A) $4xyy' = 6y^2 - x^2 \; / : x^2 \; (\neq 0)$ $u = \frac{y}{x} \Rightarrow y = ux \Rightarrow y' = u'x + u$

(1)

$4u(u'x+u) = 6u^2 - 1$

$4uu'x + 4u^2 = 6u^2 - 1$

$4uu'x = 2u^2 - 1$

szétvál.

$u = \pm\frac{1}{\sqrt{2}}$ is megoldás

$\int \frac{4u\,du}{2u^2-1} = \int \frac{1}{x}\,dx$

$\ln|2u^2-1| = \ln|x| + \ln c_1$ ($c_1 > 0$)

$|2u^2-1| = c_1|x|$

Ez a megoldás is belerakható.

$2u^2 - 1 = \pm c_1 \cdot x$

$u = \pm\sqrt{\frac{\pm c_1 x + 1}{2}} \Longrightarrow u(x) = \pm\sqrt{\frac{cx+1}{2}}, \; c \in \mathbb{R}$

Tehát $\underline{\underline{y(x) = u(x)\cdot x = \pm x\sqrt{\frac{cx+1}{2}}}}, \; c \in \mathbb{R}, \; cx > -1$

(2) $xy' + 4y = 3x^3, \quad y(1) = 3/7$

elsőrendű lin., inhomogén

$y' + \underbrace{\frac{4}{x}}_{p(x)} y = \underbrace{3x^2}_{q(x)}$ $(x \neq 0)$

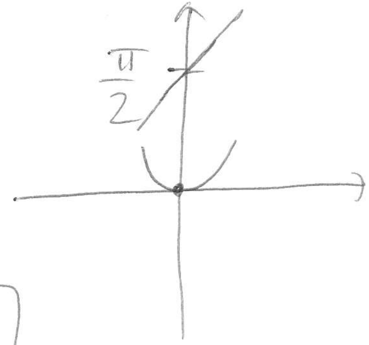
$\mu(x) = e^{\int \frac{4}{x}dx} = e^{4\ln|x|} = x^4$

$y(x) = \frac{1}{x^4}\cdot\left(\int 3x^2 \cdot x^4\,dx + c\right) = \frac{1}{x^4}\left(3\frac{x^7}{7} + c\right)$

$y(1) = \frac{3}{7} \Rightarrow c = 0 \Rightarrow$

$\boxed{y(x) = \frac{3}{7}\cdot x^3}$ $x \in \mathbb{R}$

(3) $y' = x + \sin y =: f(x,y)$



a) $x = 0$, $y = \frac{\pi}{2}$

$f(0,\frac{\pi}{2}) = 0 + \sin\frac{\pi}{2} = 1$

$y - \frac{\pi}{2} = 1(x-0) \Rightarrow \boxed{y = x + \frac{\pi}{2}}$

b) $x = 0$, $y = 0$

$f(0,0) = 0 + \sin 0 = 0$ (lehet szélsőérték)

$y'' = 1 + \cos y \cdot \underbrace{y'}_{=0} = 1 > 0$ a $(0,0)$ pontban $\Rightarrow$ lok. min van itt

(4) $y'' - 9y = 5e^{-3x}$

(H) $y'' - 9y = 0$

$\lambda^2 - 9 = 0 \Rightarrow \lambda_1 = 3,\ \lambda_2 = -3 \Rightarrow y_{há}(x) = c_1 e^{3x} + c_2 e^{-3x}$

(IH) $y_{ip}(x) = ?$ $\quad -9 \cdot /\ y_{ip}(x) = x \cdot Ae^{-3x}$ külső rezon.

$0 \cdot /\ y'_{ip}(x) = Ae^{-3x} + Ax(-3)e^{-3x}$

$1 \cdot /\ y''_{ip}(x) = -3Ae^{-3x} - 3Ae^{-3x} - 3Ax(-3)e^{-3x}$

	B.O.	J.O.
e^{-3x}	$-6A$	5
xe^{-3x}	$-9A + 9A$	0

$\Rightarrow A = -\frac{5}{6}$

OK. $\quad y_{ip}(x) = -\frac{5}{6} x e^{-3x}$

$$y_{iá}(x) = c_1 e^{3x} + c_2 e^{-3x} + \left(-\frac{5}{6}\right) x e^{-3x}$$

(5) $f_0 = 1,\ f_1 = -2,\ f_2 = 3$

$$f_n = 2f_{n-1} + 7f_{n-2} + 4f_{n-3}$$

$q^3 - 2q^2 - 7q - 4 = 0$ karakter-egyenlet

$$(q-4)\underbrace{(q^2+2q+1)}_{(q+1)^2} = 0 \Rightarrow q_1 = 4,\ q_{2,3} = -1$$

$$\Downarrow$$

$$f_n = c_1 \cdot 4^n + c_2(-1)^n + c_3 n(-1)^n$$

ez az általános feloldott alak

$f_0 = c_1 + c_2 = 1 \Rightarrow c_1 = 1 - c_2$

$f_1 = c_1 \cdot 4 + (-c_2) - c_3 = -2 \Rightarrow (1-c_2)4 - c_2 - c_3 = -2$

$f_2 = c_1 \cdot 16 + c_2 + 2c_3 = 3$

$4 - 4c_2 - c_2 - c_3 = -2$

$-5c_2 - c_3 = -6$

$\Downarrow$

$c_3 = -5c_2 + 6$

$\Downarrow$

$16(1-c_2) + c_2 + 2(-5c_2+6) = 3$

$16 - 16c_2 + c_2 - 10c_2 + 12 = 3$

$-25c_2 = -25$

$\underline{\underline{c_2 = 1}} \Rightarrow \underline{\underline{c_3 = 1}} \Rightarrow \underline{\underline{c_1 = 0}}$

$$f_n = (-1)^n + n(-1)^n$$

$$f_{1000} = (-1)^{1000} + 1000(-1)^{1000} = \underline{\underline{1001}}$$

B $\quad x^2 :/ \; 2xyy' = 3y^2 - x^2 \qquad u = \frac{y}{x}$

$x^2 \neq 0$

(1) $\quad 2u(u'x+u) = 3u^2 - 1 \qquad y = ux \Rightarrow y' = u'x + u$

$2uu'x + 2u^2 = 3u^2 - 1 \qquad \Rightarrow u = \pm 1$ is megoldás

$2uu'x = u^2 - 1$

$$\int \frac{2u\,du}{u^2-1} = \int \frac{1}{x}\,dx$$

$$\ln|u^2-1| = \ln|x| + \ln c_1 \quad (c_1 > 0)$$

$$|u^2-1| = c_1|x|$$

$$u^2 - 1 = \pm c_1 x$$

$$u^2 = 1 \pm c_1 x$$

$$u = \pm\sqrt{1 \pm c_1 x} \Rightarrow u(x) = \pm\sqrt{1+cx}, \quad c \in \mathbb{R}, \; cx > -1$$

Ez is bevonható a képletbe.

$$\Rightarrow y(x) = \pm x\sqrt{1+cx}$$

(2) $\quad xy' + 2y = 5x^3 \qquad y(1) = 1$

$(x \neq 0) \quad y' + \underbrace{\frac{2}{x}}_{p(x)} y = \underbrace{5x^2}_{q(x)}$

$$\mu(x) = e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = x^2$$

$$y(x) = \frac{1}{x^2}\left(\int x^2 \cdot 5x^2\,dx + c\right) = \frac{1}{x^2}\left(5\int x^4\,dx + c\right) =$$

$$= \frac{1}{x^2}\left(5 \cdot \frac{x^5}{5} + c\right) =$$

$$= \frac{c}{x^2} + x^3$$

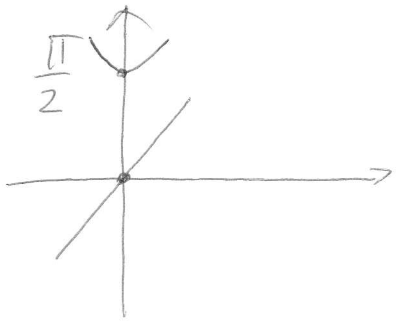
$y(1) = c + 1 \Rightarrow c = 0$

$\;\; \| $

$\;\; 1$

$$\boxed{y(x) = x^3}$$

③ $y' = x + \cos y =: f(x,y)$



a) $f(0,0) = 0 + \cos 0 = 1$

$y - 0 = 1 \cdot (x - 0)$

$\underline{\underline{y = x}}$

b) $f(0, \frac{\pi}{2}) = 0 + \cos\frac{\pi}{2} = 0$ (lehet szélsőérték)

$y'' = 1 - \sin y \cdot \underbrace{y'}_{=0} = 1$ a $(0, \frac{\pi}{2})$ pontban

$> 0 \Rightarrow$ lok. min. van itt

④ $y'' - 4y = 7e^{-2x}$ másodrendű, állandó e.h. inhomogén

(H) $y'' - 4y = 0$

$\lambda^2 - 4 = 0 \Rightarrow \lambda_1 = 2,\ \lambda_2 = -2 \Rightarrow y_{há}(x) = C_1 e^{2x} + C_2 e^{-2x}$

(IH) $y_{ip}(x) = ?$ $-4\cdot/\ y_{ip}(x) = x \cdot A e^{-2x}$ ← külső rezonancia

$0\cdot/\ y_{ip}'(x) = Ae^{-2x} + Ax(-2)e^{-2x}$

$1\cdot/\ y_{ip}''(x) = -2Ae^{-2x} - 2Ae^{-2x} + 4Axe^{-2x}$

	B.O.	J.O.	
e^{-2x}	$-4A$	7	$A = -\frac{7}{4}$
xe^{-2x}	$-4A + 4A$	0	OK.

$y_{ip}(x) = -\frac{7}{4} x e^{-2x}$

$\underline{y_{iá}(x) = C_1 e^{2x} + C_2 e^{-2x} - \frac{7}{4} x e^{-2x}}$

(5) $f_0 = 1, \ f_1 = 0, \ f_2 = -1$

$f_n = f_{n-1} + 5 f_{n-2} + 3 f_{n-3}$

$q^3 - q^2 - 5q - 3 = 0$

$$\left.\begin{array}{l}(q+1)\underbrace{(q^2-2q-3)}_{(q+1)(q-3)} = 0\end{array}\right\} \Rightarrow q_1 = 3, \ q_{2,3} = -1$$

$\Downarrow$

$f_n = c_1 \cdot 3^n + c_2(-1)^n + c_3 n(-1)^n$

$f_0 = c_1 + c_2 = 1$

$f_1 = 3c_1 - c_2 - c_3 = 0$

$f_2 = 9c_1 + c_2 + 2c_3 = -1$

$\Rightarrow 4c_1 - c_3 = 1 \Rightarrow c_3 = 4c_1 - 1$

$c_2 = 1 - c_1$

$9c_1 + (1-c_1) + 2(4c_1 - 1) = -1$

$9c_1 + 1 - c_1 + 8c_1 - 2 = -1$

$16c_1 = 0 \Rightarrow c_1 = 0$

$\Downarrow$

$c_2 = 1$

$\Downarrow$

$c_3 = -1$

$f_n = (-1)^n - n(-1)^n$

$f_{1000} = (-1)^{1000} - 1000(-1)^{1000} =$

$= 1 - 1000 = \underline{\underline{-999}}$

B3