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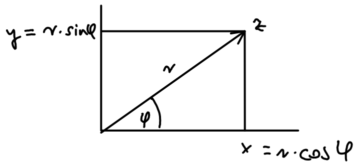
Complex



algebraic form: $z = x + jy \qquad j = \sqrt{-1}$

$\bar{z} = x - jy \qquad |z| = \sqrt{x^2+y^2} = \sqrt{z\bar{z}}$

trigonometric form: $z = r \cdot \cos\varphi + jr\sin\varphi$

$r = |z| \qquad \varphi = \operatorname{arc}(z) \qquad \operatorname{tg}\varphi = y/x$

exponential form: $z = r \cdot e^{j\varphi}$

$e^{jz} = \cos z + j \cdot \sin z$

$e^{j\varphi} = \cos\varphi + j\sin\varphi$ Euler-formula

$e^{j\pi} = -1$

Phasor concept: $x[k] = X\cos(\vartheta k + \rho) \longleftrightarrow \bar{X} = X \cdot e^{j\rho}$

$x(t) = X\cos(\omega t + \rho) \longleftrightarrow \bar{X} = X \cdot e^{j\rho}$

1, $y = ax_1 + bx_2 \longleftrightarrow \bar{Y} = a\bar{X}_1 + b\bar{X}_2$

2, DT $y[k] = x[k+1] \longrightarrow \bar{Y} = e^{j\vartheta} \cdot \bar{X} \longrightarrow \bar{X} = X \cdot e^{j\rho} = X\cos(\vartheta k + \rho)$

$x[k+1] = X \cdot \cos(\vartheta(k+1)+\rho) =$
$= X \cdot \cos(\vartheta k + \vartheta + \rho) \Rightarrow$

CT $y(t) = x'(t) \longrightarrow \bar{Y} = j\omega \cdot \bar{X}$

Transfer coefficient: $\bar{H} = \dfrac{\bar{Y}}{\bar{U}}$

$\bar{X} = X \cdot e^{j(\vartheta+\rho)} =$
$= \underbrace{X \cdot e^{j\vartheta} \cdot e^{j\rho}}_{\bar{X}}$

Transfer characteristic: DT: $H(e^{j\vartheta}) = H(\vartheta) \cdot e^{j\varphi(\vartheta)}$

CT: $H(j\omega) = \underbrace{H(\omega)}_{\text{ampl. char}} \cdot e^{j\underbrace{\varphi(\omega)}_{\text{phase char}}}$

$H(e^{j(\vartheta+2\pi)}) = H(e^{j\vartheta})$

$H(e^{j(-\vartheta)}) = H^*(e^{j\vartheta})$

$H(j(-\omega)) = H^*(j\omega)$

Normal form:

DT: $$H(e^{j\vartheta}) = \frac{b_0 + b_1 \cdot e^{-j\vartheta} + b_2 \cdot e^{-j2\vartheta} + \ldots + b_m \cdot e^{-jm\vartheta}}{1 + a_1 \cdot e^{-j\vartheta} + a_2 \cdot e^{-j2\vartheta} + \ldots + a_n \cdot e^{-jn\vartheta}}$$

CT: $$H(j\omega) = \frac{b_0(j\omega)^n + b_1(j\omega)^{n-1} + \ldots + b_n}{(j\omega)^n + a_1(j\omega)^{n-1} + \ldots + a_n}$$

Connection with state space description:

$H(e^{j\vartheta}) = \underline{C}^T \cdot (e^{j\vartheta}\underline{\underline{I}} - \underline{\underline{A}})^{-1} \cdot \underline{B} + D \qquad H(j\omega) = \underline{C}^T(j\omega\underline{\underline{I}} - \underline{\underline{A}})^{-1}\underline{B} + D$

$(e^{j\vartheta}\underline{\underline{I}} - \underline{\underline{A}})^{-1} = \dfrac{\operatorname{adj}(e^{j\vartheta}\underline{\underline{I}} - \underline{\underline{A}})}{\det(e^{j\vartheta}\underline{\underline{I}} - \underline{\underline{A}})} \longrightarrow$ check stability (Jury/Hurwitz)

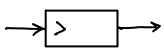
steps:
1, $e^{j\vartheta}\underline{\underline{I}} - \underline{\underline{A}}$
2, $\det(e^{j\vartheta}\underline{\underline{I}} - \underline{\underline{A}})$
3, $(e^{j\vartheta}\underline{\underline{I}} - \underline{\underline{A}})^T$
4, $\operatorname{adj}(e^{j\vartheta}\underline{\underline{I}} - \underline{\underline{A}})$: signed subdeterminants of the transposed matrix

example: $\underline{\underline{A}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \to \underline{\underline{A}}^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \to \operatorname{adj}\underline{\underline{A}} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

5, $\underline{C}^T \cdot (e^{j\vartheta}\underline{\underline{I}} - \underline{\underline{A}})^{-1} \cdot \underline{B} + D$
6, $H(e^{j\vartheta})\big|_{\vartheta=\vartheta_0} = ?$
7, $\bar{Y} = \bar{H} \cdot \bar{U}$

Connection with signal flow network:

$\bar{V} = a \cdot \bar{U}$



$u = v'$

DT: $\bar{U} = e^{j\vartheta}\bar{V} \to \bar{V} = e^{-j\vartheta}\bar{U}$

CT: $\bar{U} = j\omega\bar{V} \to \bar{V} = \dfrac{1}{j\omega}\bar{U}$

steps: (CT)
1, comment the arrows on the network drawing
2, compose one big equation with $\bar{U}$s and $\bar{Y}$s
3, $\bar{H} = \bar{Y}/\bar{U}$ (with the denominator we can check the stability)
4, if $u(t) = K + X \cdot \cos(\omega t + \rho)$, then we must calculate

5) $H(j\omega)\big|_{\omega=0}$ and $H(j\omega)\big|_{\omega=\omega_0}$

$\overline{Y} = \overline{H}\cdot\overline{U}$

Discrete Fourier Series:

$x[k] = x[k+L]$ L: period $\vartheta_0 = \frac{2\pi}{L}$

Real forms: $x[k] = X_0 + X_1\cdot\cos(\vartheta_0 k + \rho_1) + \ldots + X_K\cdot\cos(K\vartheta_0 k + \rho_K) = X_0 + \sum_{p=1}^{K} X_p\cdot\cos(p\vartheta_0 k + \rho_p)$

$x[k] = X_0 + \sum_{p=1}^{K}\left(X_p^A\cos p\vartheta_0 k + X_p^B\sin p\vartheta_0 k\right)$ $K = \begin{cases}\frac{L-1}{2} & \text{odd } L\\ L/2 & \text{even } L\end{cases}$

Complex form: $x[k] = \sum_{p=\langle L\rangle} X_p^C e^{jp\vartheta_0 k}$

$X_p^C = \frac{1}{L}\sum_{k=\langle L\rangle} x[k]\cdot e^{-jp\vartheta_0 k}$ $\langle L\rangle: \begin{cases}-\frac{L-1}{2}\,..\,\frac{L-1}{2} & \text{odd } L\\ -\frac{L}{2}+1\,..\,\frac{L}{2} & \text{even } L\end{cases}$

$k = 0\,..\,L-1$

X_p values: $p=0$: $X_0 = X_0^C$

$0<p<\frac{L}{2}$: $X_p = 2\,|X_p^C|$ $\rho_p = \mathrm{arc}(X_p^C)$

$X_p^A = 2\,\mathrm{Re}\,X_p^C$ $X_p^B = -2\,\mathrm{Im}\,X_p^C$

$p=\frac{L}{2}$: $X_{L/2} = |X_{L/2}|$ $\rho_{L/2} = \mathrm{arc}(X_{L/2}^C) <^{0}_{\pi}$

Continuous Fourier Series:

$x(t+T) = x(t)$ for any t T: period $\omega_0 = \frac{2\pi}{T}$ basic angular freq.

$x(t) = X_0 + \sum_{p=1}^{\infty} X_p\cdot\cos(p\omega_0 t + \rho_p) = X_0 + \sum_{p=1}^{\infty}(X_p^A\cos p\omega_0 t + X_p^B\sin p\omega_0 t) = \sum_{p=-\infty}^{\infty} X_p^C\cdot e^{jp\omega_0 t}$

$X_p^C = \frac{1}{T}\int_{t=\langle T\rangle} x(t)\cdot e^{-jp\omega_0 t}\,dt$

integrating through a whole period

X_p values: $p=0$: $X_0 = X_0^C$

$p\geq 1$: $X_p = 2\,|X_p^C|$ $\rho_p = \mathrm{arc}(X_p^C)$