1, a, [5]

[4]
$$y''' - 3y'' = 0$$
$$\lambda^3 - 3\lambda^2 = \lambda^2(\lambda - 3) = 0 \Rightarrow \lambda_1 = \lambda_2 = 0, \quad \lambda_3 = 3 \quad ②$$
$$y_{H,\text{ált}}(x) = C_1 + C_2 x + C_3 e^{3x} \quad ②$$

b, [4]
$$y_{I,p}(x) = A x^2 \quad ② \quad (\text{Külső rezonancia!})$$
$$y''_{I,p} = 2A; \quad y'''_{I,p} = 0; \quad -3 \cdot 2A = 2 \Rightarrow A = -\frac{1}{3}$$
$$y_{I,\text{ált}}(x) = y_{H,\text{ált}}(x) + y_{I,p}(x) = C_1 + C_2 x + C_3 e^{3x} - \frac{1}{3}x^2 \quad ②$$

c, [5]
$$y_{I,p}(x) = A \sin(3x) + B \cos(3x) \quad ② \quad (\text{Nincs külső rezonancia})$$
$$y''_{I,p}(x) = -9A \sin(3x) - 9B \cos(3x)$$
$$y'''_{I,p}(x) = -27A \cos(3x) + 27B \sin(3x)$$
$$-27A\cos(3x) + 27B\sin(3x) + 27A\sin(3x) + 27B\cos(3x) = \sin(3x)$$
$$\left.\begin{array}{l} -A + B = 0 \\ A + B = \frac{1}{27} \end{array}\right\} \quad A = B = \frac{1}{54};$$
$$y_{I,\text{ált}}(x) = C_1 + C_2 x + C_3 e^{3x} + \frac{1}{54}\left(\sin(3x) + \cos(3x)\right) \quad ③$$

Csak a b esetben van külső rezonancia! ②

——— o ———

2, a, [15] [3]
$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

[4] b, Ha f akárhányszor diff.-ható az I intervallumon, és a deriváltak egyenletesen korlátosak I-n, akkor $f(x) = T(x)$ $\forall x \in I$ esetén

[8] c, ③
$$\begin{cases} \sin'(x) = \cos x \\ \sin''(x) = -\sin x \\ \sin'''(x) = -\cos x \\ \sin^{(4)}(x) = \sin x \\ \cos(0) = 1, \quad \sin(0) = 0 \end{cases}$$
$$T(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} = \sin x \quad ③$$

A deriváltak egyenletesen korlátosak; $K = 1$. ②

$\boxed{-2-}$

3,

$\boxed{15}$ $f(x)=\frac{1}{1+2x}=\frac{1}{1-(-2x)}=\sum_{n=0}^{\infty}(-2)^n x^n$ ③, ha $|-2x|<1$

$\Longrightarrow R=\frac{1}{2}$ ③

$g(x)=\frac{-1}{2}f'(x)$ ③ $=\frac{-1}{2}\sum_{n=0}^{\infty}(-2)^n n x^{n-1}=-\sum_{n=1}^{\infty}(-2)^{n-1} n x^{n-1}$ ③

$R=\frac{1}{2}$ (nem változik) ③

— ∘ —

4, a, Legyen $\underline{r}_0=\begin{bmatrix}x_0\\y_0\end{bmatrix}$, $\underline{r}=\begin{bmatrix}x\\y\end{bmatrix}$, $\Delta\underline{r}=\underline{r}-\underline{r}_0=\begin{bmatrix}\Delta x\\\Delta y\end{bmatrix}$. Ha

$\boxed{5}$ ③
$f(x,y)=f(x_0,y_0)+\underline{A}\cdot(\underline{r}-\underline{r}_0)+\underline{\varepsilon}(\Delta\underline{r})\cdot\Delta\underline{r}$

ahol $\underline{A}\in\mathbb{R}^2$ $\Delta\underline{r}$-től független, és $\lim_{\|\Delta\underline{r}\|\to 0}\underline{\varepsilon}(\Delta\underline{r})=\underline{0}$, akkor

f totálisan diff.-ható, és $\operatorname{grad} f|_{(x_0,y_0)}=\underline{A}$

$f'_x(x_0,y_0)=\lim_{h\to 0}\frac{f(x_0+h,y_0)-f(x_0,y_0)}{h}$ ②; $f'_y(x_0,y_0)=\lim_{h\to 0}\frac{f(x_0,y_0+h)-f(x_0,y_0)}{h}$ ①

b, 1, Ha f totálisan diff.-ható, akkor léteznek a parc. deriváltak

$\boxed{4}$ is, (és ezek a tot. derivált koordinátái) (komponensei)

2, Ha a parc. deriváltak léteznek és folytonosak (x_0,y_0) egy környezetében, akkor f tot. diff. ható (x_0,y_0)-ban, és

$$\operatorname{grad} f|_{(x_0,y_0)}=\begin{bmatrix}f'_x(x_0,y_0)\\f'_y(x_0,y_0)\end{bmatrix}$$

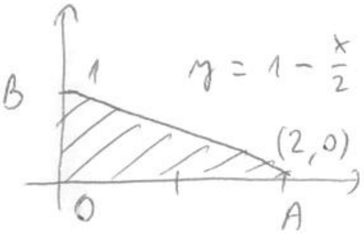
c,

$\boxed{6}$ $f(x,y)=\frac{x^2+xy}{1+y^2}$; $f'_x(x,y)=\frac{2x+y}{1+y^2}$ ①; $f'_x(1,2)=\frac{2+2}{1+4}=\frac{4}{5}$ ①

$f'_y(x,y)=\frac{x(1+y^2)-(x^2+xy)2y}{(1+y^2)^2}$ ②; $f'_y(1,2)=\frac{1\cdot 5-3\cdot 4}{25}=\frac{-7}{25}$ ①

$$\operatorname{grad} f|_{(1,2)}=\begin{bmatrix}4/5\\-7/25\end{bmatrix}=\frac{4}{5}\underline{i}+\frac{-7}{25}\underline{j}$$ ①

5^*, $\boxed{4}$ $f(x,y) = xy^2$



$y = 1 - \frac{x}{2}$; $x = 2 - 2y$

a, $\boxed{7}$ $$\int_T f\,dT = \int_{x=0}^{2}\left(\int_{y=0}^{1-\frac{x}{2}} xy^2\,dy\right)dx =$$

$$= \int_{y=0}^{1}\left(\int_{x=0}^{2-2y} xy^2\,dx\right)dy$$

b, $\boxed{7}$ $$\int_{x=0}^{2} x\left(\int_{y=0}^{1-\frac{x}{2}} y^2\,dy\right)dx = \int_{x=0}^{2} x\cdot\frac{1}{3}\left(1-\frac{x}{2}\right)^3 dx =$$

$$= \frac{1}{3}\int_{x=0}^{2}\left(x - \frac{3x^2}{2} + \frac{3x^3}{4} - \frac{x^4}{8}\right)dx = \frac{2}{3} - \frac{4}{3} + 1 - \frac{4}{15} = \frac{10-20+15-4}{15} = \underline{\underline{\frac{1}{15}}}$$

Vagy $$\int_{y=0}^{1} y^2\left(\int_{x=0}^{2-2y} x\,dx\right)dy = \int_{y=0}^{1}\frac{y^2}{2}(2-2y)^2 = 2\int_0^1\left(y^2 - 2y^3 + y^4\right)dy =$$

$$= \frac{2}{3} - \frac{4}{4} + \frac{2}{5} = \frac{10-15+6}{15} = \underline{\underline{\frac{1}{15}}}$$ ✓

6^*, a, $\boxed{7}$ $f(z)$ pontosan akkor diff.-ható a $z_0 = x_0 + iy_0$ pontban, ha u és v tot. diff.-ható (x_0, y_0)-ban, és teljesülnek a Cauchy-Riemann egyenletek:

$$u'_x(x_0,y_0) = v'_y(x_0,y_0), \text{ és } u'_y(x_0,y_0) = -v'_x(x_0,y_0)$$

Ekkor $f'(z_0) = u'_x(x_0,y_0) + i\,v'_x(x_0,y_0)$

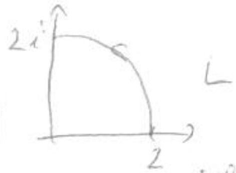
b, $\boxed{6}$ $f(z) = z\cdot\bar{z} = (x+iy)(x-iy) = x^2+y^2$; $u(x,y) = x^2+y^2$, $v(x,y) = 0$

$u'_x = 2x$; $u'_y = 2y$; $v'_x = v'_y = 0$ } C-R. egyenletek csak akkor teljesülnek, ha $x = y = 0$

$\Rightarrow$ f csak az origóban diff.-ható, sehol sem reguláris!

7* a,

$\boxed{7}$ $$\int_L |z|^2\,dz = \int_{\varphi=0}^{\pi/2} 4\cdot 2i\,e^{i\varphi}\,d\varphi = 8\cancel{i}\left[\frac{e^{i\varphi}}{\cancel{i}}\right]_0^{\pi/2} =$$



$z = 2e^{i\varphi}$, $\varphi\in[0,\frac{\pi}{2}]$

$\dot{z} = 2i\,e^{i\varphi}$

$$= 8\left(\underbrace{e^{i\frac{\pi}{2}}}_{i} - \underbrace{e^{0}}_{1}\right) = \underline{\underline{8i-8}}$$

b,

$\boxed{6}$ $$\int_L z^2\,dz = \left[\frac{z^3}{3}\right]_2^{2i} = \frac{(2i)^3 - 2^3}{3} = \underline{\underline{-\frac{8}{3}i - \frac{8}{3}}}$$

8, $y' = \frac{xy}{1+x^2}$; $\int \frac{dy}{y} = \int \frac{x\,dx}{1+x^2}$; $\ln|y| = \frac{1}{2}\ln(1+x^2) + C$

$\boxed{10}$ $y(x) = A\cdot\sqrt{1+x^2}$; $A\in\mathbb{R}$;

9, $a_{n+1} = a_n + 6a_{n-1}$; $a_0 = 6$; $a_1 = -2$

$\boxed{10}$

$\lambda^2 - \lambda - 6 = (\lambda-3)(\lambda+2) = 0$

$a_n = A\,3^n + B(-2)^n$

$$\left.\begin{aligned} a_0 &= 6 = A + B \\ a_1 &= -2 = 3A - 2B \end{aligned}\right\}\quad 10 = 5A\ ;\ A = 2\ ;\ B = 4$$

$$\underline{\underline{a_n = 2\cdot 3^n + 4(-2)^n}}$$