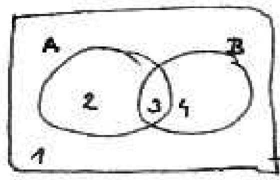
Valszám gyak

2008.09.19.
Othucsár György

Othucsár György : othi @ szit.bme.hu

2) σ algebra :
- $\Omega \in \mathcal{F}$
- $A \in \mathcal{F} \Rightarrow \bar{A} \in \mathcal{F}$
- $A_1, \dots A_i \in \mathcal{F} \Rightarrow \sum_{i=1}^{\infty} A_i \in \mathcal{F}$



Ha binárisan fogjuk fel, látjuk, hogy $16 = 2^4$ olyan van.

$$\sigma(\{A,B\}) = \{\Omega, A, B, \emptyset, \bar{A}, \bar{B}, A+B, \bar{A}+B, A+\bar{B}, \bar{A}+\bar{B}, \bar{A}\cdot\bar{B}, \bar{A}\cdot B, A\cdot\bar{B}, A\cdot B, A\cdot\bar{B}+\bar{A}B, \overline{\bar{A}\bar{B}+AB}\}$$

5) $A, B \in \mathcal{F} \Rightarrow \quad -\frac{1}{4} \leq P(AB) - P(A)\cdot P(B) \leq \frac{1}{4}$

- tfh $0 \leq P(AB) - P(A)\cdot P(B) \leq P(A) - P(A)\cdot P(B) = P(A)\cdot P(\bar{B})$

$\quad P(AB) - P(A)\cdot P(B) \leq P(B) - P(A)\cdot P(B) = P(B)\,P(\bar{A})$

$$\left(P(AB) - P(A)P(B)\right)^2 \leq P(A)\cdot P(\bar{A})\cdot P(B)\cdot P(\bar{B}) = \frac{1}{16} \quad /\sqrt{\ }$$

akkor a legnagyobb, ha egyenlők

- $0 \leq P(A)\cdot P(B) - P(AB) \leq P(A)P(B) - P(A) - P(B) - 1 = (1-P(A))(1-P(B)) =$

$\qquad = P(\bar{A})\cdot P(\bar{B})$

$P(A+B) = P(A) + P(B) - P(AB)$
$1 \geq$

de $P(A)\cdot P(B) - P(AB) \leq P(A)\cdot P(B)$

$$\left(P(A)P(B) - P(AB)\right)^2 \leq P(A)\cdot P(B)\cdot P(\bar{A})\cdot P(\bar{B}) = \frac{1}{16} \quad /\sqrt{\ } \quad /\cdot(-1)$$

12.) n elem permutációja

A_{ij} : az eredetiben az i. elem most a j. helyen van.

A: „első elem a 2.-tól balra áll"

$$\sum_{j=1}^{n-1} A_{1j}\cdot\left(\sum_{k=j+1}^{n} A_{2k}\right)$$

B: „az első elem sorszáma legfeljebb j."

$$\sum_{i=1}^{j} A_{1i}$$

16.) 3 kocka. A: összeg 7 $\qquad A\cdot B = \emptyset$
B: mind ps $\qquad B\cdot C = \emptyset$
C: nem között 3-as.

$$P(A(B+\bar{C})) = P(AB + A\bar{C}) = \underset{=0}{P(AB)} + P(A\bar{C}) - \underset{=0}{P(AB\bar{C})} = P(A\bar{C}) = \frac{9}{216}$$

115 → 3
124 → 6
9

b) $P((A+C)\bar{B}) = P(\underbrace{A\cdot\bar{B}}_{A} + \underbrace{C\cdot\bar{B}}_{C}) = P(A+C) = P(A)+P(C)-P(AC) = \frac{15}{216} + \frac{91}{216} - \frac{6}{216} = \frac{100}{216}$

$3 \leftarrow 115$ $\quad 216-125$
$6 \leftarrow 124$ $\quad = 91$
$\left\{ \begin{array}{l} 3 \leftarrow 133 \\ 3 \leftarrow 223 \end{array} \right.$

19.) 90/5 lottó

középső < 50

$$\frac{\sum_{i=3}^{49} \binom{i-1}{2}\binom{90-i}{2}}{\binom{90}{5}}$$

21.) n db kockával dobunk

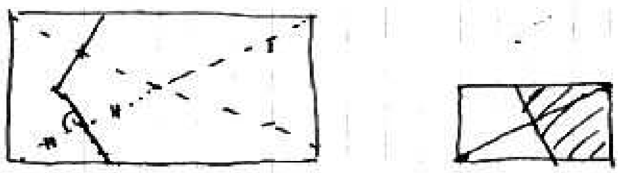
A: az összes kocka u.a.
B: legalább 1 db 6-os
C: pontosan 1 db 6-os

$P(A) = 6 \cdot \left(\frac{1}{6}\right)^n$

$P(B) = 1 - \frac{5^n}{6^n} = 1 - \left(\frac{5}{6}\right)^n$

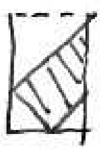
$P(C) = n \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{n-1}$

28)



$P(A) = \frac{1}{2}$

35.) $x \in [0,2]$ $\quad x+y > 1$
$y \in [0,3]$ $\quad x+1 > y$
$\quad y+1 > x$



$P(A) = 1 - \frac{2\cdot 2 + 1\cdot 1\cdot 2}{2\cdot 3}\cdot\frac{1}{2} = \frac{1}{2}$

46.) pp PP pf

A: 2 piros golyó van benne $\quad P(A) = \frac{2}{3} \Rightarrow P(\bar{A}) = \frac{1}{3}$

B: piros golyót húzunk

P(2. fehér | 1. piros)

$P(B|A) = 1$

$P(B|\bar{A}) = \frac{1}{2}$

Bayes-t.

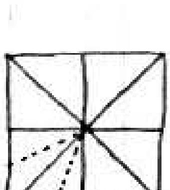
$$P(\bar{A}|B) = \frac{P(\bar{A}B)}{P(B)} = \frac{P(B|\bar{A})\cdot P(\bar{A})}{P(B|A)\cdot P(A) + P(B|\bar{A})\cdot P(\bar{A})} = \frac{\frac{1}{2}\cdot\frac{1}{3}}{1\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5}$$

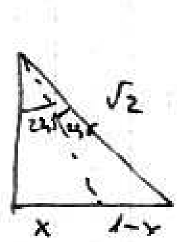
teljes valség tétel

49.) $\frac{1}{6} + \frac{4}{6}\cdot\frac{1}{6} + \left(\frac{2}{3}\right)^2\cdot\frac{1}{6} + \left(\frac{2}{3}\right)^3\cdot\frac{1}{6} + \ldots = \frac{1}{6}\cdot\sum_{i=0}^{\infty}\left(\frac{2}{3}\right)^i = \frac{1}{6}\cdot\frac{1}{1-\frac{2}{3}} = \frac{1}{2}$

55.)

$1-5\frac{\pi}{8}$?





$\frac{1}{x} = \frac{\sqrt{2}}{1-x}$

$P(A) = \frac{1-x}{1} = \frac{\sqrt{2}}{1+\sqrt{2}}$

$1-x = \sqrt{2}x \Rightarrow x(1+\sqrt{2}) = 1 \Rightarrow x = \frac{1}{1+\sqrt{2}} \Rightarrow 1-x = 1 - \frac{1}{1+\sqrt{2}}$

Valószínű gyak.　　　　2008.09.19.
Obi

84.) 5 fekete, 7 fehér
3 fekete, 8 fehér　$\downarrow 1 \uparrow 1$

P(fehér az elsőből) = ?

$= P(\text{fehér} \mid (4,8)) \cdot P(4,8) + P(\text{fehér} \mid (6,6)) \cdot P(6,6) +$

$+ P(\text{fehér} \mid (5,7)) \cdot P(5,7) = \frac{1027}{1728}$

$P(4,8) = 5/12 \cdot \frac{8}{12} = 40/144$
$P(6,6) = 7/12 \cdot 3/12 = 21/144$
$P(5,7) = 5/12 \cdot 4/12 + 7/12 \cdot 9/12 = 83/144$

94.) A, B fgt.
C kizárja A-t, B-t.

$P(A) = P(B) = P(C) = \frac{1}{3} P(\bar{A}+B+C)$

$P(AC) = 0$

$P(BC) = 0$

$P(B \mid A) = P(B)$

$P(\bar{A}+B+C) = \underbrace{P(\bar{A})}_{=\frac{2}{3}} + \underbrace{P(B)}_{=\frac{1}{3}} + P(C) - \underbrace{P(\bar{A}C)}_{P(C)} - P(\bar{A}B) - \underbrace{P(BC)}_{=0} + \underbrace{P(\bar{A}BC)}_{=0}$

$P(\bar{A}B) = P(\bar{A}) \cdot P(B) = \frac{2}{3} \cdot \frac{1}{3}$

$= \frac{7}{9}.$

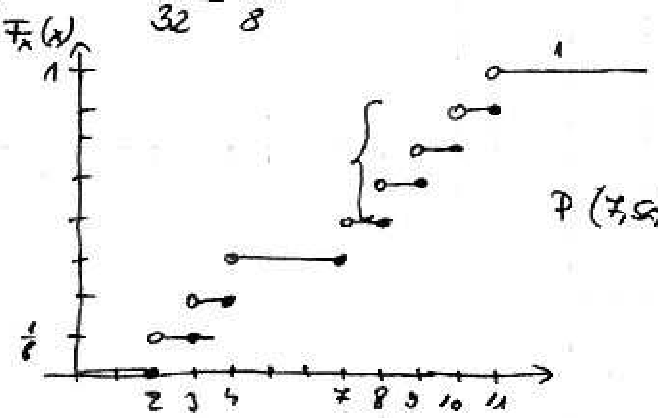
5.) K: 32-ből 5, kártya. X: lap értéke.

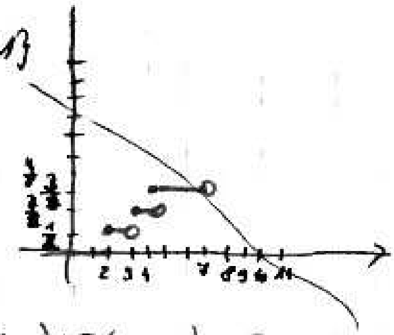
X eloszlás ?, $P(7{,}5 < X < 10{,}2) = ?$

$F_x(x) = P(X<x)$

$R_x = \{2,3,4,7,8,9,10,11\}$

$P(X=i) = \frac{4}{32} = \frac{1}{8}$





$P(7{,}5<X<10{,}2) = P(X=8)+P(X=9)+P(X=10) = \frac{3}{8}$

9.) 1% selejt X: 1 dobozban lévő hibás termékek száma

1000/doboz

véletlenszerű doboz:

$P(X\le 3)=?$

$X \in B(n,p)$

$P(X=k) = \binom{n}{k}\cdot p^k (1-p)^{n-k}$

$X \in B(100, \frac{1}{100})$

$P(X\le 3) = \binom{1000}{1}\cdot\frac{1}{100}\cdot\left(\frac{99}{100}\right)^{99} + \binom{100}{2}\cdot\left(\frac{1}{100}\right)^2\cdot\left(\frac{99}{100}\right)^{98} + \binom{100}{3}\cdot\left(\frac{1}{100}\right)^3\cdot\left(\frac{99}{100}\right)^{97} +$

$+ \binom{1000}{0}\cdot\left(\frac{99}{100}\right)^{100} = \sum_{k=0}^{3}\binom{1000}{k}\cdot\left(\frac{1}{100}\right)^k\cdot\left(\frac{99}{100}\right)^{100-k}$

14.) 20 napból 2-n nincs rekl. $P(X\ge 3) = ?$

Poisson-eloszlás $X \in Po(\lambda)$

X: reklamációk száma $Po(\lambda):\ P(X=k) = \frac{\lambda^k}{k!}\cdot e^{-\lambda}$

$P(X=0) = e^{-\lambda} = \frac{2}{20} = \frac{1}{10} \Rightarrow \lambda = \ln 10$

$P(X\ge 3) = 1 - P(X<3) = 1 - P(X=0) - P(X=1) - P(X=2) = 1 - e^{-\lambda}\left(1+\lambda+\frac{\lambda^2}{2}\right) = 0{,}404$

45.) K: kockadobás, míg 3-nál kisebbet kapunk. $P(2\le X\le 3) \overset{?}{\gtrless} P(X>3)$

X: dobások száma $X \in G(p)$

$R_x = \{1,2,3,\dots\}$ $P(X=k) = (1-p)^{k-1}\cdot p$

$p = \frac{1}{3}$

$P(X=2) = \frac{2}{3}\cdot\frac{1}{3} = \frac{2}{9} = \frac{6}{27}$

$P(X=3) = \left(\frac{2}{3}\right)^2\cdot\frac{1}{3} = \frac{4}{27}$

$P(2 \le X \le 3) = \frac{10}{27}$

$P(X>3) = 1 - p(X=1) - p(X=2) - p(X=3) =$

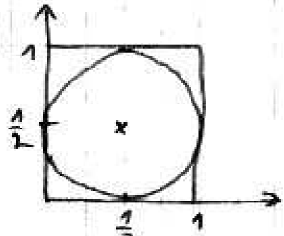
$= 1 - \frac{1}{3} - \frac{6}{27} - \frac{4}{27} = \frac{8}{27}$

51) $X \in Po(2)$ $\qquad R_X = \{0,1,2,\dots\}$

$Y = \left[\frac{X}{2}\right]$ eloszlása $\qquad R_Y = \{0,1,2,3,\dots\}$

$$P(Y=i) = P\left(\left[\frac{X}{2}\right]=i\right) = P(X=2i)+P(X=2i+1) = \frac{2^{2i}}{(2i)!}\cdot e^{-2} + \frac{2^{2i+1}}{(2i+1)!}\cdot e^{-2}$$

50.)



n pont véletlenszerűen

X azon pontok száma amelyek a körbe esnek.

$P(X \le 5) = ?$

$p = \frac{\left(\frac{1}{2}\right)^2 \pi}{1} = \frac{\pi}{4}$ egy pont a körbe esik

$X \in B(n, p) = B\left(n, \frac{\pi}{4}\right)$

$$P(X \le 5) = \sum_{i=0}^{5} \binom{n}{i} \cdot \left(\frac{\pi}{4}\right)^i \cdot \left(1-\frac{\pi}{4}\right)^{n-i}$$

67.) 90/5 lottó

a legkisebb kihúzott szám eloszlásfv-e 25 helyen.

X: legkisebb kihúzott szám $\qquad R_X = \{1,2,\dots 85\}$

$$F_X(25) = P(X<25) = \sum_{i=1}^{24} P(X=i) = \sum_{i=1}^{24} \frac{\binom{90-i}{4}}{\binom{90}{5}}$$

29.)

egységnyit ugrik, 5x.

X: az origótól mért helye

$R_X = \{-5,-3,-1,1,3,5\}$

B: balra ugrások sz.

J: jobbra ugrások száma

$$\left.\begin{array}{l} B \in B(5, 1-p) \\ J = 5 - B \end{array}\right\} X = J - B = 5 - B - B = 5 - 2B$$

$$P(X=k) = P(5-2B=k) = P\left(B = \frac{5-k}{2}\right) = \binom{5}{\frac{5-k}{2}} (1-p)^{\frac{5-k}{2}} \cdot p^{5-\frac{5-k}{2}} \qquad k \in R_X$$

Valszám gyak

2008.09.30.
Jánosváry

12) K: szabályos érmét dobunk, 2. fejig.

P(első fej után a második fejig ~~…~~ első fejig)

X_1 = az első fejig szükséges dobások száma

X_2: a második ——"—— az első fej után.

$X_1, X_2 \in G\left(\frac{1}{2}\right)$

$$P(X_1=X_2)=\sum_{i=1}^{\infty} P(X_1=i)\cdot P(X_2=i)=\sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^{i-1}\frac{1}{2}\left(\frac{1}{2}\right)^{i-1}\cdot\frac{1}{2}=\sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^{2i}=\sum_{i=1}^{\infty}\left(\frac{1}{4}\right)^{i}=\frac{\frac{1}{4}}{1-\frac{1}{4}}$$

36.) 32 lap, visszatevés nélkül

- amíg pirosat nem kapunk.
- utána amíg A'-t nem kapunk.

X: kihúzott lapok száma

$P(X=3)=?\ =\frac{1560}{29760}$

A_1: piros A', nem A', A' $\to P(A_1)$

A_2

A_3

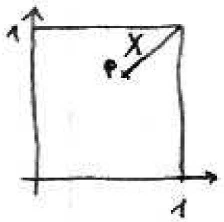
$\vdots$

A_6

$\uparrow$

harmadikra A', és hogy lehet

H7: 43

1.) 

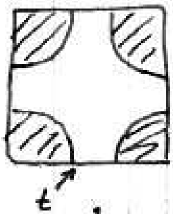
X a pont legközelebbi csúcstól vett távolsága

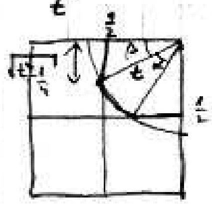
$F_X(t) = ?$, $f_X(t) = ?$

$t \in \left(0, \frac{\sqrt{2}}{2}\right) = \left(0, \frac{1}{2}\right) \cup \left[\frac{1}{2}, \frac{\sqrt{2}}{2}\right)$

$F_X(t) = P(X < t)$ $F_X(t) = 0$, $t \le 0$

$F_X(t) = 1$, $t \ge \frac{\sqrt{2}}{2}$

$t \in \left(0, \frac{1}{2}\right)$  $F_X(t) = t^2 \cdot \pi$

$t \in \left[\frac{1}{2}, \frac{\sqrt{2}}{2}\right)$  $\frac{1}{4} \cdot \sqrt{t^2 - \frac{1}{4}} \cdot 2 + 2t^2 \left(\frac{\pi}{2} - 2\arccos \frac{1}{2t}\right)$, $t \in \left[\frac{1}{2}, \frac{\sqrt{2}}{2}\right)$

$2\beta + \alpha = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{2} - 2\beta = \frac{\pi}{2} - 2\arccos \frac{1}{2t}$

$f_X(t) = F_X'(t) = \begin{cases} 2t\pi, & t \in \left(0, \frac{1}{2}\right) \\ \ldots \end{cases}$ $\cos\beta = \frac{1}{2t} \Rightarrow \beta = \arccos \frac{1}{2t}$

$2 \cdot \frac{1}{2\sqrt{t^2 - \frac{1}{4}}} \cdot 2t + 2 \cdot 2t\left(\quad\right) + 2t^2 \cdot \left(-2 \cdot \frac{-1}{\sqrt{1 - \left(\frac{1}{2t}\right)^2}} \cdot \frac{1}{2} \cdot (\,)\right)$

2)8.) $X \in E(\lambda)$, $Y = X^2$, $f_Y(t) = ?$, $EY = ?$; $X \in E(2) \to Y \in G\left(\frac{1}{3}\right)$

$F_X(t) = 1 - e^{-\lambda t}$, $t > 0$ $(\lambda > 0)$

$f_X(t) = \lambda e^{-\lambda t}$ $EX = \frac{1}{\lambda}$, $\sigma^2 X = \frac{1}{\lambda^2}$

$EY = EX^2 = \sigma^2 X + (EX)^2$

Ha $X \in E(\mu) \Rightarrow [X] + 1 \in G(1 - e^{-\mu})$

$\left(EY = EX^2 = \sigma^2 X + (EX)^2 = \frac{1}{\lambda^2} + \left(\frac{1}{\lambda}\right)^2 = \frac{2}{\lambda^2}\right)$

$F_Y(t) = P(\underset{t>0}{Y < t}) = P(X^2 < t) = P(X < \sqrt{t}) = F_X(\sqrt{t}) = 1 - e^{-\lambda\sqrt{t}}$

$f_Y(t) = \lambda e^{-\lambda\sqrt{t}} \cdot \frac{1}{2\sqrt{t}}$, $t > 0$

$EY = \int_0^\infty t \cdot f_Y(t)\, dt \ldots = \frac{1}{\lambda^2} + \left(\frac{1}{\lambda}\right)^2 = \frac{2}{\lambda^2}$

$G\left(\frac{1}{3}\right) = G(1 - e^{-\mu})$

$\Downarrow$

$\mu = \ln \frac{3}{2}$

$\underset{c=?}{c}X = E\left(\ln \frac{3}{2}\right)$

$P(c \cdot X < t) = P\left(X < \frac{t}{c}\right) = F_X\left(\frac{t}{c}\right) =$

$= 1 - e^{-2 \cdot \frac{t}{c}} = 1 - e^{-\ln\frac{3}{2} \cdot t}$

$c = \frac{2}{\ln \frac{3}{2}} \Rightarrow Y = \left[\frac{2}{\ln\frac{3}{2}} X\right] + 1 \in G\left(\frac{1}{3}\right)$

1, 2, 4, 6, 8, 12, 16, 13, 28, 32, 33, 34, 37, 40, 41, 46, 49, 56

13.) $X \in N(m, D)$, $m=?$, $D=?$ $(EX=?,\ \sigma X=?)$

$P(X<10{,}2) = \frac{1}{10}$, $P(X>13{,}6) = \frac{1}{4}$

28.) $X \in B\left(3, \frac{1}{4}\right)$, $Y = X^3$, Mi Y eloszlása? $EY=?$

13.) $F_X(t) = \Phi\left(\frac{t-m}{D}\right)$, $\Phi(-x) = 1-\Phi(x)$

x	Φx
0	$\frac{1}{2}$
⋮	⋮
$\boxed{u_{0,25}}$	$\frac{1}{4}$
⋮	⋮
$\boxed{u_{0,1}}$	0,9
$u_{0,75}$	$\frac{3}{4}$

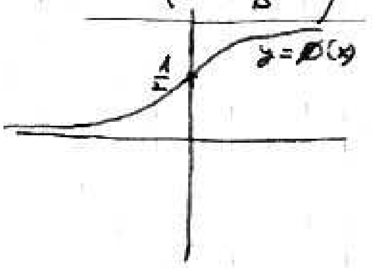
16) $X \in G(p) \Rightarrow EX = \frac{1}{p}$

28.) $P(X=k) = \binom{3}{k}\left(\frac{1}{4}\right)^k \cdot \left(\frac{3}{4}\right)^{3-k}$, $k = 0,1,2,3$

13.) $P(X<10{,}2) = F_X(10{,}2) = \frac{1}{10} = \Phi\left(\frac{10{,}2-m}{D}\right)$

$P(X>13{,}6) = 1-P(X\leq 13{,}6) = 1-P(X\leq 13{,}6) = 1-F_X(13{,}6) = 1-\Phi\left(\frac{13{,}6-m}{D}\right) = \frac{1}{4}$

$\Phi\left(\frac{13{,}6-m}{D}\right) = \frac{3}{4} \Rightarrow \frac{13{,}6-m}{D} = u_{0,75}$



$\Phi(x) = 1-\Phi(-x)$

$$\left.\begin{array}{l} \Phi\left(\frac{m-10{,}2}{D}\right) = 1-\frac{1}{10} = 0{,}9 \\ \frac{m-10{,}2}{D} = u_{0,9} \end{array}\right\} \Rightarrow m, D$$

28.) $R_Y = \{0, 1, 8, 27\}$

$P(Y=i^3) = P(X=i) = \binom{3}{i}\left(\frac{1}{4}\right)^i \cdot \left(\frac{3}{4}\right)^{3-i}$

$EY = 1 \cdot P(X=1) + 8 \cdot P(X=2) + 27\ P(X=3) = \ldots$

32.) $f_X(x) = \begin{cases} 2 \cdot e^{-2x}, & \text{ha } x \in [0,1] \\ \frac{2x}{3e^2}, & \text{ha } x \in (1,2] \end{cases}$ $\qquad EX = \int_{-\infty}^{\infty} x f(x)\,dx = \ldots$

$(f(x) = 0$ egyébként$)$

37.) $|1p, 2f, 3f|$ K: addig húzunk visszatevés nélkül, míg mindkét színt ki nem fogjuk

X: kivett golyók száma. K eloszlása, EX, $\sigma x = ?$

40.) $f_X(x) = \begin{cases} A \cdot \cos\frac{x}{2} & x \in (0, \pi) \\ 0 & \text{egyébként} \end{cases}$ $\qquad F_X(t) = \int_{-\infty}^{t} f_X(u)\,du$, $\int_{-\infty}^{\infty} f_X(u)\,du = 1$

$A=?$, F_X, $P\left(X > \frac{\pi}{2}\right) = ?$ $\qquad P\left(X > \frac{\pi}{2}\right) = 1 - F_X\left(\frac{\pi}{2}\right)$

Valószínűség

2008.10.17
Kela

32) $EX = \int_{-\infty}^{\infty} x\, f(x)\, dx = \begin{cases} 0, \text{ ha } x \in (-\infty, 0) \cup (2, \infty) \\ \int_0^1 x \cdot 2e^{-2x} = 2 \cdot x \cdot \frac{e^{-2x}}{-2} - \int_0^1 \frac{e^{-2x}}{-2} = x \cdot e^{-2x} - \left[\frac{e^{-2x}}{4}\right]_0^1, \quad x \in [1,2] \\ \quad u = x,\ v = \frac{e^{-2x}}{-2},\ u' = 1,\ v' = e^{-2x} \\ \int_1^2 \frac{2}{3e^2} x^2\, dx = \ldots \end{cases}$

40.) $A \int_0^{\pi} \cos\frac{x}{2}\, dx = 1 \Rightarrow A \cdot \left[2 \sin\frac{x}{2}\right]_0^{\pi} = A \cdot [2] = 1 \Rightarrow A = \frac{1}{2}$

$\underset{t \in [0,\pi]}{F_X(t)} = \int_0^t \frac{1}{2} \cos x\, dx = \sin\frac{t}{2}$

$P\left(X > \frac{\pi}{2}\right) = 1 - \sin\frac{\pi}{4} = 1 - \frac{\sqrt{2}}{2}$

37.) $R_X = \{3, 4, 5, 6\}$

$P(X=3) = P(fzz, fpz, zff, pfz, zpf, pzf) = 6 \cdot \frac{2 \cdot 3 \cdot 1}{6 \cdot 5 \cdot 4} = \frac{3}{10}$

$P(X=4) = P(\underbrace{ffzz}_{3}, \underbrace{ffpz}_{3}, \underbrace{zzfp}_{3}, \underbrace{zzpf}_{3}) = 3 \cdot \left[\frac{2 \cdot 1 \cdot 3 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3} \cdot 2 + \frac{3 \cdot 2 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3} \cdot 2\right] = \frac{3}{10}$

$P(X=6) = P(\underbrace{ffzzzp}_{\frac{5!}{2!\,3!} = 10}) = 10 \cdot \frac{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = \frac{1}{6}$

$P(X=5) = 1 - \frac{6}{10} - \frac{1}{6} = \ldots$

56) $\boxed{3p, 2f}$ $n = 10$-szer húzunk visszatevéssel.

X a kihúzottak között a pirosak száma.

$Z = (X+2)(X-2) \qquad EZ = ?$

$EZ = EX^2 - 4 = \sigma^2 X + (EX)^2 - 4$

$X \in B\left(10, \frac{3}{5}\right), \quad EX = \frac{30}{5} = 6, \quad \sigma^2 X = 6 \cdot \frac{2}{5} = 2{,}4$

$EZ = 2{,}4 + 36 - 4 = 34{,}4.$

III. 1, 3, 9, 14, 16, 19, 21, 23, 28, 30, 36, 43, 47, 59, 63, 67, 69, 74
grom

1) $X, Y \in G(p)$, függetlenek, $P(X=Y)=?$

$$P(X=Y)=\sum_{i=1}^{\infty}\underbrace{P(X=i,\ Y=i)}_{P(X=i)P(Y=i)}=\sum_{i=1}^{\infty} q^{i-1}\cdot p\cdot q^{i-1}\cdot p=p^2\sum_{i=1}^{\infty}\left(q^2\right)^{i-1}=\frac{p^2}{1-q^2}=$$

$$=\frac{p\cdot p}{\underbrace{(1-q)}_{p}(1+q)}=\frac{p}{1+q}$$

Pl akkor állnak meg, ha egyikük 6-ost dob $\to \frac{1}{7}$

3) $X, Y \in E(1)$, fgt, $Z=\min\{X,Y\}$, $V=\max\{X,Y\}$

Biz. be, hogy: a) $Z \in E(2)$

b) V eloszlása u.a, mint $\overbrace{X+\frac{1}{2}Y \text{ eloszlása}}^{\text{ezt majd később}}$

a.) $\underset{t>0}{F_Z(t)}=P(Z<t)=1-P(Z\geq t)=1-\underbrace{P(\min\{X,Y\}\geq t)}_{P(X\geq t,\ Y\geq t)}=1-\underbrace{P(X\geq t)}_{e^{-t}}\cdot\underbrace{P(Y\geq t)}_{e^{-t}}=$

$=e^{-2t}\in E(2)$

b.) $\underset{t>0}{F_V(t)}=P(\max\{X,Y\}<t)=P(X<t,\ Y<t)=P(X<t)\cdot P(Y<t)=F_X(t)\cdot F_Y(t)=$

$=(1-e^{-t})(1-e^{-t})=(1-e^{-t})^2=e^{-2t}-2e^{-t}+1$

14.) $f_{X,Y}(x,y)=\begin{cases} a(x^2+xy+y^2), & 0<x,y<1 \\ 0 & \text{egyébként}\end{cases}$

a) $a=?$

b) fgt-k-e? → az y szerintinél x-et integráljuk $(-\infty,\infty)$ között, és fordítva, majd összeszorozzuk

a) nümüdfgv. megnyugtató a teljes integrál 1-et ad.

$$1=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy=a\int_0^1\int_0^1 (x^2+xy+y^2)\,dx\,dy=a\int_0^1\left[\frac{x^3}{3}+x^2y+xy^2\right]_0^1 dy$$

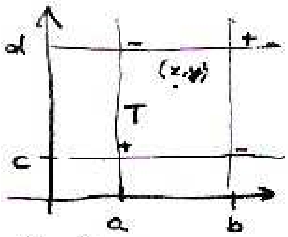
$$=a\int_0^1\left(\frac{1}{3}+\frac{y}{2}+y^2\right)dy=a\left[\frac{y}{3}+\frac{y^2}{4}+\frac{y^3}{3}\right]_0^1=a\left[\frac{1}{3}+\frac{1}{4}+\frac{1}{3}\right]=a\cdot\frac{11}{12}\Rightarrow a=\frac{12}{11}$$

$$f_X(x)=f_Y(x)=\frac{12}{11}\int_0^1(x^2+xy+y^2)\,dy=\frac{12}{11}\left[x^2y+\frac{xy^2}{2}+\frac{y^3}{3}\right]_0^1=\frac{12}{11}\left(x^2+\frac{x}{2}+\frac{1}{3}\right)$$

$$f_X(x)\cdot f_Y(y)\overset{?}{=}f_{X,Y}(x,y)$$

ha összeszorozzuk, lesz benne egy konstans rész $\left(\frac{16}{121}\right)$, az együttesben viszont nincs NEM fgt.

19.) $F_{X,Y}(x,y) = x^2 y \quad , \quad 0 \le x \le 1 \, , \; 0 \le y \le 1$



$$P\left(\underbrace{\frac{1}{4}}_{=a} \le X \le \underbrace{\frac{3}{4}}_{=b} \, , \; \underbrace{\frac{1}{4}}_{=c} \le Y \le \underbrace{\frac{1}{2}}_{=d}\right) = ?$$

$$P(\underline{X} \in T) = F_{X,Y}(a,c) + F_{X,Y}(b,d) - F_{X,Y}(a,d) - F_{X,Y}(b,c) = \dots$$

21)
$$f(x,y) = \begin{cases} \frac{1}{7}(6x^2y - 12xy + 6y - 36x + 18) \, , & x \in (0,1), \; y \in (0,1) \\ 0 & \text{különben} \end{cases}$$

Fgt-e?

$$f_X(x) = f_Y(x) = \frac{1}{7}\int_0^1 (6x^2y - 12xy + 6y - 36x + 18)\, dy = \frac{1}{7}\Big[3x^2y^2 - 6xy^2 + 3y^2 -$$

$$- 36yx + 18y\Big]_0^1 = \frac{1}{7}(3x^2 - 6x + 3 - 36x + 18) = \frac{1}{7}(3x^2 - 39x + 21)$$

$$f_Y(y) = f_X(y) = \frac{1}{7}\int_0^1 (6x^2y - 12xy + 6y - 36x + 18)\, dx = \frac{1}{7}\Big[2x^3y - 6x^2y + 6xy -$$

$$- 18x^2 + 18x\Big]_{0<x}^1 = \frac{1}{7}(2y - 6y + 6y - 18 + 18) = \frac{2}{7}y$$

$$f_X(x) \cdot f_Y(y) = \frac{6}{49}(x^2y - 13xy + 7y)$$ // vhol elszámoltam

$$f_{X,Y}(x,y) = \frac{1}{7}\underbrace{(6y+1)}_{f_Y(y)}\underbrace{(x^2 - 2x + 1)}_{f_X(x)} \Rightarrow \text{fgt-k.}$$

47) K: 3 szabályos kockával dobunk. Y a dobott számok összege.

$EY = ? \, , \; \sigma^2 Y = ?$

X_i: az i-edik kocka értéke., fgt-k

$$Y = X_1 + X_2 + X_3 \quad , \quad EY = E(X_1 + X_2 + X_3) = EX_1 + EX_2 + EX_3 = 3 \cdot EX_1 = 3 \cdot 3{,}5$$

$\frac{1}{6} \cdot (1+2+3+4+5+6)$

10,5?

$$\sigma^2 Y = E(Y^2) - (EY)^2$$

$$EY^2 = E(X_1 + X_2 + X_3)^2 = E(X_1^2 + X_2^2 + X_3^2 + 2X_1X_2 + 2X_2X_3 + 2X_1X_3) =$$

$$\underbrace{E(X_1^2) + E(X_2^2) + E(X_3^2)}_{3E(X_1^2)} + 2E(X_1X_2) + 2E(X_2X_3) + 2E(X_1X_3) =$$

$$EX_1 \cdot EX_2 = (EX_1)^2$$

$$E(Y^2) = 3E(X_1)^2 + 6(EX_1)^2$$

$$\sigma^2 Y = 3E(X_1)^2 + 6(EX_1)^2 - (3E(X_1))^2 = 3E(X_1^2) - 3(EX_1)^2 = 3\sigma^2 X_1 =$$

$$= \sigma^2 X_1 + \sigma^2 X_2 + \sigma^2 X_3 = \frac{91}{6} - (3{,}5)^2$$

① $\sigma^2(X+Y) = \sigma^2 X + \sigma^2 Y \pm 2\,\mathrm{cov}(X,Y)$

23) 32 → 8-at visszatevés nélkül.

$X = \begin{cases} 1, \text{ ha van piros} \\ 0, \text{ ha nincs} \end{cases}$ $\qquad Y = \begin{cases} 1, \text{ ha van ász} \\ 0, \text{ ha nincs.} \end{cases}$

Együttes eloszlás táblázat:

$\frac{1}{\binom{32}{8}} \cdot$

Y \ X	0	1
0	$\binom{21}{8}$	$\sum_{i=1}^{7}\binom{7}{i}\binom{21}{8-i}$
1	$\sum_{j=1}^{3}\binom{3}{j}\binom{21}{8-j}$	[illegible]

59.) $f_{X,Y}(x,y) = \frac{1}{2\pi d^2} \exp\left(-\frac{x^2+y^2}{2d^2}\right)$

$Z = \max\{|X|,|Y|\}$, $f_Z(t) = ?$

$f_X(x)\cdot f_Y(y) = \frac{1}{\sqrt{2\pi}\,d} e^{-\frac{x^2}{2d^2}} \cdot \frac{1}{\sqrt{2\pi}\,d} \cdot e^{-\frac{y^2}{2d^2}} \Rightarrow X, Y \in N(0,d)$ függetlenek

$\underset{t>0}{F_Z(t)} = P(Z<t) = \ldots = P(|X|<t, |Y|<t) = P(|X|<t)\cdot P(|Y|<t) = P(-t<X<t)\cdot$

$\cdot P(-t<Y<t) = (F_X(t) - F_X(-t))(F_Y(t) - F_Y(-t)) = \Big(\underbrace{\Phi\left(\tfrac{t}{d}\right) - \Phi\left(-\tfrac{t}{d}\right)}_{2\Phi\left(\frac{t}{d}\right)-1}\Big)^2 =$

$F_X(t) = F_Y(t) = \Phi\left(\frac{t}{d}\right)$

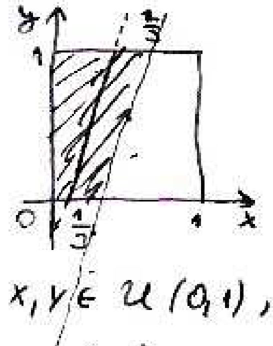
$X \in N(m,D) \Rightarrow F_X(t) = \Phi\left(\frac{t-m}{D}\right)$

$= \left(2\Phi\left(\frac{t}{d}\right) - 1\right)^2 = F_Z(t)$, $t>0$

$f_Z(t) = F'_Z(t) = 2\left(2\Phi\left(\frac{t}{d}\right) - 1\right)\cdot \varphi\left(\frac{t}{d}\right)\cdot \frac{1}{d}$

63.) $X, Y \in U(0,1)$, függetlenek. $P(3X < Y+1) = ?$

(x,y) egy pont lesz a $(0,1)$ egységnégyzeten, $\forall\, x,y \Leftrightarrow$ véletlen pont az egységnégyzeten



$T = \frac{\left(\frac{2}{3} + \frac{1}{3}\right)\cdot 1}{2} = \frac{1}{2}$

$y = 3x - 1$

$y = 0 \Rightarrow x = \frac{1}{3}$

$y = 1 \Rightarrow x = \frac{2}{3}$

67.) $X, Y \in U(0,1)$, fgt, $Z = e^{X+Y}$. $EZ = ?$, $\sigma Z = ?$

$Z = \underset{\text{fgt!}}{e^X \cdot e^Y} \Rightarrow EZ = Ee^X \cdot Ee^Y = (Ee^X)^2$

$$E e^X = \int_{-\infty}^{\infty} e^x \cdot f_X(x)\,dx = \int_0^1 e^x\,dx = e-1$$

$$E Z^2 = E(e^{2X}) \cdot E(e^{2Y}) = (E e^{2X})^2 = \left(\frac{1}{2} e^2 - \frac{1}{2}\right)^2$$

$$E e^{2X} = \int_0^1 e^{2x}\,dx = \left[\frac{1}{2} e^{2x}\right]_0^1 = \frac{1}{2} e^2 - \frac{1}{2}$$

$$\sigma^2 Z = E Z^2 - (E Z)^2 = \ldots$$

Valószín gyak. 2008. 11. 14. Ikela

(15.) K: 3x dobunk a kockával, X a 6-osok, Y a párosok száma.

$P(X=i, Y=j) = ?$ $\quad cov(X,Y) = ?$ $\quad$ Függetlenek?

$X \in B(3, \frac{1}{6})$ $\qquad Y \in B(3, \frac{1}{2})$

$\frac{1}{216}$

Y \ X	0	1	2	3	
0	27	0	0	0	27
1	54	27	0	0	81
2	36	36	9	0	81
3	8	12	6	1	27
					216

~~3~~ · ~~...~~ 3!·

$\cdot\; 3 \cdot 3 \cdot \cancel{2} \cdot 2$

pt helye, pt értéke, ps értéke

$cov(X,Y) = \underbrace{E(X\cdot Y)}_{1} - \underbrace{EX}_{\frac{1}{2}} \cdot \underbrace{EY}_{\frac{3}{2}} = 1 - \frac{1}{2}\cdot\frac{3}{2} = \frac{1}{4}$

NEM FGT.

$E(XY) = \cancel{1\cdot 54 + 1\cdot 27 + 2\cdot} \frac{1\cdot1\cdot27 + 2\cdot1\cdot36 + 2\cdot2\cdot9 + 3\cdot1\cdot12 + 3\cdot2\cdot6 + 3\cdot3\cdot1}{216} = \frac{216}{216} = 1$

(18.) $X, Y \in U(0,1)$

$\begin{pmatrix} X+Y \\ X-Y \end{pmatrix}$ várhatóérték vektora? $\begin{pmatrix} E(X+Y) \\ E(X-Y) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Kovarianciamátrix? $\begin{pmatrix} \sigma^2(X+Y) & cov(X+Y, X-Y) \\ cov(X+Y, X-Y) & \sigma^2(X-Y) \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{6} \end{pmatrix}$

$X+Y \in U(0,2)$ $\qquad E(X+Y) =$

$X-Y \in U(-1,1)$

$\sigma^2(X \pm Y) = \sigma^2 X + \sigma^2 Y = 2\sigma^2 X = 2 \cdot \frac{1}{12} = \frac{1}{6}$

$E(X+Y) = E(X) + E(Y) = 2EX = 1$

$E(X-Y) = E(X) - E(Y) = 0$

$cov(X+Y, X-Y) = cov(X, X-Y) + cov(Y, X-Y) = \overbrace{cov(X,X)}^{\sigma^2 X} - \underbrace{cov(X,Y) + cov(X,Y)}_{0} + \underbrace{cov(Y,Y)}_{\sigma^2 Y} = 0$

(39.) $X \in E(2)$, $cov(X, X^2) = EX^3 - \overset{\frac{1}{2}}{EX} \cdot \overset{\frac{1}{2}}{EX^2} = \frac{1}{2}$ $\quad EX^2 = \sigma^2 X + (EX)^2$ ($\frac{1}{4}$)

$EX^3 = \int_0^\infty x^3 \cdot 2e^{-2x}\,dx = \underbrace{\left[4x^3 \cdot e^{-2x}\right]_0^\infty}_{0} + 3\cdot\frac{1}{2}\int_0^\infty \underbrace{x^2 \cdot 2e^{-2x}\,dx}_{EX^2 = \frac{1}{2}}$

(41.) $X \in N(0,1)$, $R(X, X^3) = \frac{cov(X, X^3)}{\sigma X\, \sigma X^3} = \qquad \sigma = \frac{3}{\sqrt{15}}$

$cov(X,Y) = E(X\cdot Y) - EX \cdot EY$

$cov(X, X^3) = EX^4 - \underbrace{EX}_{0} \cdot EX^3 = \int_{-\infty}^{\infty} x^4 \varphi(x)\,dx = \int (-x^3)\cdot\overbrace{(-x\,\varphi(x))}^{\varphi'(x)} = \left[-x^3 \cdot \varphi(x)\right]_{-\infty}^{\infty} + 3\int_{-\infty}^{\infty} \underbrace{x^2 \varphi(x)\,dx}_{1} = 3$

$\sigma X = 1$

75, 73, 15, 18, 39, 48, 51, 52, 56, 58, 65, 73, 76, 78, 84, 106, 104.

$$\sigma^2 x^3 = E x^6 - \underbrace{(E x^3)^2}_{0}$$

$$E x^3 = \int_{-\infty}^{\infty} x^3 \varphi(x)\,dx = 0$$

ptl. fv

$$E X^6 = \int_{-\infty}^{\infty} x^6 \varphi(x)\,dx = \int_{-\infty}^{\infty} (-x^5)\underbrace{(-x\varphi(x))}_{\varphi'(x)}\,dx = \underbrace{\left[-x^5\varphi(x)\right]_{-\infty}^{\infty}}_{0} + 5\underbrace{\int_{-\infty}^{\infty} x^4\varphi(x)\,dx}_{3} = 15$$

$$\sigma x^3 = \sqrt{15}$$

(52) X, Y fgt-ek, $EX=4$, $EY=0$, $\sigma^2 X = 1$, $\sigma^2 Y = 2$

$$E(5X-6Y) = 5EX - 6EY = 20$$

$$E XY = \cancel{\sigma^2(XY)\ \sigma X \sigma Y = \sigma^2 X \sigma^2 Y = \sigma X \sigma Y = 1\cdot 2 = \sqrt{2}\cdot 2 = \sqrt{2}} \quad EX\cdot EY = 0$$

$$\sigma^2(5X-6Y+8) = \sigma^2 5X - \sigma^2 6Y + \overbrace{\sigma^2 8}^{0} = 25\sigma^2 X - 36\sigma^2 Y + \cancel{0} = 25\cdot 1 - 36\cdot 2 \cancel{\ldots}$$

$$\operatorname{cov}(5X, 6Y) = E(5X\cdot 6Y) - \underbrace{E(5X)\cdot E(6Y)}_{0} = E(30XY) = 30E(XY) = 0$$

$$= 30\operatorname{cov}(X,Y) = 0, \text{ mert fgt-ek.}$$

(73) $X \in E(1)$, $Y \in N(0,1)$ fgt-ek, $Z = X^2$, $W = 3X - Y$

$\operatorname{cov}(Z, W) = ?$

$$\operatorname{cov}(Z,W) = E(Z\cdot W) - EZ\cdot EW = E(\underbrace{X^2(3X-Y)}_{3X^3 - X^2Y}) - \underbrace{EX^2}_{2}\cdot\underbrace{E(3X-Y)}_{3} =$$

$$= 3\underbrace{EX^3}_{6} - \underbrace{E(X^2Y)}_{\underbrace{EX^2}_{1}\cdot\underbrace{EY}_{0}} - 6 = 12$$

$$EZ = EX^2 = \overbrace{1+1}^{\sigma^2 X + EX} = 2$$

$$EW = 3EX - EY = 3 - 0 = 3$$

$$EX^3 = \int_0^{\infty} x^3 \cdot e^{-x}\,dx = \underbrace{\left[-x^3 \cdot e^{-x}\right]_0^{\infty}}_{0} + 3\cdot\int_0^{\infty} x^2 \cdot e^{-x}\,dx = 6$$

(84) $X, Y \in Po(2)$, $R(X, X+Y-1) = \dfrac{\operatorname{cov}(X, X+Y-1)}{\sigma X \cdot \sigma(X+Y-1)}$, X, Y fgt-ek.

$$\operatorname{cov}(X, X+Y-1) = \underbrace{\operatorname{cov}(X,X)}_{\sigma^2 X} + \underbrace{\operatorname{cov}(X,Y)}_{0} + \underbrace{\operatorname{cov}(X,-1)}_{0}$$

$$\sigma^2(X+Y-1) = \sigma^2(X+Y) = \sigma^2(X) + \sigma^2 Y = 2\sigma^2 X$$

$$\sigma(X+Y-1) = \sqrt{2}\,\sigma X \rightarrow R(X, X+Y-1) = \frac{\sigma^2 X}{\sqrt{2}\,\sigma^2 X} = \frac{\sqrt{2}}{2}$$

Valszám

2008. 11. 14.
Kela

(Pl) $X \in E(1)$, $Y \in E(2)$, f_Z t.-?

$Z = X - Y = X + (-Y)$

$F_{(-Y)}(t) = P(-Y < t) = P(Y > -t) = 1 - F_Y(-t) = 1 - (1 - e^{-2(-t)}) = e^{2t}$
$t < 0$

$F_{(-Y)}(t) = 1$, ha $t \geq 0$

$f_{(-Y)}(t) = 2e^{2t}$, $t < 0$ $\quad (f_{(-Y)}(t) = 0, \ t \geq 0)$

$f_{(Z)}(t) = f_{X-Y}(t) = \int_{-\infty}^{\infty} f_X(u) \cdot f_{(-Y)}(t-u) = \int_{\max\{t,0\}}^{\infty} e^{-u} \cdot 2e^{2(t-u)} \, du =$

$u > 0$
$t - u < 0$
$t < u$

$= 2e^{2t} \cdot \int_{\max\{t,0\}}^{\infty} e^{-3u} \, du$

Ha $t < 0$: $\quad 2e^{2t} \int_0^{\infty} e^{-3u} \, du = 2e^{2t} \left[-\frac{1}{3} e^{-3u} \right]_0^{\infty} = \frac{2}{3} e^{2t}$

Ha $t \geq 0$: $\quad 2e^{2t} \int_t^{\infty} e^{-3u} \, du = 2e^{2t} \left[-\frac{1}{3} e^{-3u} \right]_t^{\infty} = \frac{2}{3} e^{2t} \cdot e^{-3t} = \frac{2}{3} e^{t}$

Mi $|Z|$ sűrűségfv-e?

$F_{|Z|}(t) = P(|Z| < t) = P(-t < Z < t) = F_Z(t) - F_Z(-t)$
$t > 0$

$F_{|Z|}(t) = 0$, $t \leq 0$

$f_{|Z|}(t) = F'_{|Z|}(t) = F'_{|Z|}(t) - F'_{|Z|}(-t) = f_{(Z)}(t) - f_{|Z|}(-t)$

$f_{|Z|}(t) = \frac{2}{3} e^{t} + \frac{2}{3} e^{2t}$
$t > 0$