2}{9n+7}} = \lim_{n\to\infty} \sqrt{\frac{1+\frac{2}{n}}{9+\frac{7}{n}}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$ (1P)

(1P)

MIVEL $\lim a_n \neq 0$ EZÉRT $\sum a_n$ DIV (1P)

b)

$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^8+1}{n^{81}}$

KONVERGENS, MERT LEIBNIZ-SOR (1P)

$\rightarrow$ VÁLTAKOZÓ ELŐJELŰ (1P)

$\rightarrow \frac{n^8+1}{n^{81}} \to 0$ (1P)

$\rightarrow \frac{n^8+1}{n^{81}} = \frac{n^8}{n^{81}} + \frac{1}{n^{81}} = \underbrace{\frac{1}{n^{73}}}_{\text{CSÖKK}} + \underbrace{\frac{1}{n^{81}}}_{\text{CSÖKK}}$ (1P)

$\boxed{\text{7 PONT}}$