1, $\boxed{12}$ $f(x,y)=\ln(x^4+y^2)+e^{xy^4}$

$$f'_x(x,y)=\frac{4x^3}{x^4+y^2}+y^4e^{xy^4}\ (2)\ ;\quad f'_x(0,1)=1\ (1)$$

$$f'_y(x,y)=\frac{2y}{x^4+y^2}+4y^3xe^{xy^4}\ (2)\ ;\quad f'_y(0,1)=2\ (1)$$

Érintő sík:
$$z=\underbrace{f(0,1)}_{1\ (1)}+f'_x(0,1)(x-0)+f'_y(0,1)(y-1)\ (1)=1+x+2(y-1)\ (1)$$
$$\underline{\underline{z=x+2y-1}}$$

Iránymenti derivált: $|\underline{v}|=\sqrt{3^2+4^2}=\sqrt{9+16}=5$; $\underline{e}=\frac{\underline{v}}{|\underline{v}|}$ (1)

$$\left.\frac{df}{d\underline{e}}\right|_P=\operatorname{grad}f(P)\cdot\underline{e}\ (1)=\frac{1\cdot3+2\cdot4}{5}=\underline{\underline{\frac{11}{5}}}\ (1)$$

2, $\boxed{13}$ $f(x,y)=4xy-x^2y-y^2$

$$f'_x(x,y)=4y-2xy\ (1)=2y(2-x)\Rightarrow y=0 \text{ vagy } x=2$$
$$f'_y(x,y)=4x-x^2-2y\ (1)=0$$

Ha $y=0$, $4x-x^2=x(4-x)=0\Rightarrow A(0,0)$, $B(4,0)$

Ha $x=2$, $8-4-2y=4-2y=0\Rightarrow C(2,2)$

$A(0,0)$, $B(4,0)$, $C(2,2)$: vizsgálandó pontok. (5)

$$H(x,y)=\begin{vmatrix}-2y & 4-2x\\ 4-2x & -2\end{vmatrix}=4y-4(2-x)^2\ (3)$$

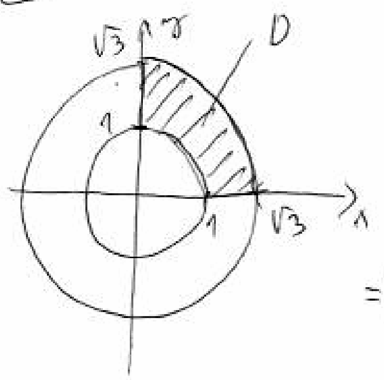
A: $H(0,0)=-16<0\Rightarrow$ Nyeregpont (1)
B: $H(4,0)=-16<0\Rightarrow$ — " — (1)
C: $H(2,2)=8>0$; $f''_{xx}(2,2)=-4<0\Rightarrow$ lokális max. (1)

3, $\boxed{6}$ Az integrálás sorrendjét felcseréljük:

$$\int_0^1\left(\int_1^2 x\sqrt{xy+1}\,dx\right)dy=\int_{x=1}^2\left(x\int_{y=0}^1\sqrt{xy+1}\,dy\right)dx\ (1)=\int_{x=1}^2 \cancel{x}\left[\frac{(xy+1)^{3/2}}{(3/2)\cdot \cancel{x}}\right]_{y=0}^1dx=\ (2)$$

$$=\int_{x=1}^2\frac{2}{3}\left((x+1)^{3/2}-1\right)dx\ (1)=\frac{2}{3}\left[\frac{(x+1)^{5/2}}{5/2}\right]_{x=1}^2-\frac{2}{3}\ (1)=\underline{\underline{\frac{4}{15}\left(3^{5/2}-2^{5/2}\right)-\frac{2}{3}}}\ (1)$$

6, $\boxed{10}$



$0 \le \varphi \le \pi/2$

$1 \le r \le \sqrt{3}$

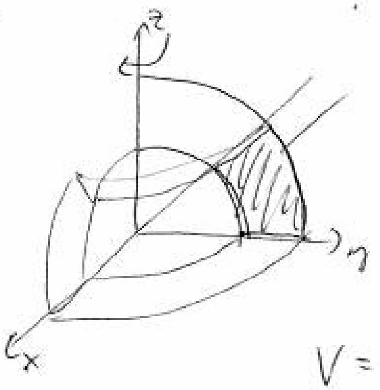
$$\iint\limits_D \frac{y}{x^2+y^2+1}\,dx\,dy = \int_{\varphi=0}^{\pi/2}\int_{r=1}^{\sqrt{3}} \frac{r\sin\varphi}{r^2+1}\, r\,dr\,d\varphi = \quad ③$$

$$= \left(\int_0^{\pi/2} \sin\varphi\,d\varphi\right)\cdot\left(\int_{r=1}^{\sqrt{3}} \frac{r^2}{r^2+1}\,dr\right) ① = \Big[-\cos\varphi\Big]_0^{\pi/2} ② \cdot \int_{r=1}^{\sqrt{3}}\left(1-\frac{1}{1+r^2}\right)dr =$$

$$= 1\cdot\Big[r - \operatorname{arctg} r\Big]_1^{\sqrt{3}} ③ = \sqrt{3}-1-\underbrace{\operatorname{arctg}\sqrt{3}}_{\pi/3}+\underbrace{\operatorname{arctg} 1}_{\pi/4} = \sqrt{3}-1-\frac{\pi}{12} \quad ①$$

— o —

5, $\boxed{9}$



Ezt forgatjuk el 90°-kal z-körül

$$\left.\begin{aligned} &1 \le r \le 2 \\ &0 \le \varphi \le \frac{\pi}{2} \\ &\frac{\pi}{4} \le \vartheta \le \frac{\pi}{2} \end{aligned}\right\} ③$$

$$V = \int_{r=1}^{2}\int_{\vartheta=\pi/4}^{\pi/2}\int_{\varphi=0}^{\pi/2} \underbrace{r^2\sin\vartheta}_{\text{Jacobi}}\,d\varphi\,d\vartheta\,dr = \quad ②$$

$$= \underbrace{\left(\int_{r=1}^{2} r^2\,dr\right)}_{\left[\frac{r^3}{3}\right]_1^2 = \frac{8-1}{3} = \frac{7}{3}}\cdot\underbrace{\left(\int_{\vartheta=\pi/4}^{\pi/2}\sin\vartheta\,d\vartheta\right)}_{\Big[-\cos\vartheta\Big]_{\pi/4}^{\pi/2} = 0+\frac{1}{\sqrt{2}}}\cdot\underbrace{\left(\int_{\varphi=0}^{\pi/2} d\varphi\right)}_{\frac{\pi}{2}} = \underline{\underline{\frac{7}{3}\cdot\frac{1}{\sqrt{2}}\cdot\frac{\pi}{2}}} \quad ④$$