2008.02.11

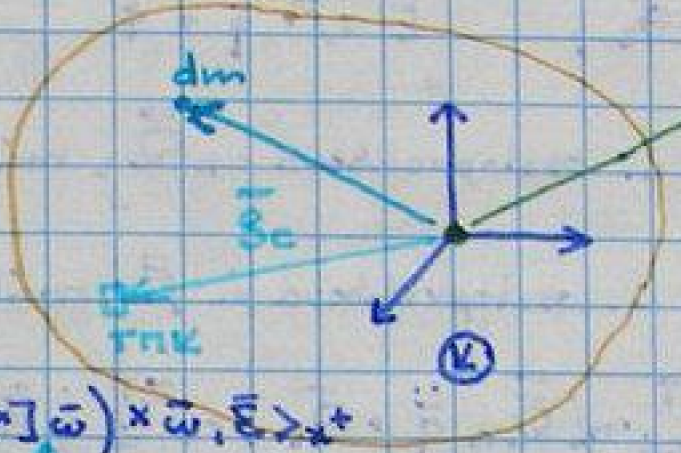
Meghajtó motor figyelembevétele

$$K_{rotor,i} = \frac{1}{2}\Theta_{rotor,i}\cdot(\nu_i\cdot\dot q_i)^2$$

$$\frac{d}{dt}\frac{\partial K_{rotor,i}}{\partial \dot q_i} = \frac{d}{dt}\Theta_{rotor,i}\,\nu_i^2\cdot\dot q_i = \Theta_{rotor,i}\,\nu_i^2\,\ddot q_i$$

$$\boxed{D_{ii} := D_{ii} + \Theta_{rotor,i}\,\nu_i^2}$$

Gyorsulás „energia" = Gibbs-függvény, $\frac{1}{2} m a^2$



$\bar v, \bar a, \bar\omega, \bar\varepsilon, \bar{\bar K}_c$

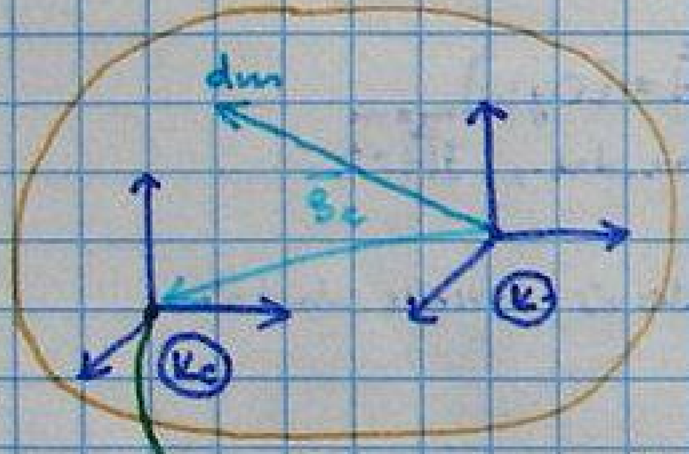
$$\bar a_S = \bar a + \bar\varepsilon\times\bar s + \bar\omega\times(\bar\omega\times\bar s)$$

$$G = \int_{(S)} \frac{1}{2}\,\bar a_S^2\, dm$$

$$\bar a_S^2 = \langle \bar a + \bar\varepsilon\times\bar s + \bar\omega\times(\bar\omega\times\bar s),\ \bar a + \bar\varepsilon\times\bar s + \bar\omega\times(\bar\omega\times\bar s)\rangle =$$
$$= \langle\bar a,\bar a\rangle + \langle[\bar s\times]^T[\bar s\times]\bar\varepsilon - 2([\bar s\times]^T[\bar s\times]\bar\omega)\times\bar\omega,\ \bar\varepsilon\rangle +$$
$$+ 2\langle\bar s,\ \bar a\times\bar\varepsilon + \bar\omega\times(\bar\omega\times\bar a)\rangle + \ldots$$

·dm ·dm ·dm ·dm

$$G = \frac{1}{2}\langle\bar a,\bar a\rangle m + \frac{1}{2}\langle\bar{\bar K}\bar\varepsilon - 2(\bar{\bar K}\bar\omega)\times\bar\omega,\ \bar\varepsilon\rangle + 2\langle m\bar s_c,\ \bar a\times\bar\varepsilon + \bar\omega\times(\bar\omega\times\bar a)\rangle + \ldots$$



$\bar v_c, \bar a_c, \bar\omega, \bar\varepsilon, \bar{\bar K}_c$

$$\boxed{G = \frac{1}{2}\langle\bar a_c,\bar a_c\rangle m + \frac{1}{2}\langle\bar{\bar K}_c\bar\varepsilon - 2(\bar{\bar K}_c\bar\omega)\times\bar\omega,\ \bar\varepsilon\rangle}$$

$$\bar v_c = \bar v + \bar\omega\times\bar s_c = \bar v - [\bar s_c\times]\bar\omega = \bar{\bar\Omega} - [\bar s_c\times]\bar{\bar\Pi} = \bar{\bar\Omega}_c\cdot\dot q$$

$$\bar a_c = \bar a + \bar\varepsilon\times\bar s_c + \bar\omega\times(\bar\omega\times\bar s_c) = \bar{\bar\Omega}_c\,\ddot q + \bar\Theta_c$$

$$\bar a = \bar{\bar\Omega}\,\ddot q + \bar\Theta$$

$$\bar\varepsilon = \bar{\bar\Pi}\,\ddot q + \bar\Theta$$

$$\boxed{\begin{aligned}\bar\Theta_c &= \bar\Theta + \bar\Phi\times\bar s_c + \bar\omega\times\cancel{\bar s_c}(\bar\omega\times\bar s_c)\\ \bar{\bar\Omega}_c &= \bar{\bar\Omega} - [\bar s_c\times]\bar{\bar\Pi}\end{aligned}}$$

$$\boxed{G = \frac{1}{2}\langle\bar a_c,\bar a_c\rangle m + \frac{1}{2}\langle\bar{\bar K}_c\bar\varepsilon - 2(\bar{\bar K}_c\bar\omega)\times\bar\omega,\ \bar\varepsilon\rangle}$$

Appell-egyenletek: m-DOF robot

$$\frac{\partial G}{\partial \ddot{q}_i} + \frac{\partial D}{\partial q_i} = \tau_i, \quad i = 1, \dots, m$$

$$\frac{\partial G}{\partial \ddot{q}_i} = \frac{1}{2}\left\langle \frac{\partial \bar{a}_c}{\partial \ddot{q}_i}, \bar{a}_c \right\rangle m + \frac{1}{2}\left\langle \bar{a}_c, \frac{\partial \bar{a}_c}{\partial \ddot{q}_i} \right\rangle m + \frac{1}{2}\left\langle \bar{\bar{K}}_c \frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i}, \bar{\varepsilon} \right\rangle + \frac{1}{2}\left\langle \bar{\bar{K}}_c \bar{\varepsilon} - 2(\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega}, \frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i} \right\rangle$$

$$\boxed{\frac{\partial G}{\partial \ddot{q}_i} = \left\langle \frac{\partial \bar{a}_c}{\partial \ddot{q}_i}, \bar{a}_c \right\rangle m + \left\langle \frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i}, \bar{\bar{K}}_c \bar{\varepsilon} - (\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega} \right\rangle}$$

$$\bar{\bar{A}}\bar{x} = [\bar{a}_1 \ \bar{a}_2 \ \dots] \begin{pmatrix} x_1 \\ x_2 \\ \vdots \end{pmatrix} = \bar{a}_1 x_1 + \bar{a}_2 x_2 + \dots$$

$$\bar{a}_c = \bar{\bar{\Omega}}_c \ddot{q} + \bar{\Theta}_c, \qquad \bar{\varepsilon} = \bar{\bar{\Gamma}} \ddot{q} + \bar{\Phi}$$

$\frac{\partial \bar{a}_c}{\partial \ddot{q}_i} = \bar{\Omega}_{c,i}$ $\quad$ i. oszlopvektora az $\bar{\bar{\Omega}}_c$ mátrixnak

$\frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i} = \bar{\Gamma}_i$ $\quad$ i. oszlopvektora a $\bar{\bar{\Gamma}}$ mátrixnak

$$\frac{\partial G}{\partial \ddot{q}_i} + \frac{\partial P}{\partial q_i} = \bar{\Omega}_{c,i}^T \cdot m(\bar{\bar{\Omega}}_c \ddot{q} + \bar{\Theta}_c) + \bar{\Gamma}_i^T \{\bar{\bar{K}}_c(\bar{\bar{\Gamma}}\ddot{q} + \bar{\Phi}) - (\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega}\}$$

$$\frac{\partial G}{\partial \ddot{q}_i} + \frac{\partial P}{\partial \bar{q}} = \bar{\tau}; \quad (\bar{\bar{\Omega}}_c^T \bar{\bar{\Omega}}_c m + \bar{\bar{\Gamma}}^T \bar{\bar{K}}_c \bar{\bar{\Gamma}})\ddot{q} + \bar{\bar{\Omega}}_c^T \bar{\Theta}_c m + \bar{\bar{\Gamma}}^T [\bar{\bar{K}}_c \bar{\Phi} - (\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega}] + \frac{\partial P}{\partial \bar{q}} = \bar{\tau}$$

$\bar{\bar{H}}(\bar{q}) = \sum_{s=1}^{m} \{\bar{\bar{\Omega}}_{cs}^T \bar{\bar{\Omega}}_{cs} m_s + \bar{\bar{\Gamma}}_s^T \bar{\bar{K}}_{cs} \bar{\bar{\Gamma}}\}$ ált. inerciamátrix, $\bar{\bar{H}} = [D_{jk}]_{m \times m}$

szimmetrikus és poz.def., $\exists \bar{\bar{H}}^{-1}$

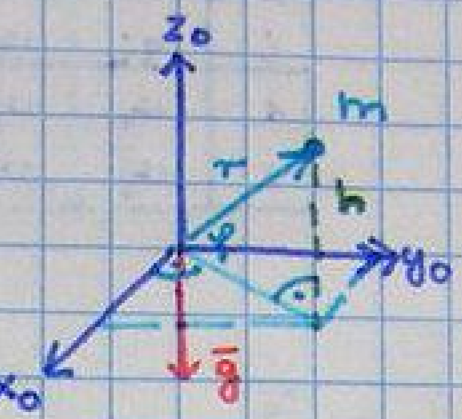
$\bar{h}_{cc} = \sum_{s=1}^{m} \{\bar{\bar{\Omega}}_{cs}^T \bar{\Theta}_{cs} m_s + \bar{\bar{\Gamma}}_s^T [\bar{\bar{K}}_{cs} \bar{\Phi}_s - (\bar{\bar{K}}_{cs} \bar{\omega}_s) \times \bar{\omega}_s]\}$ centripetális és Coriolis-hatás

$$\bar{h}_g = \frac{\partial P}{\partial \bar{q}}, \quad h_{g_i} = D_i = \frac{\partial P}{\partial q_i}$$

Mozgásegyenlet: $\boxed{\bar{\bar{H}}(\bar{q})\ddot{\bar{q}} + \bar{h}_{cc}(\bar{q}, \ddot{\bar{q}}) + \bar{h}_g(\bar{q}) = \bar{\tau}}$

Potenciális energia:

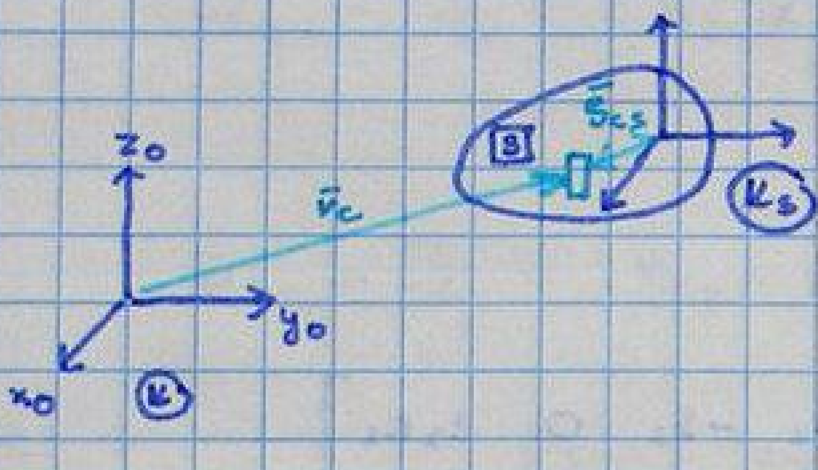
$P = mgh$



$h = |\bar r| \sin\varphi$

$\langle \bar r, \bar g \rangle = |\bar r| \cdot |\bar g| \cdot \underbrace{\cos(90° + \varphi)}_{-\sin(\varphi)}$

$P = -m \langle \bar g, \bar r \rangle$



$$P = -\sum_{s=1}^{m} m_s (\bar g^T \; 0) \underline{\bar{\bar T}_{0,s}} \begin{pmatrix} \bar s_{cs} \\ 0 \end{pmatrix}$$

$$\bar{\bar T}_{0,i-1}, \underbrace{\bar{\bar T}_{i-1,i}}_{q_i} \bar{\bar T}_{i,s}$$

$$\frac{\partial P}{\partial q_i} = D_i = -\sum_{s=i}^{m} (\bar g^T \; 0) \bar{\bar T}_{0,i-1} \underbrace{\frac{\partial \bar{\bar T}_{i-1,i}}{\partial q_i}}_{\bar{\bar \Delta}_{i-1} \bar{\bar T}_{i-1,i}} \bar{\bar T}_{i,s} \begin{pmatrix} \bar s_{cs} \\ 1 \end{pmatrix} m_s =$$

$\bar{\bar \Delta}_{i-1}$: rotációs $\rightarrow \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

transzlációs $\rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$= \underbrace{-(\bar g^T \; 0) \bar{\bar T}_{0,i-1} \bar{\bar \Delta}_{i-1}}_{\bar G_i^T} \underbrace{\sum_{s=i}^{m} \bar{\bar T}_{i-1,s} \begin{pmatrix} m_s \bar s_{cs} \\ m_s \end{pmatrix}}_{\begin{pmatrix} \bar R_i \\ M_i \end{pmatrix} = ?}$$

$\bar G_i^T$: rotációs $\rightarrow (-\bar g \cdot \bar m_{0,i-1}, \bar g \cdot \bar l_{0,i-1}, 0, 0)$

transzlációs $\rightarrow (0, 0, 0, -\bar g \cdot \bar m_{0,i-1})$

$$\bar R_i = \sum_{s=i}^{m} \{ \bar{\bar A}_{i-1,s} \, m_s \bar s_{cs} + \bar p_{i-1,s} \, m_s \}$$

$$M_i = \sum_{s=i}^{m} m_s$$

$$\bar R_i = \sum_{s=i+1}^{m} \{ \underbrace{\bar{\bar A}_{i-1,s}}_{\bar{\bar A}_{i-1,i} \bar{\bar A}_{i,s}} m_s \bar s_{cs} + \underbrace{\bar p_{i-1,s}}_{\bar p_{i-1,i} + \bar{\bar A}_{i,s} \bar p_{i,s}} m_s \} + \bar{\bar A}_{i-1,i} m_i \bar s_{ci} + \bar p_{i-1,i} \, m_i$$

$$\bar R_i = \bar{\bar A}_{i-1,i} \{ \bar R_{i+1} + m_i \bar s_{ci} \} + M_i \bar p_{i-1,i}$$

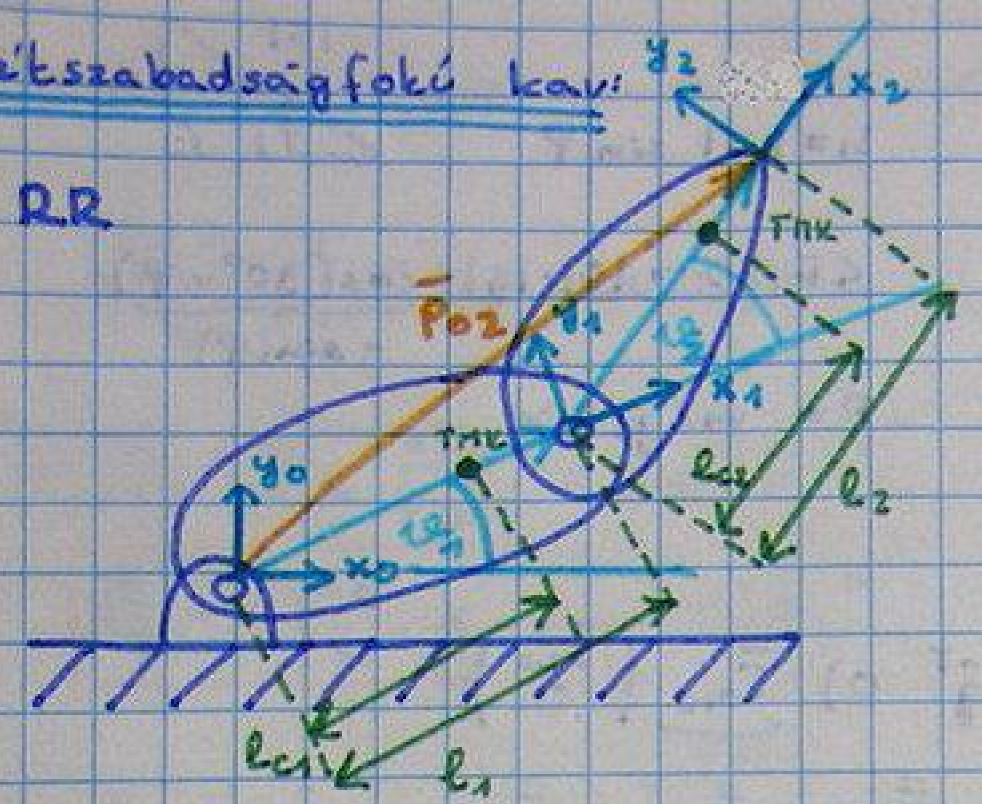
$$M_i = M_{i+1} + m_i$$

Hátratartó rekurzió: $\bar R_{m+1} = \bar 0, \quad M_{m+1} = 0$

$\bar{\bar T}_{0,0} = \bar{\bar I}_4, \quad h_{g_i}(\bar q) = D_i(\bar q) = \bar G_i^T \begin{pmatrix} \bar R_i \\ M_i \end{pmatrix}$

Kétszabadságfokú kar:

RR



i	q_i	ϑ_i	d_i	a_i	α_i
1	ϑ_1	ϑ_1	0	l_1	0°
2	ϑ_2	ϑ_2	0	l_2	0°

$$\bar{\bar{T}}_{0,1} = \begin{bmatrix} C_1 & -S_1 & 0 & l_1C_1 \\ S_1 & C_1 & 0 & l_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \bar{\bar{T}}_{1,2} = \begin{bmatrix} C_2 & -S_2 & 0 & l_2C_2 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{02} = \bar{\bar{T}}_{01} \cdot \bar{\bar{T}}_{12} = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1C_1 + l_2C_{12} \\ S_{12} & C_{12} & 0 & l_1S_1 + l_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0\bar{\omega}_1 = {}^1\bar{\omega}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \dot{q}_1 = \bar{\bar{\Gamma}}_1 \dot{\bar{q}}$$

$${}^0\bar{\omega}_2 = {}^2\bar{\omega}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (\dot{q}_1 + \dot{q}_2) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \bar{\bar{\Gamma}}_2 \dot{\bar{q}}$$

$$\bar{p}_{01} = \begin{pmatrix} l_1C_1 \\ l_1S_1 \\ 0 \end{pmatrix} \rightarrow {}^0\bar{v}_1 = \frac{d}{dt}\bar{p}_{01} = \begin{pmatrix} -l_1S_1\dot{q}_1 \\ l_1C_1\dot{q}_1 \\ 0 \end{pmatrix} = \begin{pmatrix} -l_1S_1 \\ l_1C_1 \\ 0 \end{pmatrix} \dot{q}_1$$

$$\rightarrow {}^1\bar{v}_1 = \bar{\bar{A}}_{01}^T \, {}^0\bar{v}_1 = \begin{bmatrix} C_1 & S_1 & 0 \\ -S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} -l_1S_1 \\ l_1C_1 \\ 0 \end{pmatrix} \dot{q}_1 = \begin{pmatrix} 0 \\ l_1 \\ 0 \end{pmatrix} \dot{q}_1 = \bar{\bar{\Omega}}_1 \dot{\bar{q}}, \quad \bar{\bar{\Omega}}_1 = \begin{pmatrix} 0 \\ l_1 \\ 0 \end{pmatrix}$$

$$\rightarrow {}^0\bar{v}_2 = \frac{d}{dt}\bar{p}_{02} = \begin{pmatrix} -l_1S_1\dot{q}_1 - l_2S_{12}(\dot{q}_1 + \dot{q}_2) \\ l_1C_1\dot{q}_1 + l_2C_{12}(\dot{q}_1 + \dot{q}_2) \\ 0 \end{pmatrix}$$

$$\rightarrow {}^2\bar{v}_2 = \bar{\bar{A}}_{02}^T \, {}^0\bar{v}_2 = \begin{bmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -l_1S_1 - l_2S_{12} & -l_2S_{12} \\ l_1C_1 + l_2C_{12} & l_2C_{12} \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} =$$

$$= \underbrace{\begin{bmatrix} l_1S_2 & 0 \\ l_1C_2 + l_2 & l_2 \\ 0 & 0 \end{bmatrix}}_{\bar{\bar{\Omega}}_2} \dot{\bar{q}} = \bar{\bar{\Omega}}_2 \dot{\bar{q}}$$