13 2020.01.09
(Idmon) 1. vizsga

(1) $y'' + 4y' + 5y = 10x + 3$

$\lambda^2 + 4\lambda + 5 = 0$

$\lambda_{1,2} = \frac{-4 \pm \sqrt{16 - 4\cdot 5}}{2} = \frac{-4 \pm 2i}{2}$ $\begin{cases} -2+i \\ \underset{\alpha}{-2} - \underset{\beta}{i} \end{cases}$

$y_{in} = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)$

$y_{in} = e^{-2x}(C_1 \cos x + C_2 \sin x)$

$\left\{ \begin{array}{l} y_u: Ax + B \\ y_u': A \\ y_u'': 0 \end{array} \right.$

$0 + 4A + 5(Ax + B) = 10x + 3$

$5Ax + 4A + 5B = 10x + 3$

1) $5A = 10 \Rightarrow \boxed{A = 2}$

$4A + 5B = 3 \Rightarrow \boxed{B = -1}$

$y_{ált}: e^{-2x}(C_1 \cos x + C_2 \sin x) + 2x - 1$

(2) $\int_F \frac{(xy, x)}{\sqrt{x^2+y^2}}\, df = ?$ Felületi integrál definíciójából:

mivel a felület külső irány ott

$r(t) = -R(\cos t, \sin t) \quad t \in [0, 2\pi]$

$v = \begin{pmatrix} \frac{xy}{\sqrt{x^2+y^2}} \\ \frac{x}{\sqrt{x^2+y^2}} \end{pmatrix}$

$\dot{r}(t) = \begin{pmatrix} -R \sin t \\ R \cos t \end{pmatrix}$ $\quad CROSS(\dot{r}(t)) = \begin{pmatrix} -R\cos t \\ -R \sin t \end{pmatrix}$

$v(r(t)) = \begin{pmatrix} \frac{R^2 \sin t \cos t}{R^2} \\ \frac{R \cos t}{R^2} \end{pmatrix} \begin{pmatrix} \sin t \cos t \\ -\frac{\cos t}{R} \end{pmatrix}$

$\int_0^{2\pi} v(r(t)) \cdot CROSS(\dot{r}(t))\, dt = \int_0^{2\pi} \left(\sin t \cos t, -\frac{\cos t}{R}\right)(-R\cos t, -R\sin t)\, dt$

$f' \cdot f^{\alpha} = \frac{f^{\alpha+1}}{\alpha+1}$

$= \int_0^{2\pi} -R \sin t \cos^2 t + \sin t \cos t\, dt = -R \cdot \frac{\cos^3 t}{3} + \frac{\sin^2 t}{2} \Big|_0^{2\pi} = \underline{\underline{0}}$

③ a) $\exists$ pot fgv ? $\Rightarrow$ rot(v) = 0 ?

$$\text{rot}(v) = \det\begin{pmatrix} i & j & k \\ d_x & d_y & d_z \\ 2xye^{x^2}+y\cos(xy) & x\cos(xy)+e^{x^2} & 2z \end{pmatrix} =$$

$$= (0-0)\underline{i} + (0-0)\underline{j} + \left(\sin(xy)+2xe^{x^2}-\left(2xe^{x^2}+\sin(xy)\right)\right)\underline{k} \Rightarrow$$

$$\text{rot}(v) = (0,0,0) \Rightarrow \underline{\exists \text{ pot fgv}}$$

$$F = \int 2xye^{x^2}+y\cos(xy)\,dx = ye^{x^2}+xy\sin(xy)+C(y,z)$$

$$F'_y = e^{x^2}+x\cos(xy)+C'_y(y,z) = x\cos(xy)+e^{x^2}$$

$$\underline{C'_y(y,z) = 0}$$

$$(C_y = 0)$$

$$F'_z = 0+0+C'_z(z) = 2x$$

$$C(z) = \int 2z\,dz = \underline{z^2+C}$$

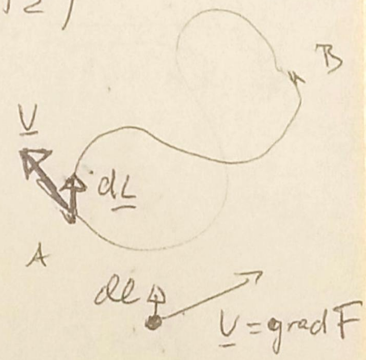
$$\underline{F = ye^{x^2}+xy\sin(xy)+z^2+C}$$

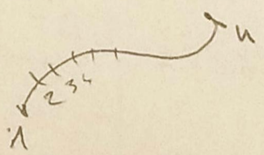
b) $\int_L v \, dr = ?$ ha L origótól $\left(1, \frac{\pi}{2}, \sqrt{2}\right)$

$$\int_L v\cdot dr = F\left(1,\frac{\pi}{2},\sqrt{2}\right) - F(0,0,0) =$$

$$+C-C$$

$$= \frac{\pi}{2}e^{1^2}+1\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right)+2 = \underline{\frac{\pi}{2}(e+1)+2}$$





$F_2 - F_1 + F_3 - F_2 \ldots$

$F_n - F_1$

④ $0 \neq r \in \mathbb{R}^3$

$$\operatorname{div}\left(\frac{r}{|r|^2}\right) = \left(\operatorname{grad}\frac{1}{|r|^2}\right)\cdot \underline{r} + \frac{1}{|r|^2}\cdot \operatorname{div}\underline{r} =$$

$$= -2\,|r|^{-4}\cdot \underbrace{\underline{r}\cdot r}_{|r|^2} + |r|^{-2}\cdot 3 = -2|r|^{-2} + 3|r|^{-2} = \underline{\underline{\frac{1}{|r|^2}}}$$

$$\operatorname{grad}|r|^{-2} = -2\,|r|^{-3}\cdot\frac{r}{|r|}$$