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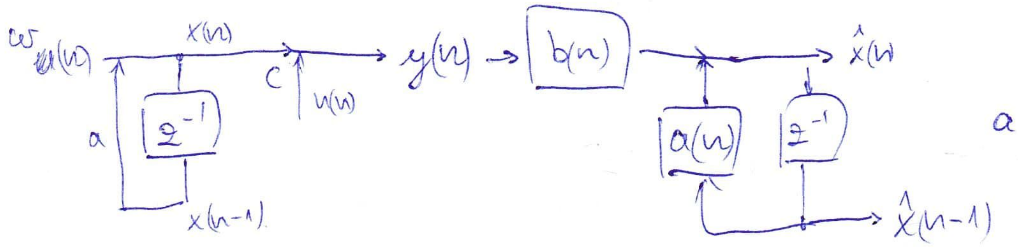
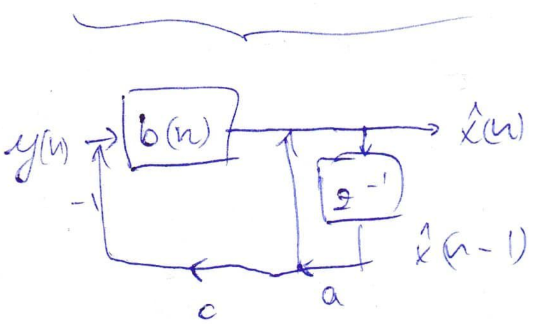
2. ZH megoldások

(1) $x(n) = a\,x(n-1) + w(n)$

$y(n) = c\,x(n) + n(n)$



$a(n) = a[1 - b(n)c]$



rendszerzaj: $E[w(n)] = 0$, σ_w^2, $E[w(n)\,w^T(n)] = Q(n)$

megfigyelési zaj: $E[n(n)] = 0$, σ_n^2, $E[n(n)\,n^T(n)] = R(n)$

$e(n) = x(n) - \hat{x}(n)$ $\quad E\left[\left(x(n) - \hat{x}(n)\right)^2\right] \to$ min.

$\hat{x}(n) = a\,\hat{x}(n-1) + b(n)\,y(n)$

$$\frac{\partial}{\partial a(n)} = -2E\left[e(n)\,\hat{x}(n-1)\right] = 0$$

$$\frac{\partial}{\partial b(n)} = -2E\left[e(n)\,y(n)\right] = 0$$

(2) $\hat{x}(n) = \frac{1}{N}\sum_{\ell=n-N}^{n-1} y(\ell)$ $\qquad \hat{x}(n+1) = \frac{1}{N}\sum_{\ell=n-N+1}^{n} y(\ell) = \hat{x}(n) + \frac{1}{N}\left[y(n) - y(n-N)\right]$

$$z\,\hat{X}(z) = \hat{X}(z) + \frac{1}{N}\left(1 - z^{-N}\right)Y(z)$$

$$H(z) = \frac{\hat{X}(z)}{Y(z)} = \frac{z^{-1}}{N}\,\frac{1-z^{-N}}{1-z^{-1}}$$

$$\left|H(z)\Big|_{z=e^{j\omega T}}\right| = \frac{e^{-j\omega T}}{N}\cdot\frac{1-e^{-j\omega NT}}{1-e^{-j\omega T}} = \frac{e^{-j\omega T}\,e^{-j\frac{N}{2}\omega T}\left(e^{j\frac{N}{2}\omega T} - e^{-j\frac{N}{2}\omega T}\right)}{N\,e^{-j\frac{1}{2}\omega T}\left(e^{j\frac{1}{2}\omega T} - e^{-j\frac{1}{2}\omega T}\right)} =$$

$$= \frac{e^{-j\frac{N+1}{2}\omega T}}{N} \frac{\sin\left(\frac{N}{2}\omega T\right)}{\sin\left(\frac{1}{2}\omega T\right)}$$

$$\left|H(e^{j\omega T})\right| = \frac{1}{N}\left|\frac{\sin\left(\frac{N}{2}\omega T\right)}{\sin\left(\frac{1}{2}\omega T\right)}\right| = \frac{1}{N}\left|\frac{\sin\frac{3\pi}{2}}{\sin\frac{3\pi}{20}}\right| \cong 0{,}22$$

③

$1+\sin\omega t+\cos\omega t$ $\qquad N=4;\ \frac{2\pi}{N}$

$1+\sin\frac{2\pi}{4}n+\cos\frac{2\pi}{4}n$

$$y=\begin{pmatrix}2\\2\\0\\0\end{pmatrix} \qquad F\cdot y=\frac{1}{4}\begin{pmatrix}1&1&1&1\\1&-j&-1&j\\1&-1&1&-1\\1&j&-1&-j\end{pmatrix}\cdot\begin{pmatrix}2\\2\\0\\0\end{pmatrix}=\begin{pmatrix}1\\0{,}5-0{,}5j\\0\\0{,}5+0{,}5j\end{pmatrix}$$

$$W\cdot y=\frac{1}{4}\begin{pmatrix}1&1&1&1\\1&1&-1&-1\\1&-1&-1&1\\1&-1&1&-1\end{pmatrix}\begin{pmatrix}2\\2\\0\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$$

④ $(1-z^{-N})\left(\frac{z_m z^{-1}}{1-z_m z^{-1}}+\frac{z_m^{-1} z^{-1}}{1-z_m^{-1}z^{-1}}\right)$

$$(-z^{-N})\frac{z_m z^{-1}}{1-z_m z^{-1}} = z_m z^{-1}+(z_m z^{-1})^2+(z_m z^{-1})^3+\dots+(z_m z^{-1})^N+(z_m z^{-1})^{N+1}+\dots$$

$$-(z_m z^{-1})z^{-N}$$

$\downarrow \qquad\qquad \downarrow$

$z_m \longrightarrow$ kiesik $\longleftarrow$ $z_m^N z_m$

$\overset{?}{=}1$, mert N-edik egységgyök

$H(z)=a_1 z^{-1}+a_2 z^{-2}+\dots+a_N z^{-N}$

$a_k \neq z_m^{k} = e^{j\frac{2\pi}{N}ml}$

$a_k = z_m^k + z_m^{-k} = e^{j\frac{2\pi}{N}mk}+e^{-j\frac{2\pi}{N}mk}=2\cos\left(\frac{2\pi}{N}mk\right)$

$y(0)=0$

$y(1)=2\cos\left(\frac{2\pi}{N}m\right)$ $\qquad \begin{matrix}N=5\\m=2\end{matrix} \Rightarrow \cos\left(\frac{2\pi}{5}\cdot 2\cdot m\right)$ ...

$y(2)=2\cos\left(\frac{2\pi}{N}m\cdot 2\right)$ ...

$y(N) = 2\cos\left(\frac{2\pi}{N} m N\right) = 2$

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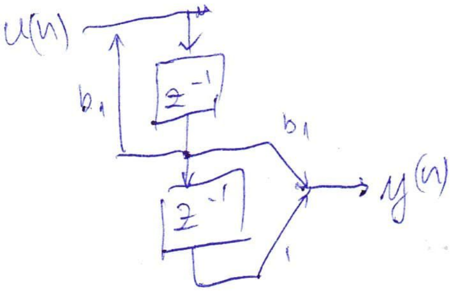
(5) $z^{-1}\dfrac{z^{-1}+b_1}{1+b_1 z^{-1}}$ ...

Mindentáteresztő: $|H(z)| = 1$

Konjugálttal szorzás

$$z^{-1}\frac{z^{-1}+b_1}{1+b_1 z^{-1}} \cdot z\frac{z+b_1}{1+b_1 z} =$$

$$= \frac{1+b_1(z^{-1}+z)+b_1^2}{1+b_1(z^{-1}+z)+b_1^2} = 1 \checkmark$$



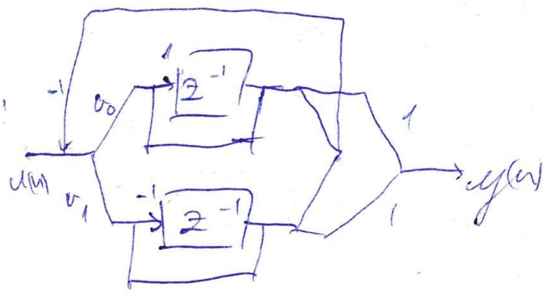
$z_0 = 1 \quad H_0(z) = \dfrac{r_0 z^{-1}}{1-z^{-1}} \quad \omega_0 = 1$

$z_1 = -1 \quad H_1(z) = \dfrac{r_1 z^{-1}}{1+z^{-1}} \quad \omega_1 = 1$

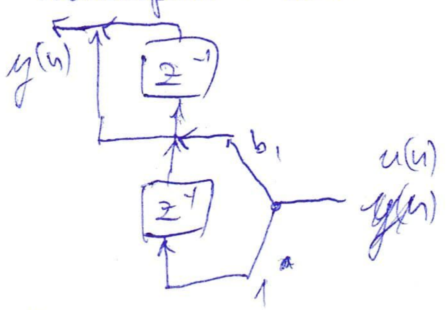
$$\frac{H_0(z)\omega_0 + H_1(z)\omega_1}{1+H_0(z)\omega_0 + H_1(z)\omega_1} = \frac{\frac{r_0 z^{-1}}{1-z^{-1}} + \frac{r_1 z^{-1}}{1+z^{-1}}}{1+\frac{r_0 z^{-1}}{1-z^{-1}} + \frac{r_1 z^{-1}}{1+z^{-1}}} = \frac{r_0 z^{-1} + r_0 z^{-2} - r_1 z^{-1} + r_1 z^{-2}}{1 - z^{-2} + r_0 z^{-1} + r_0 z^{-2} - r_1 z^{-1} + r_1 z^{-2}} =$$

$$= \frac{(r_0-r_1)z^{-1} + (r_0+r_1)z^{-2}}{1+(r_0-r_1)z^{-1} + (\underbrace{-1}_{r_0+r_1})z^{-2}} = \frac{b_1 z^{-1} + z^{-2}}{1+b_1 z^{-1}}$$

$$\left.\begin{aligned} r_0 - r_1 &= b_1 \\ r_0 + r_1 &= 1 \\ r_0 - r_1 &= b_1 \\ r_0 + r_1 - 1 &= 0 \end{aligned}\right\} \Rightarrow r_0 = \frac{1+b_1}{2} \quad r_1 = \frac{1-b_1}{2}$$



Transzponált struktúra:



(6) $\left|\dfrac{a+bj}{1+abj}\right| \leq 1$

$a^2+b^2 \leq (1+a)^2+b^2$

$0 \leq 1+2a \qquad a \geq -\dfrac{1}{2}$

$$a = \mathrm{Re}\left\{\sum_{n=0}^{N-1} \frac{r_n z_n z^{-1}}{1-z_n z^{-1}}\right\} = \frac{1}{2}\sum_{n=0}^{N-1} r_n \underbrace{\left(\frac{z_n z^{-1}}{1-z_n z^{-1}} + \frac{z_n^{-1} z^{-1}}{1-z_n^{-1} z^{-1}}\right)}_{\frac{z_n z^{-1} - 1 + z_n^{-1} z - 1}{1-z_n z^{-1} + z_n^{-1} z + 1} = -1} = -\frac{1}{2}\sum_{n=0}^{N-1} r_n$$

$$-\frac{1}{2}\sum_{n=0}^{N-1} r_n \geq -\frac{1}{2}$$

$$\sum_{n=0}^{N-1} r_n \leq 1$$

Ortogonális r..: $\begin{bmatrix} x(n+1) \\ y(n) \end{bmatrix} = T \begin{bmatrix} x(n) \\ u(n) \end{bmatrix}$ $\quad T^T = T^{-1}$

$$\begin{bmatrix} x^T(n+1) & y(n) \end{bmatrix} \begin{bmatrix} x(n+1) \\ y(n) \end{bmatrix} = \begin{bmatrix} x^T(n) & u(n) \end{bmatrix} T^T T \begin{bmatrix} x(n) \\ u(n) \end{bmatrix}$$

$$x^T(n+1)\,x(n+1) - x^T(n)\,x(n) = -y^T(n)\,y(n) \quad \text{ha } u(n) = 0$$

Ortogonális rezonátor:

$$A = \begin{bmatrix} \cos & -\sin \\ \sin & \cos \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2r_n \cos \\ 2r_n \sin \end{bmatrix}$$

$$\begin{bmatrix} x(n+1) \\ y(n) \end{bmatrix} = \begin{bmatrix} 0 & -\sin & \cos \\ 0 & \cos & \sin \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(n) \\ u(n) \end{bmatrix}$$

$\underbrace{A-GC}_{\begin{bmatrix} \cos & 0 \\ \sin & 0 \end{bmatrix}}$; $r_n = \frac{1}{2}$ az ortogonális beállításhoz

$$\begin{bmatrix} 0 & 0 & 1 \\ -\sin & \cos & 0 \\ \cos & \sin & 0 \end{bmatrix} \begin{bmatrix} 0 & -\sin & \cos \\ 0 & \cos & \sin \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} x(n+1) \\ y(n) \end{bmatrix} = T \begin{bmatrix} x(n) \\ u(n) \end{bmatrix} \quad T = \left[\begin{array}{c|c} & \\ \hline & \end{array}\right]$$

Energia: $x^T(n)\, P\, x(n)$

1. Mindentáteresztő $|H(z)| = 1$ $\quad y^2 - u^2$

$$A^T P A + C^T C = P$$

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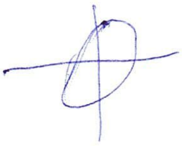
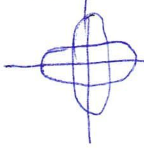
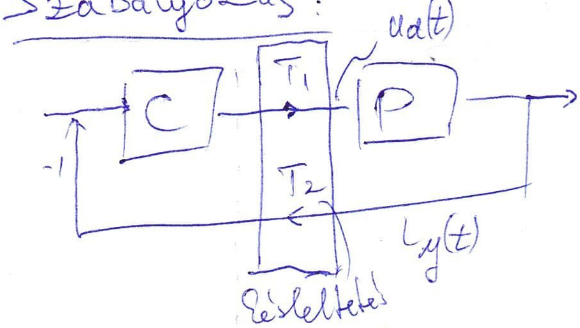
$$H(z) = z^{-1}\,\frac{z^{-1}+b_1}{1+b_1 z^{-1}}$$

$$P_D = \begin{bmatrix}1 & b_1\\ b_1 & 1\end{bmatrix} \qquad P_R = \begin{bmatrix}\frac{2}{1+b_1} & 0 \\ 0 & \frac{2}{1+b_1}\end{bmatrix}$$



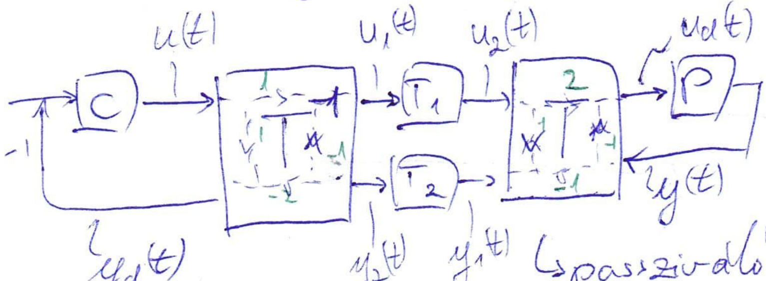


Szabályozás:



→ a hálózatot disszipatívnak tekintjük

P disszipatív: $\int_0^t P_{in}(\tau)\,d\tau = \int_0^t u_d(\tau)\, y(\tau)\,d\tau \ge 0$



↳ passziváló transzformáció

Hálózatban tárolt energia: $U_N = \int_0^t u_1^T(\tau)\,u_1(\tau) + y_1^T(\tau)\,y_1(\tau) - u_2^T(\tau)\,u_2(\tau) - y_2^T(\tau)\,y_2(\tau)\,d\tau$

$u_2(t) = u_1(t-T_1)$

$y_2(t) = y_1(t-T_2)$

$$= \underbrace{\int_{t-T_1}^{t} u_1^T(\tau)\,u_1(\tau)\,d\tau}_{\ge 0} + \underbrace{\int_{t-T_2}^{t} y_1^T(\tau)\,y_1(\tau)\,d\tau}_{\ge 0} \ge 0$$

$$\int_0^t u_1^T(\tau)\,u_1(\tau) - y_2^T(\tau)\,y_2(\tau)\,d\tau \ge \int_0^t u_2^T(\tau)\,u_2(\tau) - y_1^T(\tau)\,y_1(\tau)\,d\tau$$

$$\begin{bmatrix}u_d(t)\\ y(t)\end{bmatrix} = \begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}u_2(t)\\ y_1(t)\end{bmatrix} = \begin{bmatrix}u_2(t)+y_1(t)\\ u_2(t)-y_1(t)\end{bmatrix};\ \begin{bmatrix}u_d(t)\\ y_1(t)\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & -1\end{bmatrix}\begin{bmatrix}u_2(t)\\ y(t)\end{bmatrix}$$

$$\begin{bmatrix} u(t) \\ y_d(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} u_1(t) + y_2(t) \\ u_1(t) - y_2(t) \end{bmatrix} ; \begin{bmatrix} u_1(t) \\ y_d(t) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} u(t) \\ y_2(t) \end{bmatrix}$$

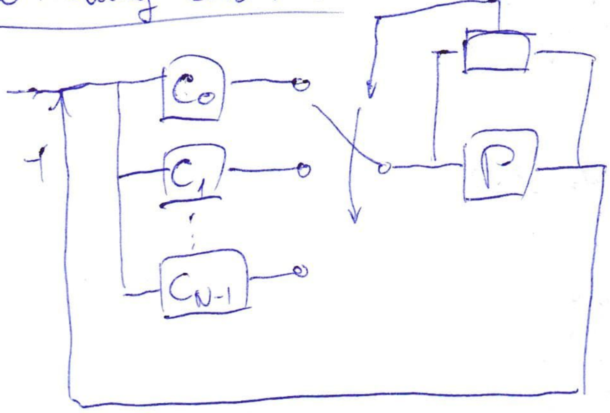
$$P(s) \,\cancel{\#}: \quad P_1(s) = 1 - 2\frac{P(s)}{1+P(s)} = \frac{1-P(s)}{1+P(s)}$$

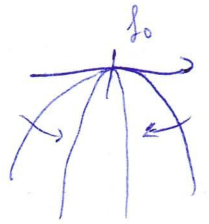
$$K(s) = e^{-sT_1} + e^{-sT_2}$$

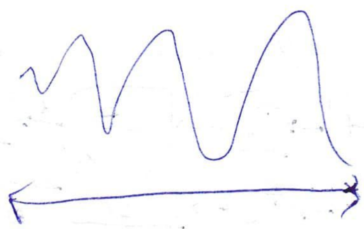
$$P_2(s) = 1 - 2 \cdot \frac{K(s)\, P_1(s)}{1 + K(s)\, P_1(s)} = \frac{1 - K(s) P_1(s)}{1 + K(s) P_1(s)} = \frac{1 + P(s) - K(s)(1-P(s))}{1 + P(s) + K(s)(1-P(s))} =$$

$$= \frac{1 - K(s) + P(s)(1+K(s))}{1 + K(s) + P(s)(1-K(s))}$$

Switching control



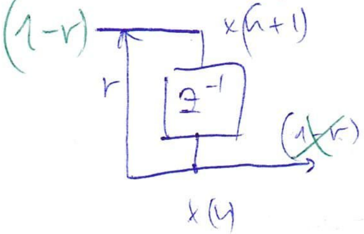


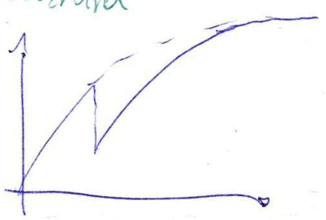


$$H(z) = \frac{(1-r)z^{-1}}{1-rz^{-1}} \qquad H(z)\Big|_{z=1} = 1$$

$$H(z)\Big|_{z=-1} = \frac{-\cancel{1}(1-r)}{1+r}$$

transzponált struktúra





$x(0) = 0$

$x(1) = 1$

$x(2) = 1 + r$

$x(n) = 1 + r + r^2 + \ldots + r^{n-1} \xrightarrow[n\to\infty]{} \frac{1}{1-r}$