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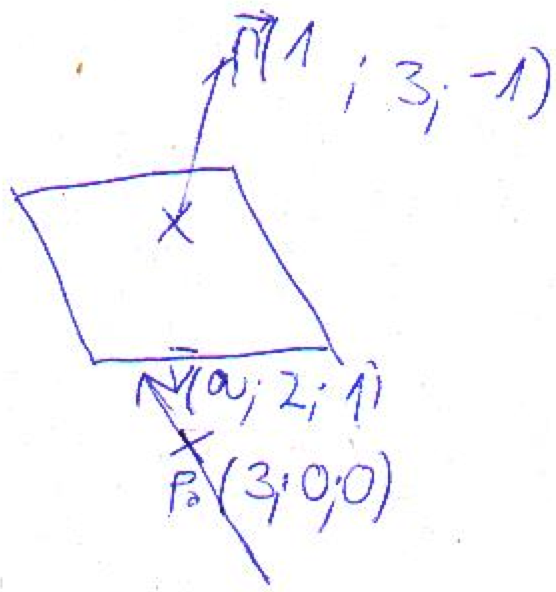
$\boxed{M}$
malek
#include <math.h>

1.



akkor nem metszi, ha a $\bar v$ és $\bar n$ merőlegesek egymásra.
Ez akkor van, ha skaláris szorzatuk 0.

$$1\cdot a + 3\cdot 2 - 1 = 0$$
$$a = -5$$

2.

(i)
$$\frac{\sqrt[3]{8n^4+3n^2+\sin 2n}}{3n^{\frac{4}{3}}+2n+\cos^2 n}\cdot\frac{n^{-\frac{4}{3}}}{n^{-\frac{4}{3}}} = \frac{\sqrt[3]{8+3\overset{\to 0}{n^{-2}}+\overset{[-1;1]}{\sin(2n)}\,\overset{\to 0}{n^{-4}}}}{3+2\underset{\downarrow\, 0}{n^{-\frac{1}{3}}}+\underset{\downarrow\, 0}{n^{-\frac{4}{3}}}\underset{\downarrow\,[-1;1]}{\cos^2 n}} = \frac{2}{3}$$

(ii) failsafe solution: számológép-pel $n=500$-ra :-)

minden esetre $\sqrt[n]{n}\to 1$

3.

(i)
$$\overset{\text{sor}}{\sum_{n=1}^{\infty} a q^{n-1}} = \frac{a}{1-q}$$

$$a = 3$$
$$q = \frac{x-1}{2}$$

akkor konvergens ha $|q|<1$

$$-1 < \frac{x-1}{2} < 1$$
$$-2 < x-1 < 2$$
$$-1 < x < 3$$

(ii)
$$\sum = \frac{a}{1-q} = \frac{3}{\underbrace{1}_{=\frac{2}{2}} - \frac{x-1}{2}} = \frac{6}{3-x}$$

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4. (i) $\frac{1}{1+x}$ Taylor sora

$$T_a^f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$ ← Taylor-sorfejtéshez a képlet

$$\left((1+x)^{-1}\right)'=-(1+x)^{-2}$$

$$f^{(2)}(x)=2(1+x)^{-3}$$

$$f^{(3)}(x)=-6(1+x)^{-4}$$

$$f^{(n)}(x)=(-1)^n n!(1+x)^{-(n+1)}$$

$$T_0^f(x)\sum_{n=0}^{\infty}\frac{(-1)^n\cancel{n!}}{\cancel{n!}}x^n=\sum_{n=0}^{\infty}(-1)^n x^n=1-x+x^2-\dots+(-1)^n x^n$$

(ii) $\ln(x+1)$

$$(\log_a x)'=\frac{1}{x\ln(a)}$$

$$(\ln(x+1))'=\frac{(x+1)'}{(x+1)\underbrace{\ln(e)}_{=1}}=\frac{1}{x+1}=(x+1)^{-1}$$ az előző feladatból ismerős...

$$T_0^f(x)=\underbrace{\frac{\ln(1)}{1}}_{0}x^0+\frac{1}{1}x+\frac{-1}{2}x^2+\frac{2}{6}x^3+\frac{-6}{24}x^4\dots$$
($n=0$, $n=1$, $n=2$, $n=3$, $n=4$)

$$T_0^f(x)=\sum_{n=1}^{\infty}\frac{x^n(-1)^{n-1}}{n}$$

okay, ez ugyanaz mint a megoldásban, csak n-nel megyünk 0-tól:

$$T_0^f(x)=\sum_{n=0}^{\infty}\frac{x^{n+1}(-1)^n}{n+1}$$

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5. $f^{(n)}(x) = (n-1)! \cdot (-1)^{n-1} (1+x)^{-n}$

$f^{(100)}(x) = 99! \, (-1) (1+x)^{-99}$

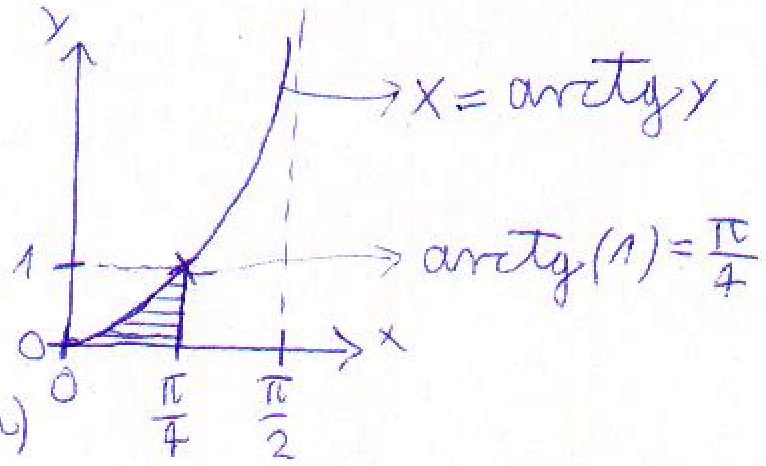
$f^{(100)}(0) = -99!$

6. http://tinyurl.com/B3XG9DN

$$\int_0^1 \int_{\operatorname{arctg} y}^{\frac{\pi}{4}} \frac{1}{1+y^2} \, dx \, dy$$

① $\operatorname{arctg} y < x < \frac{\pi}{4}$

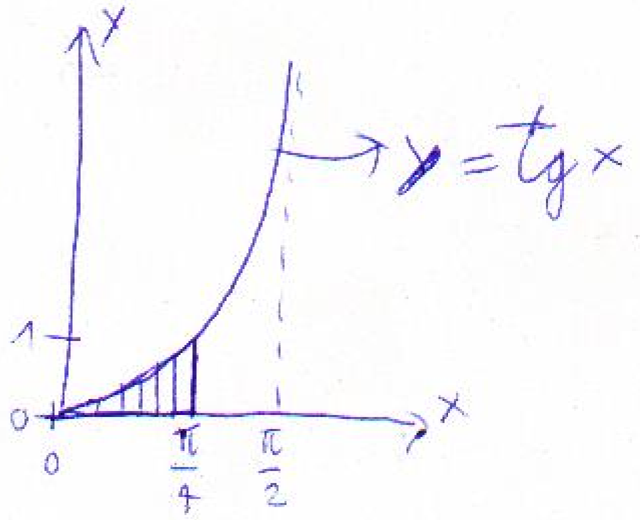
(ez van vonalkázva)



② $0 < y < 1$

① $0 < y < \operatorname{tg} x$

② $0 < x < \frac{\pi}{4}$



(ii) $$\int_0^{\frac{\pi}{4}} \int_0^{\operatorname{tg} x} \frac{1}{1+y^2} \, dy \, dx = \int_0^{\frac{\pi}{4}} \Big[\operatorname{arctg} y\Big]_0^{\operatorname{tg} x} dx = \int_0^{\frac{\pi}{4}} x \, dx = \left[\frac{x^2}{2}\right]_0^{\frac{\pi}{4}} = \frac{\pi^2}{16 \cdot 2} - \frac{0^2}{2} = \frac{\pi^2}{32}$$

7. (i) $x \in \mathbb{R}$ $y \in \mathbb{R}$ valóban.

de mivel $\sqrt{-5} = (-5)^{\frac{1}{2}}$ pl. problémás, ezért $x > 0$

(ii) ha $\left.\frac{\partial f(x,y)}{\partial x}\right|_{\substack{x=1 \\ y=2}}$ -re gondol, akkor valóban.

$$\frac{\partial f(x,y)}{\partial x} = y \cdot x^{(y-1)} = 2 \cdot 1^1 = 2$$

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7. (iii) Iránymenti derivált

skaláris szorzat

$$\frac{\text{grad}_p f \cdot \vec{v}}{|\vec{v}|}$$

$\vec{v} = (1; 1)$

$\text{grad} f = \left(\frac{\partial f}{\partial x}; \frac{\partial f}{\partial y}; \frac{\partial f}{\partial z}\right)$ definíció szerint

$\text{grad} f = \left(\frac{\partial x^y}{\partial x}; \frac{\partial x^y}{\partial y}\right)$

$\text{grad} f = \left(y x^{y-1}; x^y \ln(x)\right)$

$\text{grad}_p f = (2 \cdot 1; 1^2 \underbrace{\ln(1)}_{0}) = (2; 0)$

$$\frac{(2; 0) \cdot (1; 1)}{\sqrt{1^2 + 1^2}} = \frac{2 \cdot 1 + 0 \cdot 1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$