1, $\boxed{8}$ Szeparálható. $y \equiv 0$ mo. (1)

$$\int sh^{-4}(3y)\,ch(3y)\,dy = \int (5+2x)^{1/3}\,dx \quad (1)$$

$$(3) \quad \frac{sh^{-3}(3y)}{(-3)\cdot 3} = \frac{(5+2x)^{4/3}}{\frac{4}{3}\cdot 2} \ (3) + C$$

2, $\boxed{10}$ (H): $y' = -\frac{3}{x}y$; $\int \frac{dy}{y} = -3\int \frac{dx}{x}$; $\ln|y| = -3\ln|x| + C$

$$\Rightarrow y_{H,ált}(x) = K x^{-3} \ ; \ K \in \mathbb{R} \quad (4)$$

$y_{I,p}(x) = K(x)\,x^{-3}$; Beírva:

$$K'(x)\cdot x^{-3} + \cancel{K(x)\cdot(-3)x^{-4}} + \cancel{\frac{3}{x}K(x)x^{-3}} = 2x^3$$

$$K'(x) = 2x^6 \ ; \ K(x) = \int 2x^6\,dx = \frac{2}{7}x^7 \ ; \ y_{I,p}(x) = \frac{2}{7}x^4$$

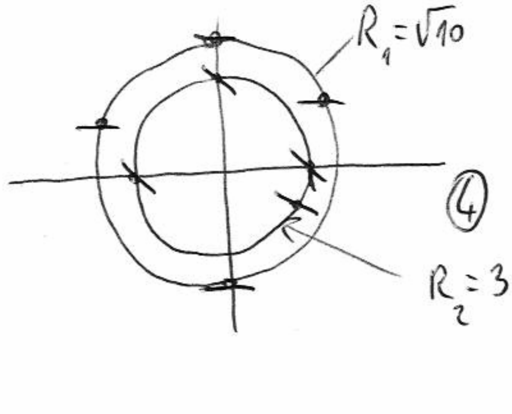
$$y_{I,ált}(x) = K x^{-3} + \frac{2}{7}x^4 \ ; \ K \in \mathbb{R} \quad (4)$$

$$3 = K\cdot 1^{-3} + \frac{2}{7}\cdot 1^4 \Rightarrow K = 3 - \frac{2}{7} = \frac{19}{7} \ ; \ y_{kezd}(x) = \frac{19}{7}x^{-3} + \frac{2}{7}x^4 \quad (2)$$

3, a, $\boxed{4}$ $x^2 + y^2 - 10 = K$

$K = 0$: $x^2 + y^2 = 10$; $\sqrt{10}$ sugarú kör

$K = -1$: $x^2 + y^2 = 9$; 3 — " —



(4)

b, $\boxed{4}$ $y'(1) = (-3)^2 + 1^2 - 10 = 0$ (2)

$$y''(x) = 2y(x)y'(x) + 2x$$

$$y''(1) = 2\cdot(-3)\cdot 0 + 2\cdot 1 = 2 > 0 \Rightarrow \text{lokális minimum} \quad (2)$$

4, $\boxed{12}$ (H) $\lambda^2-5\lambda+6=(\lambda-2)(\lambda-3)=0$; $y_{H,ált}(x)=C_1e^{2x}+C_2e^{3x}$ (4)

$4\,sh(2x)=2e^{2x}-2e^{-2x}$ (1) rezonancia!

$y_{I,p}(x)=Axe^{2x}+Be^{-2x}$ (2) $/\cdot 6$

$y'_{I,p}(x)=A(1+2x)e^{2x}-2Be^{-2x}$ $/\cdot(-5)$

(+) $y''_{I,p}(x)=A(4+4x)e^{2x}+4Be^{-2x}$

$$2e^{2x}-2e^{-2x}=Ae^{2x}\underbrace{(\cancel{6x}-5-\cancel{10x}+4+\cancel{4x})}_{-1}+Be^{-2x}\underbrace{(6+10+4)}_{20}$$

$2=-A\Rightarrow A=-2$; $-2=20B\Rightarrow B=-\frac{1}{10}$

$y_{I,p}(x)=-2xe^{2x}-\frac{1}{10}e^{-2x}$ (3) ; $y_{I,ált}(x)=C_1e^{2x}+C_2e^{3x}-2xe^{2x}-\frac{1}{10}e^{-2x}$ (2)

— o —

5, $\boxed{12}$ a, $\boxed{3}$ $a_n=\frac{n^2-3n+2}{(n+2)^2}\xrightarrow{n\to\infty}1\neq0\Rightarrow\sum_n a_n$ divergens (3)

b, $\boxed{5}$ $\sum_{n=1}^{\infty}(-1)^{n+1}\underbrace{\frac{n-3}{n^2}}_{c_n}$ Leibniz:
- váltakozó előjelű (1)
- $c_n=\frac{n-3}{n^2}\xrightarrow{n\to\infty}0$ (1)
- $c_{n+1}=\frac{n+1-3}{(n+1)^2}\overset{?}{\leq}\frac{n-3}{n^2}=c_n$

$n^2(n-2)\overset{?}{\leq}(n^2+2n+1)(n-3)$

$\cancel{n^3}-2n^2\overset{?}{\leq}\cancel{n^3}-n^2-5n-3$

$0\overset{?}{\leq}n^2-5n-3$ ✓, ha $n\geq6$ (2)

Leibniz, tehát konvergens. (1)

c, $\boxed{4}$ $a_n=\frac{n+2}{n^{4/3}}\geq\frac{n}{n^{4/3}}=\frac{1}{n^{1/3}}=b_n$ (2) ; $\sum_n b_n=\infty$ (1) $(\alpha=\frac{1}{3}<1)$

Minoráns krit. értelmében a sor divergens. (1)