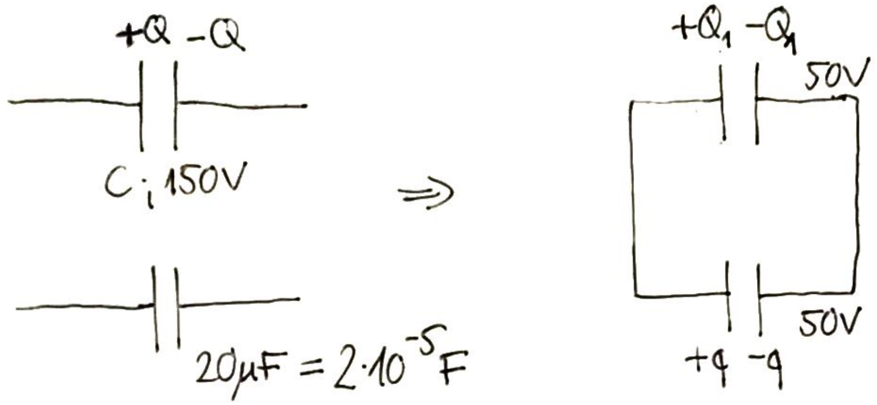
F1.



töltésmegmaradás miatt:
$$Q_1 = Q - q \quad (1)$$

$$q = 2\cdot 10^{-5}\,F \cdot 50V = 10^{-3}\,C.$$

$$\left.\begin{array}{l} Q = C\cdot 150V \\ Q_1 = C\cdot 50V \end{array}\right\} \quad Q_1 = \frac{Q}{3} \longrightarrow (1):\quad q = \frac{2Q}{3} \Rightarrow Q = \frac{3}{2}q = 1{,}5\cdot 10^{-3}\,C$$

Tehát: $C = \dfrac{Q}{150V} = 10^{-5}\,F = 10\,\mu F$

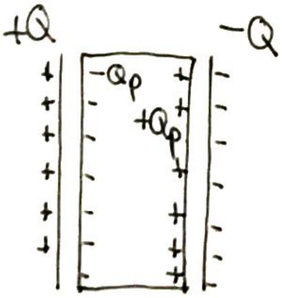
F2.

$d = 2{,}0$ mm
$A = 0{,}30\ m^2$
$\varepsilon_r = 3{,}0$
$U_0 = 12V$

a) $$C = \frac{\varepsilon_0\varepsilon_r A}{d} = \frac{Q}{U_0} \rightarrow Q = \frac{\varepsilon_0\varepsilon_r A U_0}{d} = 4{,}8\cdot 10^{-8}\,C = \approx 48\,nC$$

b) Dielektrikum nélkül: $E = \dfrac{U_0}{d} = \dfrac{Q_0}{\varepsilon_0 A}$
Dielektrikummal: $E = \dfrac{U_0}{d} = \dfrac{Q - Q_p}{\varepsilon_0 A}$

$$Q_p = Q - Q_0 = (C - C_0)U_0 = (\varepsilon_r - 1)\frac{\varepsilon_0 A}{d}U_0$$



$$\sigma_p = \frac{Q_p}{A} = (\varepsilon_r - 1)\frac{\varepsilon_0 U_0}{d} = 1{,}1\cdot 10^{-7}\,\frac{C}{m^2}$$

vagy: polarizáció: $P = \varepsilon_0 E_p = \varepsilon_0(\varepsilon_r - 1)E$

Gauss-törvény: $E_p \cdot A = \frac{1}{\varepsilon_0} Q_p \Rightarrow \frac{Q_p}{A} = \varepsilon_0 E_p = P = \varepsilon_0(\varepsilon_r - 1)\cdot\frac{U_0}{d}$

c) Mivel a feszültségforrásról leválasztottuk, a kondenzátor töltése megmarad

$$U = \frac{Q}{C_0} = \frac{\cancel{\varepsilon_0}\varepsilon_r \cancel{A} U_0}{\cancel{d}} \cdot \frac{\cancel{d}}{\cancel{\varepsilon_0 A}} = \varepsilon_r U_0 = 36\,V$$

d) A lassú kihúzás során a kondenzátor energiája megváltozott.

$$W_1 = \frac{1}{2} C U_0^2 \; ; \; W_2 = \frac{1}{2} C_0 U^2$$

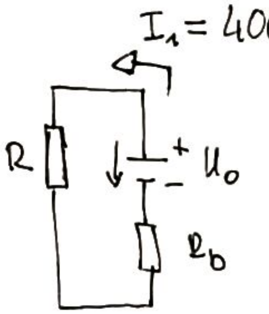
$$\Delta W = W_2 - W_1 = \frac{1}{2} C_0 \cdot \varepsilon_r^2 U_0^2 - \frac{1}{2} C_0 \cdot \varepsilon_r \cdot U_0^2 = \frac{1}{2} C_0 U_0^2 \varepsilon_r(\varepsilon_r - 1) =$$

$$= \frac{1}{2} \frac{\varepsilon_0 A}{d} U_0^2 \varepsilon_r(\varepsilon_r - 1) = 5{,}7 \cdot 10^{-7}\,J$$

F3.

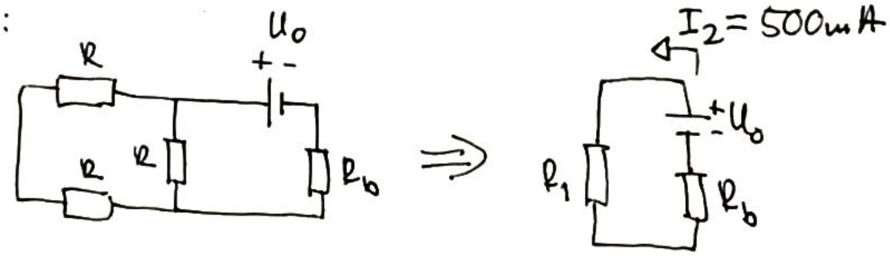
nyitott kapcsoló:

$\underline{R = 9\,\Omega}$



$$U_0 - R_b \cdot I_1 = R I_1 \quad (1)$$

zárt kapcsoló:



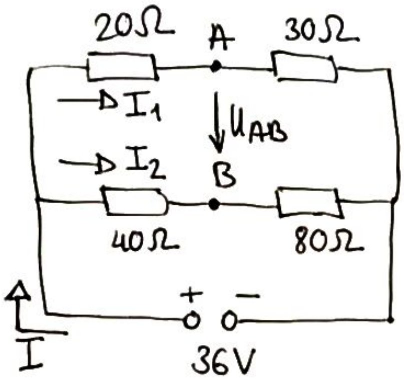
$$R_1 = \frac{2R \cdot R}{2R + R} = \frac{2}{3} R = 6\,\Omega$$

$$U_0 - R_b \cdot I_2 = R_1 \cdot I_2 \quad (2)$$

(1) − (2): $R_b (I_2 - I_1) = R I_1 - R_1 I_2$

$$R_b = \frac{R I_1 - R_1 I_2}{I_2 - I_1} = 6\,\Omega$$

F4.



a, $50\Omega \cdot I_1 = 120\Omega \cdot I_2 = 36V \rightarrow$

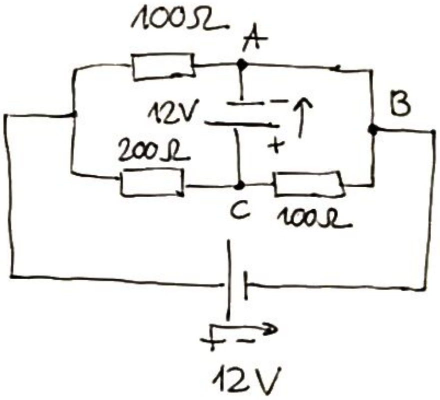
$\rightarrow I_1 = 0{,}72A; \; I_2 = 0{,}3A$

$U_{AB} = 40\Omega \cdot I_2 - 20\Omega \cdot I_1 = -2{,}4V$

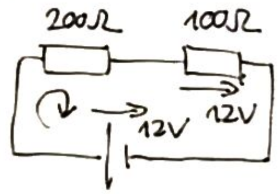
b, Wheatstone-híd: $20\Omega \cdot 80\Omega = 40\Omega \cdot R \rightarrow R = 40\Omega$

vagy: $\left.\begin{aligned} 20\Omega \cdot I_1 &= 40\Omega \cdot I_2 \\ R I_1 &= 80\Omega \cdot I_2 \end{aligned}\right\} \quad \frac{R}{20\Omega} = \frac{8}{4} \rightarrow R = 40\Omega$

F5.



a, A CB 100Ω-on 12V esik:

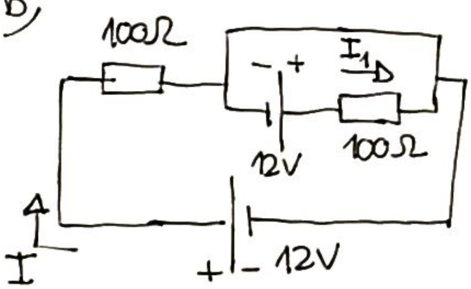


$U_{200\Omega} = 0\,V$

$\downarrow$

$I_{200\Omega} = 0.$

b,



$$I_1 = \frac{12V}{100\Omega} = 0{,}12A$$

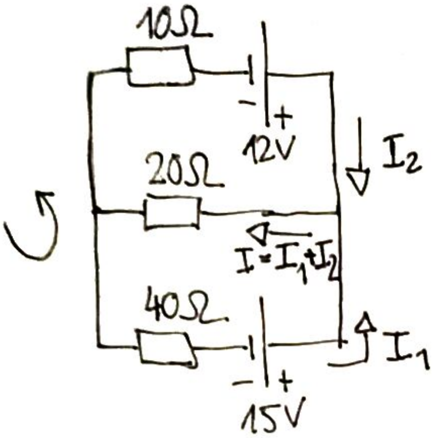
A „külső" 100Ω-on is 12V esik, így:

$I = I_1 = 0{,}12A \rightarrow I_{AB} = 0$

c, $P = 100\Omega \cdot (0{,}12A)^2 \cdot 2 = 2{,}88W.$

F6.

a)



$$40\Omega\cdot I_1 - 15V + 12V - 10\Omega\cdot I_2 = 0$$

$$40I_1 - 10I_2 = 3 \quad (1)$$

$$12V - 10\Omega\cdot I_2 - 20\Omega\,(I_1+I_2) = 0$$

$$20I_1 + 30I_2 = 12 \quad (2)$$

(1) és (2):

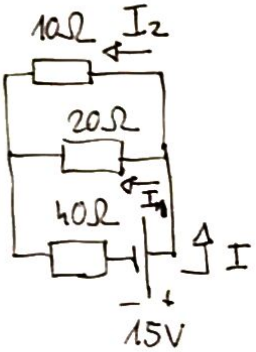
$$\left.\begin{aligned} 40I_1 - 10I_2 &= 3 \\ 40I_1 + 60I_2 &= 24 \end{aligned}\right\}$$

$$70I_2 = 21 \rightarrow I_2 = 0{,}3A$$

$$I_1 = 0{,}15A$$

$$I = 0{,}45A$$

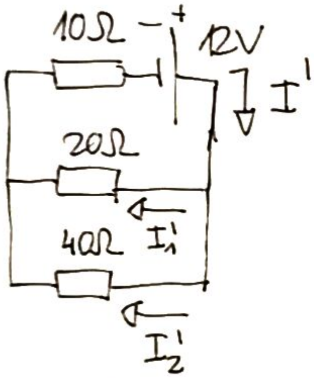
b)



$$I = \frac{15V}{40\Omega + \frac{10\cdot 20}{30}\Omega} = \frac{9}{28}A$$

$$I_1 = \frac{15V - 40\Omega\cdot I}{20\Omega} = \frac{3}{28}A$$

$$I_2 = I - I_1 = \frac{3}{14}A$$



$$I' = \frac{12V}{10\Omega + \frac{20\cdot 40}{60}\Omega} = \frac{18}{35}A$$

$$I_1' = \frac{12V - 10\Omega\cdot I'}{20\Omega} = \frac{12}{35}A$$

$$I_2' = I' - I_1' = \frac{6}{35}A$$

szuperpozíció:

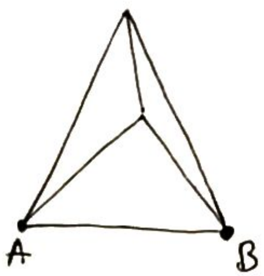
10Ω: $I' - I_2 = 0{,}3A$

20Ω: $I_1' + I_1 = 0{,}45A$

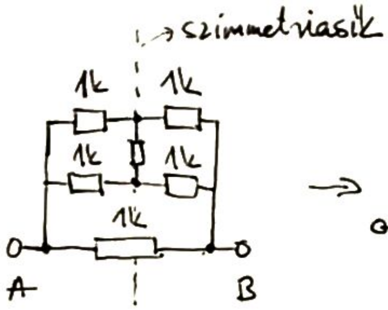
40Ω: $I - I_2' = 0{,}15A$

F7.

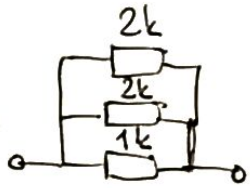
a)



$\rightarrow$

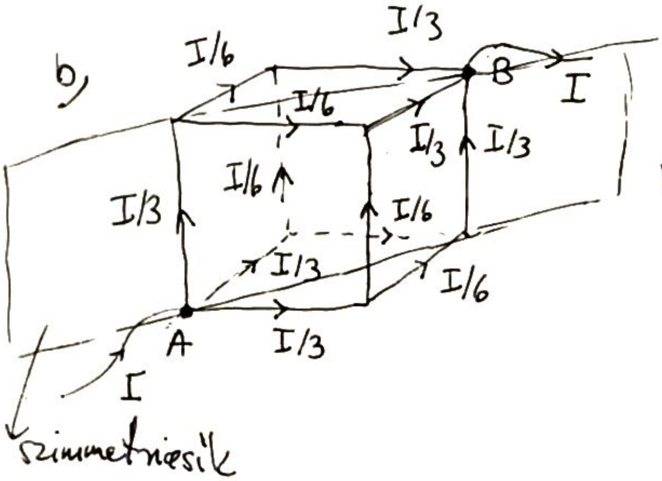


$\rightarrow$



$$R_{AB} = \frac{1}{\frac{1}{2}+\frac{1}{2}+\frac{1}{1}} = \frac{1}{2}\,k\Omega$$

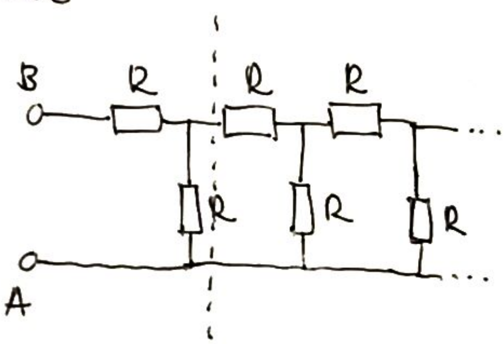
b)



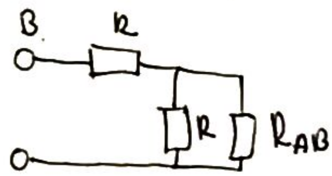
$$R_{AB}\cdot I = 1k\Omega\cdot\frac{I}{3} + 1k\Omega\cdot\frac{I}{6} + 1k\Omega\cdot\frac{I}{3}$$

$$R_{AB} = 2\cdot\frac{1}{3}+\frac{1}{6} = \frac{5}{6}\,k\Omega$$

F8.



Leválasztva az első egységet, továbbra is végtelen láncot kapunk:



$$R+\frac{R_{AB}\cdot R}{R_{AB}+R} = R_{AB} \Rightarrow R\cdot R_{AB}+R^2+\cancel{R\cdot R_{AB}} = \cancel{R\cdot R_{AB}}+R_{AB}^2$$

$$R_{AB}^2 - R\cdot R_{AB} - R^2 = 0 \rightarrow R_{AB} = \frac{1+\sqrt{5}}{2}R$$