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Useful constants: $h = 6.63 \cdot 10^{-34} \text{Js}$, $e = 1.6 \cdot 10^{-19} \text{C}$, $m_e = 9.1 \cdot 10^{-31} \text{kg}$,
 $L_A = 6.022 \cdot 10^{23} \text{1/mol}$

1. In a 3D spherical quantum harmonic oscillator the energy of the 3rd excited state is 0.73eV . What is the frequency ω ? **(2 points)**

Solution:

$$E_{n,m,l} = \hbar\omega \left(n + m + l + \frac{3}{2} \right), \text{ where } n, m, l = 0, 1, 2, \dots$$

The energies of the states:

ground state : $E_0 = 1.5 \cdot \hbar\omega$,

First excited state: $E_1 = \hbar\omega(1 + 0 + 0 + 1.5)$,

2nd exc. state $E_2 = \hbar\omega(1 + 1 + 0 + 0.5)$

3rd excited state $E_3 = \hbar\omega(1 + 1 + 1 + 1.5)$

$$\omega = \frac{E_3}{\hbar \left(3 + \frac{3}{2} \right)} = \frac{0.73 \cdot 1.66 \cdot 10^{-19}}{6.63 \cdot 10^{-34} \cdot 4.5} = 2.46 \cdot 10^{14} \text{Hz}$$

2. Calculate the energy of the ground state of H, if the wavelength of the light emitted in transition $E_4 \rightarrow E_2$ is 486 nm **(2 points)**

Solution:

The wavelength of the emitted light is determined by the energy difference $E_4 - E_2$:

$$h\nu = h \frac{c}{\lambda} = \Delta E$$

Let's denote the ground state with E_1 . For H

$$E_n = E_1 \frac{1}{n^2}$$

so

$$\Delta E = E_1 \left(\frac{1}{16} - \frac{1}{4} \right)$$

$$h \frac{c}{\lambda} = E_0 \left(\frac{1}{16} - \frac{1}{4} \right)$$

$$E_0 = \frac{h \frac{c}{\lambda}}{\left(\frac{1}{16} - \frac{1}{4} \right)} = 2.18 \cdot 10^{-18} \text{J} = -13.6 \text{eV}$$

3. An electron is confined into a three dimensional cubic potential box with sides of $L = 3.6 \mu\text{m}$. What is the wavelength of photons emitted during an electronic transition between level 3 and the ground state? How would this value change if the size of the box was doubled? **(2 points)**

Solution:

1D potential box: $L = n \cdot \lambda/2$, where $n = 1, 2, \dots$. So $p_n = h/\lambda_n = h \cdot n/L$, i.e.

$$E_n = \frac{p^2}{2m_e} = \frac{h^2}{2m_e(2L)^2} n^2 \quad n = 1, 2, \dots$$

For a 3D cubic potential box:

$$E_{n,m,l} = \frac{h^2}{8m_e L^2} (n^2 + m^2 + l^2) \quad n, m, l = 1, 2, \dots$$

The energy of the ground level is

$$E_{111} = 3 \frac{h^2}{8m_e L^2}$$

For the 3rd level in 3 D

$$E_{122} = E_{212} = E_{221} = 9 \frac{h^2}{8m_e L^2} = 3E_{111}$$

The energy of the emitted photon then is

$$E_{\text{photon}} = h\nu = \frac{hc}{\lambda} = \Delta E = 6 \frac{h^2}{8m_e L^2} = 2.79 \cdot 10^{-26} \text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = 7.12 \text{ m}$$

If the size of the box is doubled the energy levels will be 4 times smaller and the phonon wavelength 4 times larger.

4. Sketch the conventional, primitive and Wigner-Seitz unit cells of a 2 dimensional fcc lattice **(2 points)**

Solution:

5. The dispersion relation of electrons in a semiconductor valence band near the band edge is approximated by the following function:

$$E(k) = -6.048 \cdot 10^{-20} (k - 2.4510^8)^2 + 13 \quad [\text{eV}]$$

In the same semiconductor the energy near the conduction band edge is

$$E(k) = 9.29 \cdot 10^{-20} k^2 + 13.7 \quad [\text{eV}]$$

Determine the effective masses of the electrons and the holes. **(2 points)**

Solution:

The energy formula in J for electrons in the valence band:

$$E_{e,v}(k) = (-9.29 \cdot 10^{-20} (k - 2.4510^8)^2 + 13) \cdot 1.6 \cdot 10^{-19} \quad [\text{J}]$$

In the conduction band:

$$E_{e,c}(k) = (9.29 \cdot 10^{-20} k^2 + 13.7) \cdot 1.6 \cdot 10^{-19} \quad [\text{J}]$$

Therefore the hole dispersion relation in the valence band is:

$$E_{h,c}(k) = -E_{e,v}(k) = (9.29 \cdot 10^{-20} (k - 2.4510^8)^2 + 13) \cdot 1.6 \cdot 10^{-19} \quad [\text{J}]$$

The definition of the effective mass:

$$\frac{1}{m_{eff}} = \frac{1}{\hbar^2} \frac{d^2 E(k)}{dk^2}$$

From this:

$$\frac{1}{m_c} = \frac{2.98 \cdot 10^{-38}}{\hbar^2} = 1.74 \cdot 10^{30} \text{ kg}^{-1} \quad \Rightarrow \quad m_c = 3.73 \cdot 10^{-31} \text{ kg} = 0.41 m_e$$

$$\frac{1}{m_v} = \frac{1.94 \cdot 10^{-38}}{\hbar^2} = 2,67 \cdot 10^{30} \text{ kg}^{-1} \quad \Rightarrow \quad m_v = 5.74 \cdot 10^{-31} \text{ kg} = 0.63 m_e$$