Info anal2 2. Zh. MINTAMEGOLDÁS (α - verzió) 2013. 04. 18.

$\boxed{-1-}$

1, $\boxed{10}$

$$\sum_{n=0}^{\infty} \underbrace{\frac{(-1)^n}{\sqrt{n}\cdot 3^{2n}}}_{a_n} (x-2)^n \qquad x_0 = 2$$

$$\sqrt[n]{|a_n|} = \frac{1}{\sqrt{\underbrace{\sqrt[n]{n}}_{\downarrow\, 1}}\cdot 3^2} \xrightarrow{n\to\infty} \frac{1}{9} \implies R = 9 \quad 5$$

Végpontok:

$x = x_0 - 9$ -ben $\displaystyle\sum_{n=0}^{\infty} \frac{\cancel{(-1)^n}}{\sqrt{n}\;\cancel{3^{2n}}} \cancel{(-9)^n} = \sum_{n=0}^{\infty} \frac{1}{n^{1/2}} = \infty$ 2 $\left(\frac{1}{2} \le 1\right)$

$x = x_0 + 9$ -ben $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n}\;\cancel{3^{2n}}} \cancel{9^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^{1/2}}$ konvergens, 2 mert Leibniz típusú

Tehát a konvergenciatartomány: $KT = (-7, 11]$ 1

2, a, $\boxed{3}$ $\displaystyle f(x) = \sin(3x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} 3^{2n+1} x^{4n+2}$, $\underline{x \in \mathbb{R}}$

b, $\boxed{4}$ $\displaystyle g(x) = \operatorname{ch}(x+2) = \sum_{n=0}^{\infty} \frac{1}{(2n)!} (x+2)^{2n}$, $\underline{x \in \mathbb{R}}$

c, $\boxed{8}$ $\displaystyle g(x) = \frac{1}{2} e^{x+2} + \frac{1}{2} e^{-x-2} = \frac{e^2}{2} e^x + \frac{e^{-2}}{2} e^{-x}$ 4 $=$

$$= \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{e^2}{2} + (-1)^n \frac{e^{-2}}{2}\right) x^n \quad 3 \qquad , \ \underline{x \in \mathbb{R}} \ 1$$

d, $\boxed{7}$ $\displaystyle h(x) = \frac{1}{2+3x} = \frac{1}{2}\cdot\frac{1}{1-\left(\frac{-3x}{2}\right)} = \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{-3}{2}\right)^n x^n$, 5 ha $\left|\frac{-3x}{2}\right| < 1$,

azaz $\underline{|x| < \frac{2}{3}}$ 2

2, (folyt.)

$\boxed{7}$ e, $h(x)=\frac{1}{2+3x}=\frac{1}{2+3(x-1)+3}=\frac{1}{5}\cdot\frac{1}{1-\left(\frac{-3}{5}(x-1)\right)}=$

$=\frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{-3}{5}\right)^{n}(x-1)^{n}$, ⑤ ha $\left|\frac{-3}{5}(x-1)\right|<1$, ②

azaz $|x-1|<\frac{5}{3}$, $\underline{x\in\left(-\frac{2}{3},\frac{8}{3}\right)}$

———∘———

3, a, $f(x)=\frac{1}{\sqrt[5]{2+7x^{3}}}=\left(2\cdot\left(1+\frac{7}{2}x^{3}\right)\right)^{-1/5}=\frac{1}{\sqrt[5]{2}}\sum_{n=0}^{\infty}\binom{-1/5}{n}\left(\frac{7}{2}\right)^{n}x^{3n}$, ⑤

$\boxed{8}$

ha $\left|\frac{7}{2}x^{3}\right|<1$, azaz $|x|<\sqrt[3]{\frac{2}{7}}=R$ ③

b, $\boxed{6}$ $f^{(9)}(0)=9!\cdot a_{9}=9!\cdot\frac{1}{\sqrt[5]{2}}\cdot\binom{-1/5}{3}\left(\frac{7}{2}\right)^{3}=\frac{9!}{\sqrt[5]{2}}\cdot\frac{\left(-\frac{1}{5}\right)\left(-\frac{6}{5}\right)\left(-\frac{11}{5}\right)}{3\cdot 2}\cdot\left(\frac{7}{2}\right)^{3}$ ④

$9=3n\Rightarrow n=3$

$f^{(10)}(0)=10!\cdot a_{10}=0$, mert $10\neq 3n$, ha $n\in\mathbb{N}$. ②

———∘———

4, a, $x=\rho\cos\varphi$; $y=\rho\sin\varphi$

$\boxed{6}$ $\lim_{(x,y)\to(0,0)}f(x,y)=\lim_{\rho\to 0+}\frac{\rho^{\cancel{3}}\cos^{3}\varphi+2\rho^{\cancel{3}}\sin^{3}\varphi+\cancel{\rho^{2}}\sin\varphi\cos\varphi}{\cancel{\rho^{2}}}=$

$=0+0+\sin\varphi\cos\varphi$ függ φ-től, tehát <u>nem létezik</u> a határérték.

$\boxed{7}$ b, $f'_{x}(0,0)=\lim_{x\to 0}\frac{f(x,0)-f(0,0)}{x}=\lim_{x\to 0}\frac{\frac{x^{3}+0+0}{x^{2}+0}-0}{x}=1$ ③

$$f'_y(0,0) = \lim_{y \to 0} \frac{f(0,y) - f(0,0)}{y} = \lim_{y \to 0} \frac{\frac{0 + 2y^3 + 0}{y^2} - 0}{y} = 2 \quad ②$$

A gradiens az origóban nem létezik, mert a függvény nem folytonos. (Határértéke sem létezik.) ②

5, $f(x,y) = \sqrt{2x^2 + y^2}$

a, [8]

$$f'_x(x,y) = \frac{4x}{2\sqrt{2x^2+y^2}} = \frac{2x}{\sqrt{2x^2+y^2}} \quad ②$$

$$f'_y(x,y) = \frac{2y}{2\sqrt{2x^2+y^2}} = \frac{y}{\sqrt{2x^2+y^2}} \quad ②$$

$(x,y) \neq (0,0)$ esetén f'_x, f'_y $\exists$ és folytonos $\Rightarrow$ $(x,y) \neq (0,0)$ esetén $\exists \operatorname{grad} f(x,y) = \begin{bmatrix} f'_x(x,y) \\ f'_y(x,y) \end{bmatrix}$ ②

Az origóban $\nexists \operatorname{grad} f$, mert itt a parciális deriváltak sem léteznek. ($f(x,0) = \sqrt{2}\,|x|$, aminek törése van $x=0$-ban.) ②

b, [6] Általában $z - z_0 = f'_x(x_0,y_0)(x-x_0) + f'_y(x_0,y_0)(y-y_0)$ ②

Most $z_0 = f(2,1) = \sqrt{2\cdot 2^2 + 1^2} = 3$ ①, $f'_x(2,1) = \frac{4}{3}$ ①, $f'_y(2,1) = \frac{1}{3}$ ①

Tehát $z - 3 = \frac{4}{3}(x-2) + \frac{1}{3}(y-1)$ ①

c, [4] $\underline{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$; $\|\underline{v}\| = 5$; $\underline{e} = \frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ ①

$$\left.\frac{df}{d\underline{e}}\right|_{(2,1)} = \underline{e} \cdot \operatorname{grad} f(2,1) \overset{②}{=} \frac{1}{5} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 4/3 \\ 1/3 \end{bmatrix} = \frac{1}{5}\left(\frac{12}{3} + \frac{4}{3}\right) = \underline{\underline{\frac{16}{15}}} \quad ①$$

d, [4] $\dfrac{\operatorname{grad} f(2,1)}{\|\operatorname{grad} f(2,1)\|}$ irányban maximális② az iránymenti derivált,

és a maximum: $\|\operatorname{grad} f(2,1)\| = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \frac{\sqrt{17}}{3}$ ②

6, $\boxed{12}$ $f(x,y) = g(e^{3x} + 2y^2)$

$$f'_x(x,y) = g'(e^{3x} + 2y^2) \cdot 3 \cdot e^{3x} \quad (3)$$

$$f'_y(x,y) = g'(e^{3x} + 2y^2) \cdot 4y \quad (3)$$

$$f''_{xx}(x,y) = \underbrace{9 \cdot e^{6x}}_{(3 \cdot e^{3x})^2} g''(e^{3x} + 2y^2) + 9 \cdot e^{3x} g'(e^{3x} + 2y^2) \quad (3)$$

$$f''_{xy}(x,y) = 4y \cdot 3 \cdot e^{3x} \cdot g''(e^{3x} + 2y^2) \quad (3)$$

Pótfeladatok:

7, a, $\boxed{3}$ $$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

b, $\boxed{7}$ $f(x) = tg(2x)$ $\qquad f\left(\frac{\pi}{8}\right) = tg\left(\frac{\pi}{4}\right) = 1 \quad (1)$

$f'(x) = \frac{2}{\cos^2(2x)} = 2 \cdot \cos^{-2}(2x) \qquad f'\left(\frac{\pi}{8}\right) = \frac{2}{\cos^2\left(\frac{\pi}{4}\right)} = 4 \quad (2)$

$$f''(x) = 2 \cdot (-2) \cdot \cos^{-3}(2x) \cdot (-\sin(2x)) \cdot 2 = 8 \frac{\sin(2x)}{\cos^3(2x)}$$

$$f''\left(\frac{\pi}{8}\right) = 8 \frac{\sin(\pi/4)}{\cos^3(\pi/4)} = 16 \quad (3)$$

$$T_2(x) = 1 + 4\left(x - \frac{\pi}{8}\right) + \frac{16}{2}\left(x - \frac{\pi}{8}\right)^2 \quad (1)$$

8, $\boxed{10}$ $f(x,y) = \left(3x - \frac{1}{y}\right)^2$

$f'_x(x,y) = 3 \cdot 2 \cdot \left(3x - \frac{1}{y}\right); \quad (3) \qquad f'_x(2,1) = 6(6-1) = 30 \quad (1)$

$f'_y(x,y) = 2 \cdot \left(3x - \frac{1}{y}\right) \cdot \frac{1}{y^2}; \quad (3) \qquad f'_y(2,1) = 2 \cdot (6-1) \cdot 1 = 10 \quad (1)$

$$df((2,1),(h,k)) = f'_x(2,1) \cdot h + f'_y(2,1) \cdot k = 30h + 10k \quad (2)$$