ANALÍZIS SZIGORLAT | −1− | (2018. 05. 30.

α VARIÁNS

1, T.: Ha $(a_n)_{n\in\mathbb{N}}$ konvergens, akkor korlátos. (3)

B.: $\varepsilon = 1$ esetén $\exists N \in \mathbb{N}: \ \forall n > N$-re $|a_n - A| < 1$, ahol $A = \lim_{n\to\infty} a_n$ (7)

Így $\max\{|a_0|, |a_1|, .. |a_N|, |A|+1\}$ jó korlát.

2 a, [6]

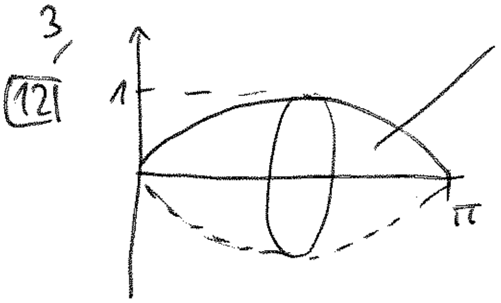
$$\left(\frac{n+4}{2n+3}\right)^{2n} = \left(\frac{1}{2}\right)^{2n} \cdot \frac{(1+4/n)^{2n}}{(1+3/2n)^{2n}} \ (2) \longrightarrow 0 \cdot \frac{e^8}{e^3} = 0 \ (2)$$

(2) (kiemelés)

[6] b,

$$\lim_{x\to 0+} x\ln(4x) = \lim_{x\to 0+} \frac{\ln(4x)}{1/x} \ (2) \overset{L'H}{=} \lim_{x\to 0+} \frac{1/x}{-1/x^2} \ (2) = \lim_{x\to 0+}(-x) = 0 \ (2)$$

3, [12]



$$V = \int_{x=0}^{\pi} \sin^2(x)\cdot\pi \, dx \ (4) = \pi\int_0^{\pi} \frac{1-\cos(2x)}{2} dx \ (4) = \frac{\pi^2}{2} \ (4)$$

4, [12] $y = 3e^{2x}(\sin(3x)+5) = 3e^{2x}\sin(3x) + 15e^{2x}$

$\Rightarrow \lambda_1 = 2 \ ; \ \lambda_{2,3} = 2 \pm 3i$ (3)

Kar. pol.: $(\lambda-2)(\lambda-2-3i)(\lambda-2+3i) \ (2) = (\lambda-2)\underbrace{\left((\lambda-2)^2+9\right)}_{\lambda^2-4\lambda+13} =$

$= \lambda^3 - 6\lambda^2 + 21\lambda - 26$

$\Rightarrow \boxed{y''' - 6y'' + 21y' - 26y = 0}$ (3)

$y_{H,\text{ált}}(x) = Ae^{2x} + Be^{2x}\sin(3x) + Ce^{2x}\cos(3x)$ (4)

$A, B, C \in \mathbb{R}$

5, a, Ha $\forall n \in \mathbb{N}$ esetén $f_n \in R[a,b]$ (Riemann-int.-ható)

$\boxed{4}$ és $\sum_{n=1}^{\infty} f_n \rightrightarrows f$ $[a,b]$-n, akkor $f \in R[a,b]$, és

$$\int_a^b f(x)\,dx = \sum_{n=1}^{\infty} \left(\int_a^b f_n(x)\,dx \right)$$

$\boxed{10}$ b, $f'(x) = \frac{1}{1+x^2}$ (2) $= \sum_{n=0}^{\infty} (-x^2)^n$ (3) $= \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n}$, ha $|x| < 1$

(Geometriai sor)

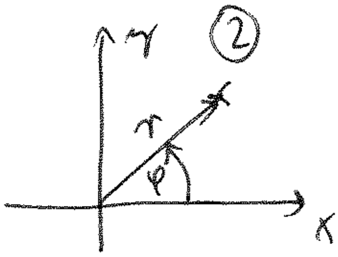
$$f(x) = f(0) + \int_{t=0}^{x} f'(t)\,dt = 0 + \int_{t=0}^{x} \left(\sum_{n=0}^{\infty} (-1)^n t^{2n} \right) dt = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad (4)$$

$R = 1$ (marad) (1)

$\boxed{4}$ c, $x = 1$ helyen a sor konvergens,

$$f(1) = \operatorname{arctg} 1 = \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

6* a, $\boxed{8}$



(2)

$x = r\cos\varphi$, $y = r\sin\varphi$ (2) $\quad \det(J(r,\varphi)) = \begin{vmatrix} x'_r & x'_\varphi \\ y'_r & y'_\varphi \end{vmatrix} =$

$$= \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} \ (2) = r(\cos^2\varphi + \sin^2\varphi) = r \ (2)$$

$\boxed{6}$ b, $V = \int_{r=0}^{R} \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} 1 \cdot r^2 \sin\vartheta \, d\varphi \, d\vartheta \, dr =$ (3) (az integrál felírása)

$$= \left[\frac{r^3}{3}\right]_0^R \cdot 2\pi \cdot \left[-\cos\vartheta\right]_0^\pi = \frac{R^3}{3} \cdot 2\pi \cdot 2 = \frac{4R^3\pi}{3}$$ (3) (kiszámolás)

7. $\boxed{12}$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx \overset{(2)}{=} \frac{2}{\pi}\int_0^{\pi} \sin x\,dx = \frac{2}{\pi}\Big[-\cos x\Big]_0^{\pi} = \frac{4}{\pi} \quad (2)$$

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f(x)\cos x}_{\text{páros}}\,dx \overset{(2)}{=} \frac{2}{\pi}\int_0^{\pi} \sin x \cos x\,dx = \frac{1}{\pi}\int_0^{\pi} \sin(2x)\,dx = 0 \quad (2)$$

$$b_5 = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f(x)\sin(x)}_{\text{páratlan}}\,dx \overset{(2)}{=} 0 \quad (2)$$

— . —

8. $\boxed{10}$

a) $$\mathcal{F}[f](\omega) = \int_{-\infty}^{\infty} e^{-i\omega x} f(x)\,dx \quad (3)$$

b) $$(f * g)(x) = \int_{t=-\infty}^{\infty} f(t)\, g(x-t)\,dt \quad (3)$$

c) $$\mathcal{F}[f * g] = \mathcal{F}[f]\cdot\mathcal{F}[g] \quad (4)$$

— . —

B - VARIÁNS

1, – mint A.

2, a, $\boxed{6}$ $\left(\frac{3n+5}{n+3}\right)^{2n} = \underbrace{3^{2n}}_{\downarrow\ \infty} \cdot \frac{\left(1+5/n\right)^{2n} \to e^{10}}{\left(1+3/n\right)^{2n} \to e^{6}} \longrightarrow \infty \cdot \frac{e^{10}}{e^{6}} = \infty$ (2)

b, $\boxed{6}$ $\lim_{x\to\infty} \frac{1}{x}\cdot \ln\left(\frac{3}{x}\right) = \lim_{x\to\infty} \frac{\ln 3 - \ln x}{x} \overset{l'H}{=} \lim_{x\to\infty} \frac{-1/x}{1} = 0$ (2)

3, $\boxed{12}$ $V = \int_{-\pi/2}^{\pi/2} \pi \cos^2 x \, dx = \frac{\pi}{2} \int_{-\pi/2}^{\pi/2} (1+\cos(2x))\, dx = \frac{\pi^2}{2}$ (4)

4, $\boxed{12}$ $\lambda_1 = 3, \quad \lambda_{2,3} = 3 \pm 2i$; (3)

Kar. pol.: $(\lambda-3)(\lambda-3-2i)(\lambda-3+2i) = (\lambda-3)\overbrace{((\lambda-3)^2+4)}^{\lambda^2-6\lambda+13} =$

$= \lambda^3 - 9\lambda^2 + 31\lambda - 39$

$\Rightarrow y''' - 9y'' + 31y' - 39y = 0$ (3); $\quad y_{H,ált}(x) = Ae^{3x} + Be^{3x}\sin(2x) + Ce^{3x}\cos(2x)$ (4)

5, – mint A; 6* mint A;

7*, $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x|\, dx = \frac{4}{\pi}\int_0^{\pi/2} \cos x\, dx = \frac{4}{\pi}$ (2)

$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{\cos x \cdot |\cos x|}_{\text{páros}}\, dx = \frac{2}{\pi}\int_0^{\pi} \cos x \cdot |\cos x|\, dx = \frac{2}{\pi}\left(\int_0^{\pi/2} \cos^2 x\, dx - \int_{\pi/2}^{\pi} \cos^2 x\, dx\right) = 0$ (2)

$b_3 = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{\sin(3x)|\cos x|}_{\text{páratlan}}\, dx = 0$ (2)

8*, a, $\mathcal{F}^{-1}[F](x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega x} F(\omega)\, d\omega$ (3)

b, c, – mint A.