DIFFERENCIÁLSZÁMÍTÁS – eredmények

(1) Differenciálhatóság.

1. $a=3;\ b=-2$

2. $f\,'(x)=\sin\frac{1}{x}-\frac{1}{x}\cos\frac{1}{x},\ \text{ha } x\neq 0$

3. $f\,'(x)=\begin{cases}4x^3\sin\frac{1}{x}-x^2\cos\frac{1}{x},\ \text{ha } x\neq 0\\ 0,\ \text{ha } x=0\end{cases}$; $f\,'$ folytonos $\mathbb{R}$-en

4. $f\,'(x)=\arctan\frac{1}{x}-\frac{x}{1+x^2},\ \text{ha } x\neq 0$

5. $f\,'(2)=\lim\limits_{x\to 2}\frac{\sqrt{2x-3}-1}{x-2}=1$

(2) Deriváltak meghatározása a differenciálási szabályok segítségével.

1. $f\,'(x)=\frac{3\cos(3x+2)\sqrt{1+x^2}-\frac{x}{\sqrt{1+x^2}}\sin(3x+2)}{1+x^2}$

2. $f\,'(x)=-\sin x\cdot e^{\cos x}\left(x^{\frac{3}{2}}+x^{-4}\right)+e^{\cos x}\left(\frac{3}{2}x^{\frac{1}{2}}-4x^{-5}\right)$

3. $f\,'(x)=2\ln(5x+6)\cdot\frac{5}{5x+6}$

4. $f\,'(x)=\frac{1}{2}\left(\arcsin\frac{1}{x}\right)^{-\frac{1}{2}}\cdot\frac{1}{\sqrt{1-\frac{1}{x^2}}}\cdot\left(-\frac{1}{x^2}\right)$

5. $f\,'(x)=3^{5x+2}\cdot 5\ln 3\cdot\tan^3\left(1+x^2\right)+3^{5x+2}\cdot 3\tan^2\left(1+x^2\right)\cdot\frac{2x}{\cos^2\left(1+x^2\right)}$

6. $f\,'(x)=\frac{\cosh x}{1+\sinh x}\cdot\frac{1}{\ln 3}$

7. $f\,'(x)=\left(e^{x\ln x}\right)'=e^{x\ln x}\cdot(\ln x+1)$

8. $f\,'(x)=e^{\sin x\cdot\ln(1+x)}\cdot\left[\cos x\cdot\ln(1+x)+\frac{\sin x}{1+x}\right]$

9. $f\,'(x)=e^{(2+3x)\ln(1+\cosh x)}\cdot\left[3\ln(1+\cosh x)+(2+3x)\cdot\frac{\sinh x}{1+\cosh x}\right]$

(3) Igazolja az alábbi egyenlőtlenségeket!

1. $f(x)=\sqrt{1+x};\ [a,b]=[0,x];\ x>0$

2. $f(x)=\arctan x;\ [a,b]=[0,x];\ x>0$

3. $f(x)=\ln x;\ [a,b]=[1,x];\ x>1$

1, 2, 3: Lagrange–féle középértéktétel alkalmazása.

4. $f(x)=\mathrm{e}^x-1-x-\frac{x^2}{2}$ monotonitásának vizsgálata $(0,\infty)$-en.

5. $f(x)=x-\sin x$ és $g(x)=\sin x-x+\frac{x^3}{6}$ monotonitásának vizsgálata $(0,\infty)$-en.

6. $f(x)=\arctan x-x+\frac{x^3}{3}$ monotonitásának vizsgálata $(0,\infty)$-en.

(4) Számolja ki az alábbi határértékeket!

1. $-\frac{1}{12}$	**2.** 2	**3.** 0
4. 1	**5.** $\lim_{x\to 0^+} f(x)=\infty;\ \lim_{x\to 0^-} f(x)=0$	**6.** 0
7. 0	**8.** $\mathrm{e}^{-\frac{1}{2}}$	**9.** e

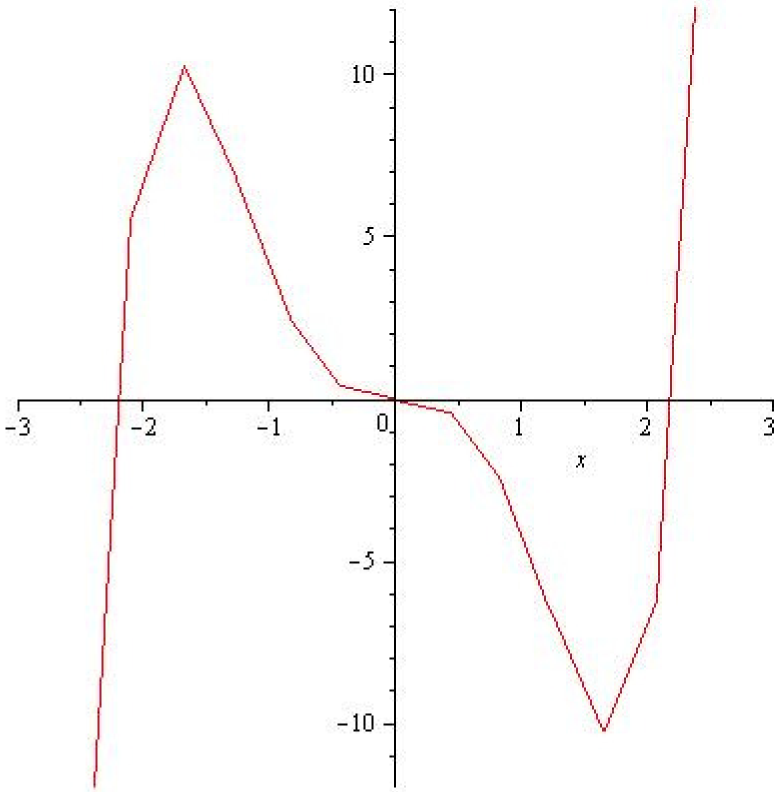
(5) Ábrázolja az alábbi függvényeket!

1.

$f(x) = x^5 - 5x^3$ Do f = Rg $f = \mathbb{R}$ $\lim_{x \to \pm\infty} f(x) = \pm\infty$

$f'(x) = 5x^2\left(x^2 - 3\right)$

$f''(x) = 10x\left(2x - 3\right)$



2.

$f(x) = x^2 - 2\ln x$

$\text{Do } f = (0, \infty)$

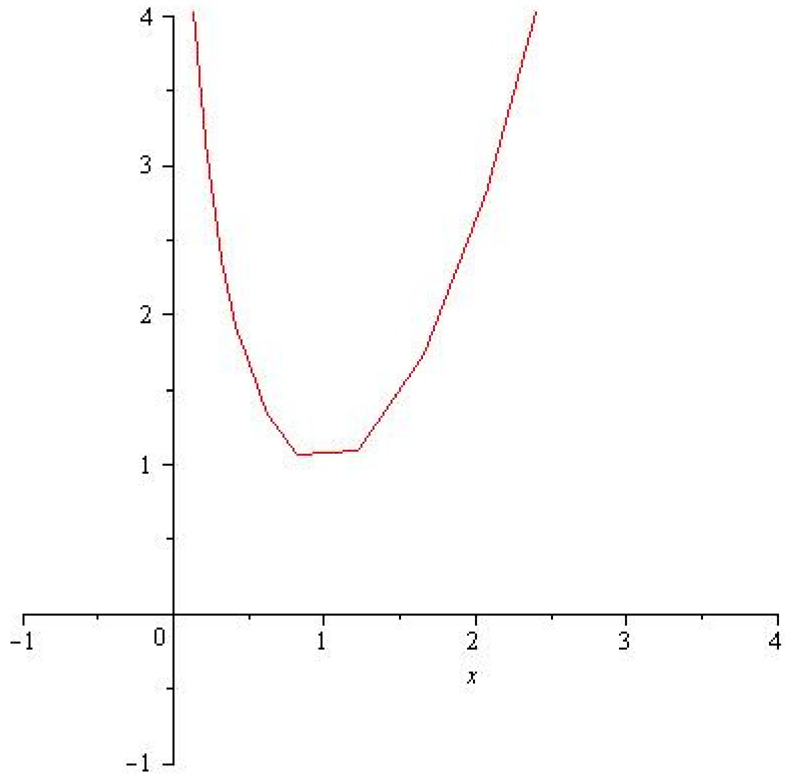
$\lim_{x \to 0^+} f(x) = \infty$

$f'(x) = 2\frac{x^2 - 1}{x}$

$\text{Rg } f = [1, \infty)$

$\lim_{x \to \infty} f(x) = \infty$

$f''(x) = 2 + \frac{2}{x^2}$



3.

$$f(x)=\frac{x}{(x-1)^2}$$

$$f'(x)=\frac{-x-1}{(x-1)^3}$$

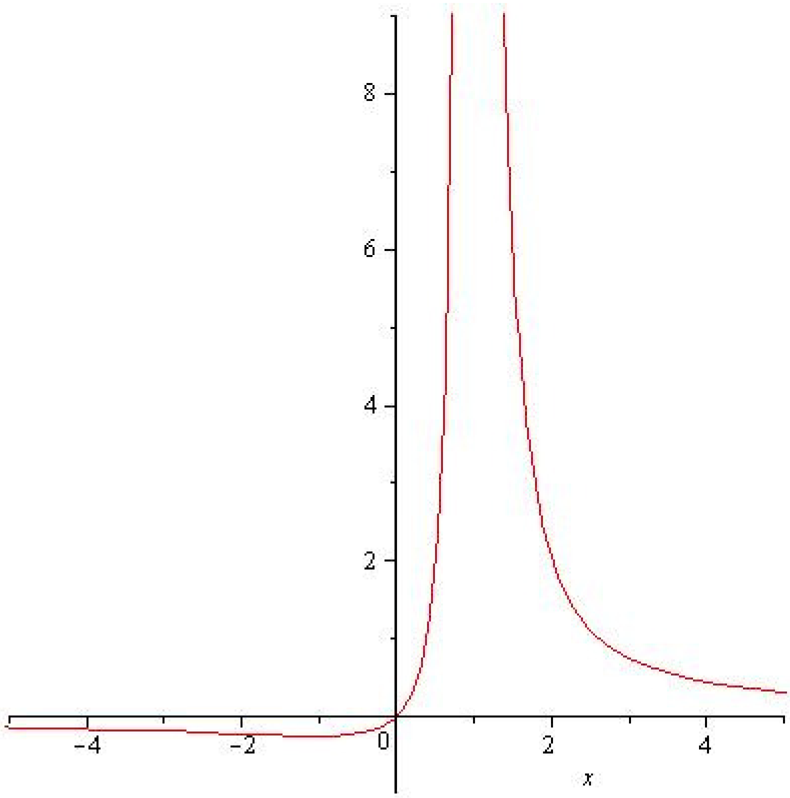
$$f''(x)=\frac{2x+4}{(x-1)^4}$$

$$\text{Do}\, f=\mathbb{R}\setminus\{1\}$$

$$\text{Rg}\, f=\left[-\frac{1}{4},\infty\right)$$

$$\lim_{x\to\pm\infty} f(x)=0$$

$$\lim_{x\to 1^{+}_{-}} f(x)=\infty$$



4.

$$f(x)=\frac{x^2}{(x-1)^2}$$

$$f'(x)=\frac{-2x}{(x-1)^3}$$

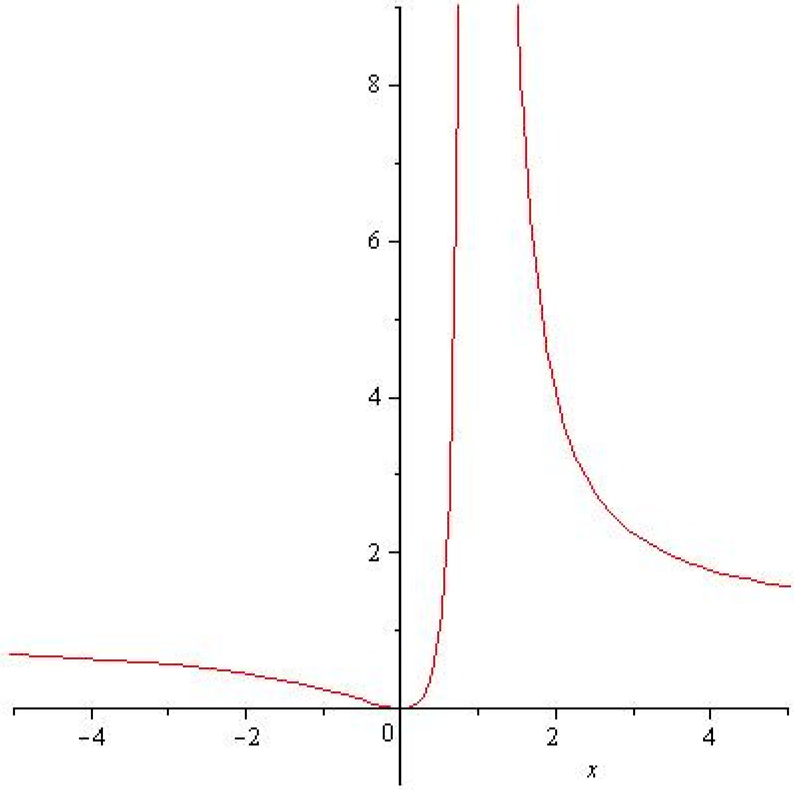
$$f''(x)=\frac{4x+2}{(x-1)^4}$$

$\mathrm{Do}\, f=\mathbb{R}\setminus\{1\}$

$\mathrm{Rg}\, f=[0,\infty)$

$\lim_{x\to\pm\infty} f(x)=1$

$\lim_{x\to 1^{+}_{-}} f(x)=\infty$



5.

$f(x)=\dfrac{x^3}{(x-1)^2}$

Do $f=\mathbb{R}\setminus\{1\}$

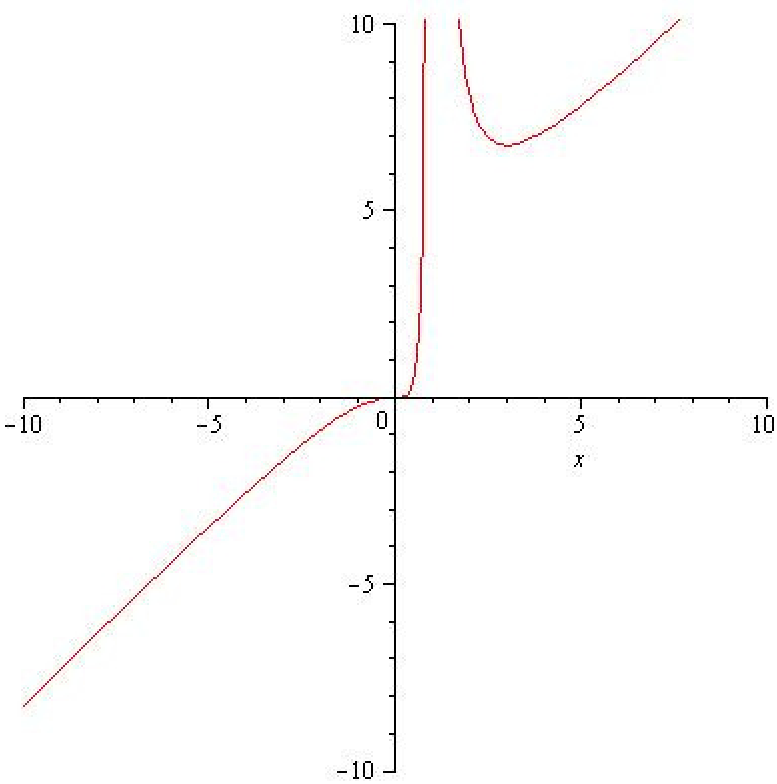
Rg $f=\mathbb{R}$

$\lim\limits_{x\to\pm\infty} f(x)=\pm\infty$

$\lim\limits_{x\to 1^{+}_{-}} f(x)=\infty$

$f'(x)=\dfrac{x^3-3x^2}{(x-1)^3}$

$f''(x)=\dfrac{6x}{(x-1)^4}$



6.

$$f(x)=\frac{1-\ln x}{x^2}$$

$$f'(x)=\frac{-3+2\ln x}{x^3}$$

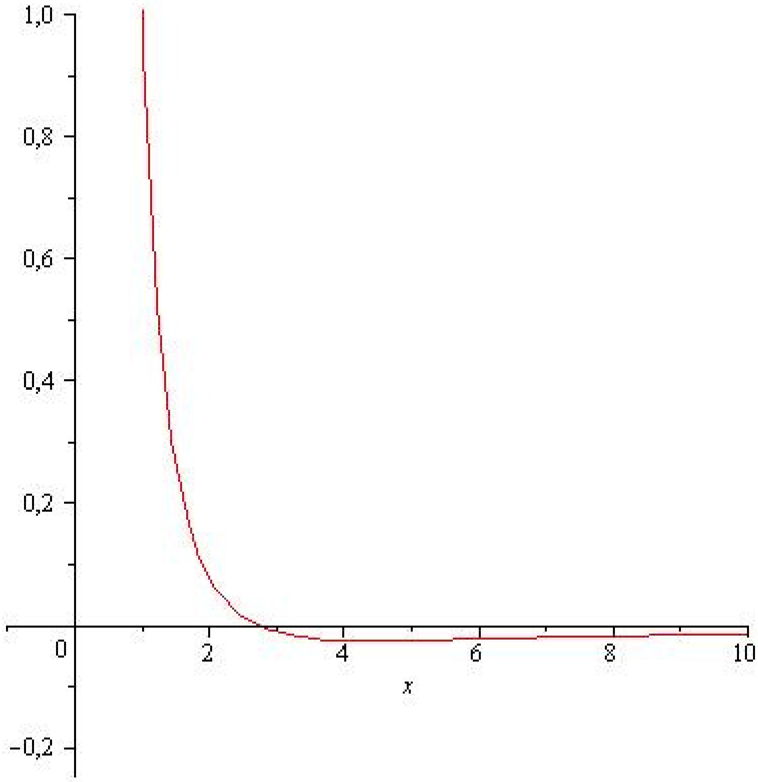
$$f''(x)=\frac{11-6\ln x}{x^4}$$

$$\text{Do } f=(0,\infty)$$

$$\text{Rg } f=\left[-\frac{1}{2e^3},\infty\right)$$

$$\lim_{x\to\infty} f(x)=0$$

$$\lim_{x\to 0^+} f(x)=\infty$$



7.

$f(x) = (x-6)\mathrm{e}^{-x}$

$f'(x) = \mathrm{e}^{-x}(-x+7)$

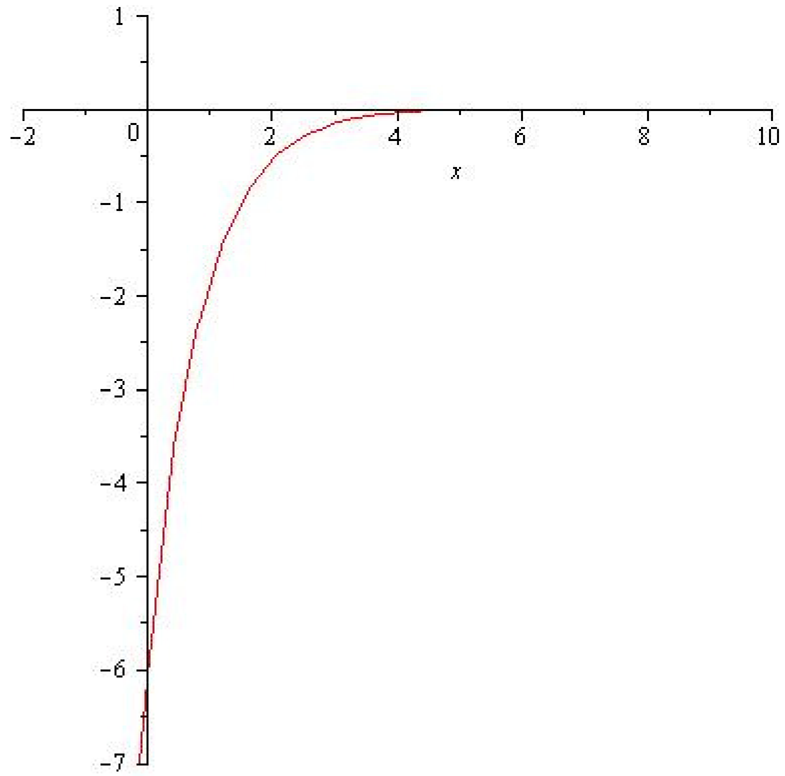
$f''(x) = \mathrm{e}^{-x}(x-8)$

$\mathrm{Do}\, f = \mathbb{R}$

$\mathrm{Rg}\, f = \left(-\infty, \mathrm{e}^{-7}\right]$

$\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = -\infty$



8.

$$f(x) = \mathrm{e}^{\frac{1}{x}}$$

$$f'(x) = -\frac{1}{x^2}\mathrm{e}^{\frac{1}{2}}$$

$$f''(x) = \frac{2x+1}{x^4}\mathrm{e}^{\frac{1}{x}}$$

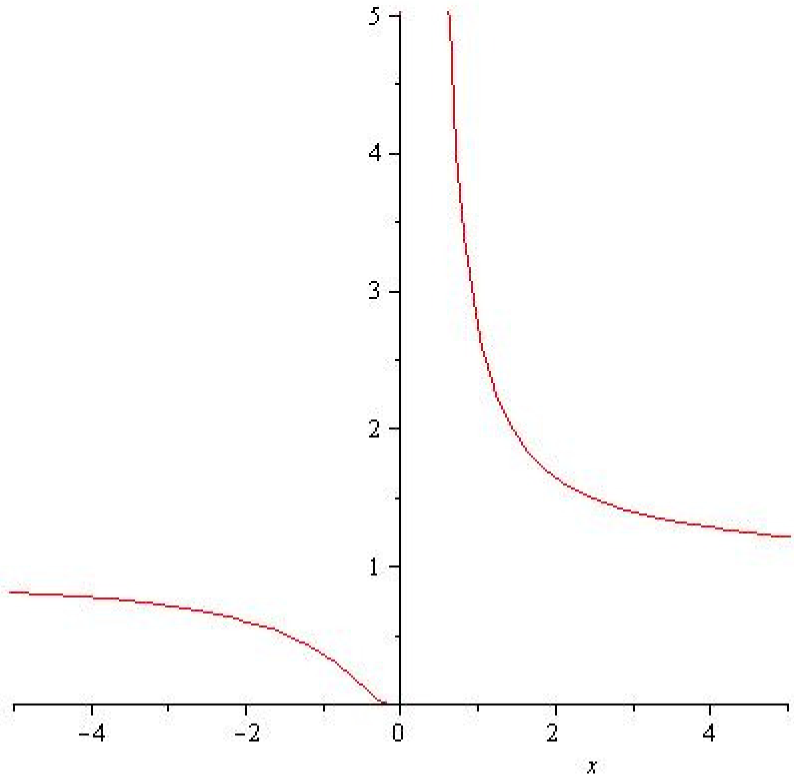
$\mathrm{Do}\, f = \mathbb{R} \setminus \{0\}$

$\mathrm{Rg}\, f = (0, \infty) \setminus \{1\}$

$$\lim_{x \to \pm\infty} f(x) = 1$$

$$\lim_{x \to 0^+} f(x) = \infty$$

$$\lim_{x \to 0^-} f(x) = 0$$



9.

$f(x) = x e^{\frac{1}{x}}$

$f'(x) = e^{\frac{1}{x}}\left(1 - \frac{1}{x}\right)$

$f''(x) = \frac{1}{x^3} e^{\frac{1}{x}}$

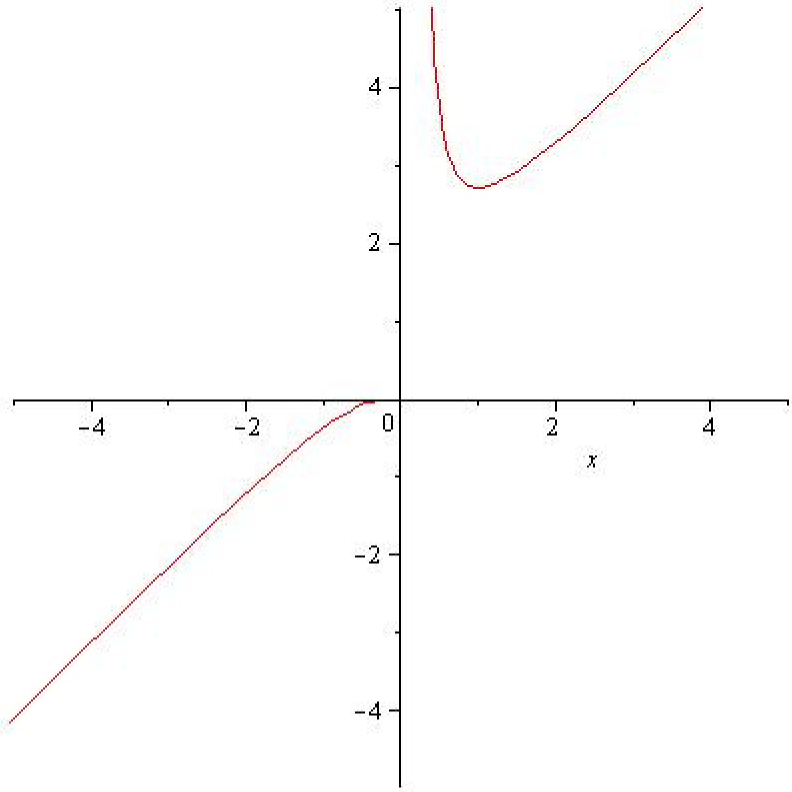
$\mathrm{Do}\, f = \mathbb{R} \setminus \{0\}$

$\mathrm{Rg}\, f = (-\infty, 0)$ és $[e, \infty)$

$\lim_{x\to\pm\infty} f(x) = \pm\infty$

$\lim_{x\to 0^+} f(x) = \infty$

$\lim_{x\to 0^-} f(x) = 0$



10.

a)

$f(x) = x^2 \ln x$

$f'(x) = x(2\ln x + 1)$

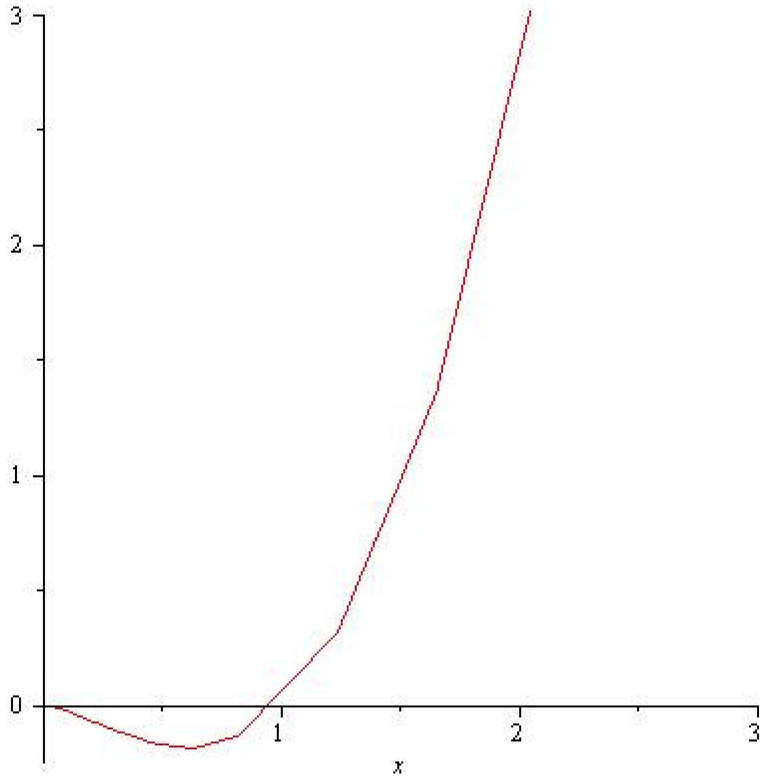
$f''(x) = 2\ln x + 3$

$\mathrm{Do}\, f = (0, \infty)$

$\mathrm{Rg}\, f = \left[-\frac{1}{2\mathrm{e}}, \infty\right)$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to 0^+} f(x) = 0$



b)

$x^2 \ln x = k$

nincs megoldás, ha: $k < -\frac{1}{2\mathrm{e}}$

$\exists$ egy megoldás, ha: $k = -\frac{1}{2\mathrm{e}} \vee k \geq 0$

$\exists$ két megoldás, ha: $-\frac{1}{2\mathrm{e}} < k < 0$

c)

$\exists$ inverz függvény a következő intervallumokon: $\left(0, \mathrm{e}^{-\frac{1}{2}}\right)$; $\left(\mathrm{e}^{-\frac{1}{2}}, \infty\right)$

11.

a)

$f(x) = xe^{-x}$

$f'(x) = e^{-x}(1-x)$

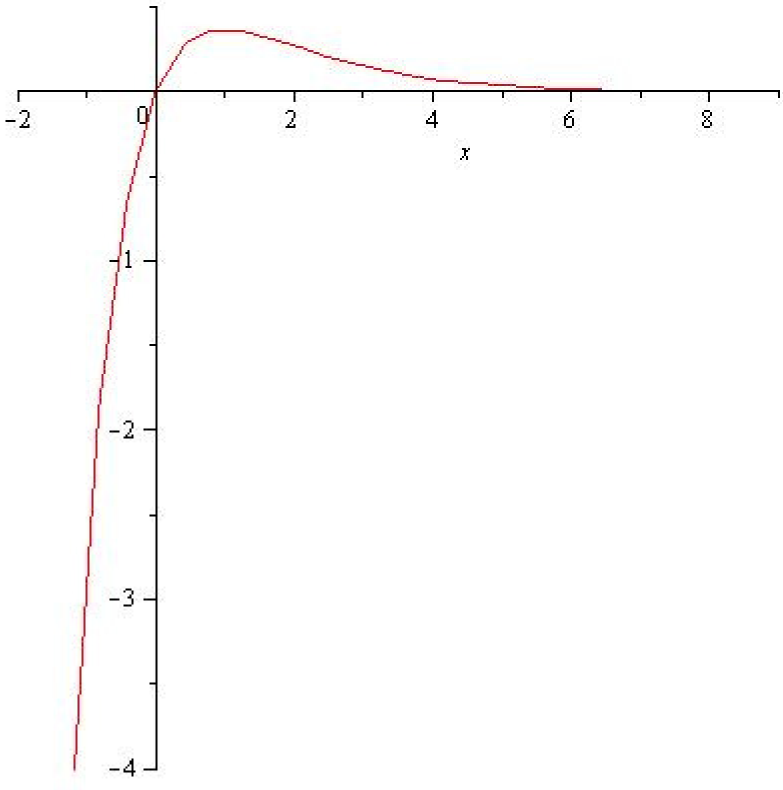
$f''(x) = e^{-x}(x-2)$

$\mathrm{Do}\, f = \mathbb{R}$

$\mathrm{Rg}\, f = \left(-\infty, e^{-1}\right]$

$\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = -\infty$



b)

$\exists$ inverz függvény a következő intervallumokon: $(-\infty,1)$; $(1,\infty)$

c)

$\max_{[0,5]} f(x) = f(1) = \frac{1}{e}$

$\max_{[0,\infty)} f(x) = \frac{1}{e}$

$\max_{(0,\infty)} f(x) = \frac{1}{e}$

$\min_{[0,5]} f(x) = f(0) = 0$

$\min_{[0,\infty)} f(x) = 0$

$\min_{(0,\infty)} f(x)$ nem létezik