1, $\boxed{12}$ $f(x,y) = \ln(x^2+y^4) + e^{xy^2}$

$f'_x(x,y) = \dfrac{2x}{x^2+y^4} + y^2 e^{xy^2}$ 2 ; $f'_x(0,1) = 1$ 1

$f'_y(x,y) = \dfrac{4y^3}{x^2+y^4} + 2xy\, e^{xy^2}$ 2 ; $f'_y(0,1) = 4$ 1

Érintősík: $z = \underbrace{f(0,1)}_{1} + f'_x(0,1)(x-0) + f'_y(0,1)(y-1) = 1 + x + 4(y-1)$ 1 1 1

$$z = x + 4y - 3$$

Iránymenti derivált: $|\underline{v}| = \sqrt{5^2+12^2} = \sqrt{25+144} = 13$; $\underline{e} = \dfrac{\underline{v}}{|\underline{v}|}$ 1

$\left.\dfrac{df}{d\underline{e}}\right|_P = \operatorname{grad} f(P)\cdot \underline{e} = \dfrac{1\cdot 5 + 4\cdot 12}{13} = \dfrac{53}{13}$ 1 1

2, $\boxed{13}$ $f(x,y) = 4xy - 2x^2y - y^2$

$f'_x(x,y) = 4y - 4xy = 4y(1-x) = 0 \Rightarrow y = 0$ vagy $x = 1$ 1

$f'_y(x,y) = 4x - 2x^2 - 2y = 0$ 1

Ha $y = 0$, $4x - 2x^2 = 2x(2-x) = 0 \Rightarrow A(0,0)$, $B(2,0)$

Ha $x = 1$, $4 - 2 - 2y = 2 - 2y = 0 \Rightarrow C(1,1)$

$A(0,0)$, $B(2,0)$, $C(1,1)$ Vizsgálandó pontok. 5

$H(x,y) = \begin{vmatrix} -4y & 4-4x \\ 4-4x & -2 \end{vmatrix} = 8y - 16(1-x)^2$ 3

A: $H(0,0) = -16 < 0 \Rightarrow$ nyeregpont 1

B: $H(2,0) = -16 < 0 \Rightarrow$ — " — 1

C: $H(1,1) = 8 > 0$; $f''_{xx}(1,1) = -4 < 0 \Rightarrow$ lokális maximum. 1

3, $\boxed{6}$ Az integrálás sorrendjét felcseréljük:

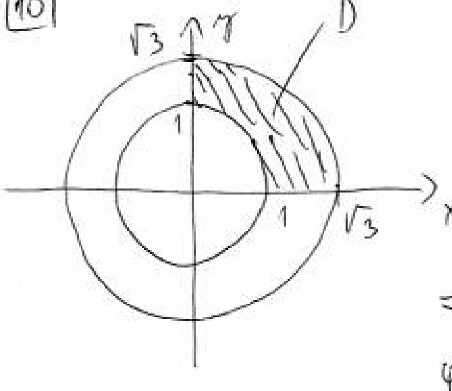
$$\int_0^1\left(\int_1^2 x e^{xy}\,dx\right)dy = \int_{x=1}^{2}\left(\int_{y=0}^{1} x e^{xy}\,dy\right)dx = \int_{x=1}^{2}\left[\cancel{x}\frac{e^{xy}}{\cancel{x}}\right]_{y=0}^{1}dx =$$

1 2

$$= \int_{x=1}^{2}(e^x - 1)\,dx = \left[e^x - x\right]_{x=1}^{2} = e^2 - e - 1$$

1 1 1

4, $\boxed{10}$



$0 \leq \varphi \leq \frac{\pi}{2}$

$1 \leq r \leq \sqrt{3}$

$$\iint_D \frac{x}{x^2+y^2+1}\,dx\,dy = \int_{\varphi=0}^{\pi/2} \int_{r=1}^{\sqrt{3}} \frac{r\cos\varphi}{1+r^2}\, r\,dr\,d\varphi = \quad (3)$$

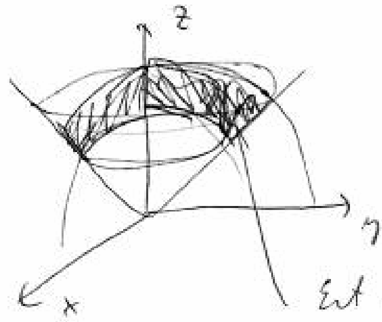
$$= \left(\int_{\varphi=0}^{\pi/2} \cos\varphi\, d\varphi\right) \cdot \left(\int_{r=1}^{\sqrt{3}} \frac{r^2}{1+r^2}\,dr\right) (1) = \Big[\sin\varphi\Big]_0^{\pi/2} (2) \cdot \int_{r=1}^{\sqrt{3}} \left(1-\frac{1}{1+r^2}\right) dr =$$

$$= 1 \cdot \Big[r - \operatorname{arctg} r\Big]_1^{\sqrt{3}} (3) = \sqrt{3} - 1 - \underbrace{\operatorname{arctg}\sqrt{3}}_{\frac{\pi}{3}} + \underbrace{\operatorname{arctg} 1}_{\frac{\pi}{4}} = \underline{\underline{\sqrt{3}-1-\frac{\pi}{12}}} \quad (1)$$

— o —

5, $\boxed{9}$ $\quad x, y > 0; \quad \sqrt{x^2+y^2} \leq z; \quad 4 \leq x^2+y^2+z^2 \leq 9$

Gömbi polár koordinátákban:



$$\left.\begin{aligned} 2 &\leq r \leq 3 \\ 0 &\leq \vartheta \leq \frac{\pi}{4} \\ 0 &\leq \varphi \leq \frac{\pi}{2} \end{aligned}\right\} (3)$$

Ezt forgatjuk el $90°$-kal z körül

$$V = \int_{r=2}^{3} \int_{\vartheta=0}^{\pi/4} \int_{\varphi=0}^{\pi/2} 1 \; \underbrace{r^2 \sin\vartheta}_{\text{Jacobi}} \, d\varphi\, d\vartheta\, dr = \quad (2)$$

$$= \underbrace{\left(\int_{r=2}^{3} r^2\,dr\right)}_{\left[\frac{r^3}{3}\right]_2^3 = \frac{27-8}{3} = \frac{19}{3}} \cdot \underbrace{\left(\int_{\vartheta=0}^{\pi/4} \sin\vartheta\, d\vartheta\right)}_{\left[-\cos\vartheta\right]_0^{\pi/4} = 1-\frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}} \cdot \underbrace{\left(\int_{\varphi=0}^{\pi/2} d\varphi\right)}_{\frac{\pi}{2}} = \underline{\underline{\frac{19}{3} \cdot \frac{\sqrt{2}-1}{\sqrt{2}} \cdot \frac{\pi}{2}}} \quad (4)$$