

Űrkommunikáció Space Communication 2023/1.

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Organization Issues

- Lectures (14 Semester week)
 - Recording is forbidden
 - Questions during lecture are welcome
 - Slides will be uploaded after the lectures
- 1 Midterm Exam (Score > 50%) and Final Exam
 - probably on VIK Moodle tool
 - Multiple choice questions and numerical tasks
- References:
 - J. G. Proakis, M. Salehi: Communication Systems Engineering (Prentice Hall, 2002)
 - Th. M. Cover, J. A. Thomas: Elements of Information Theory (Wiley, 2006)
 - H. L. Van Trees: Detection, Estimation, and Modulation Theory, Vol I (Wiley)

Hungarian references

- Dallos György: Tantárgyi segédlet a Hírközléelmélet című tárgyhoz (2006)
<http://www.hit.bme.hu/~dallos/hirkelm/>
- Frigyes I.: Hírközlő rendszerek (Műegyetemi Kiadó, 2001)
- Csibi S.: Információ közlése és feldolgozása (Tankönyvkiadó, 1980)

The goal of Communication

To transfer *information / news* from the

- information **Source** to the
- **Sink** of information get through a
- communication **Channel**.

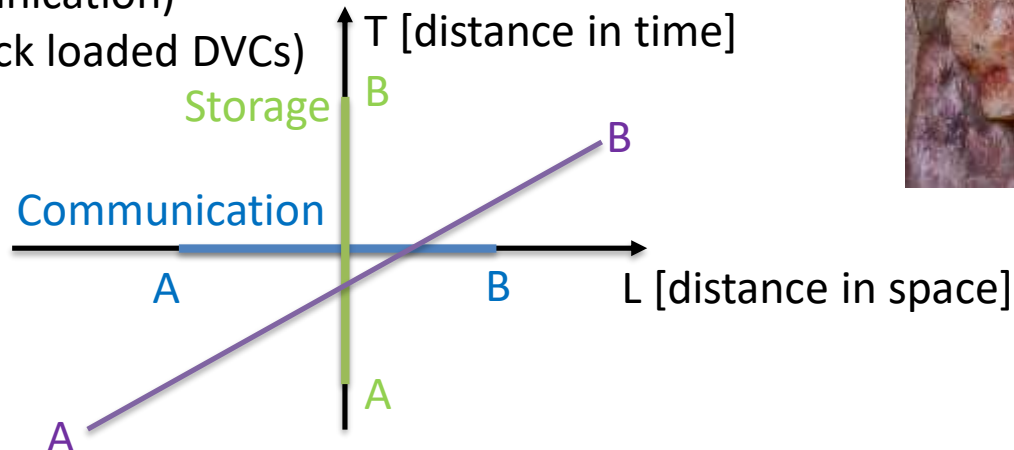
The channel is influenced / corrupted by natural and/or artificial

- **Noise**
- **Interference**
- **Fading** of the signal bearing the information
- Sketch on a disc, etc.

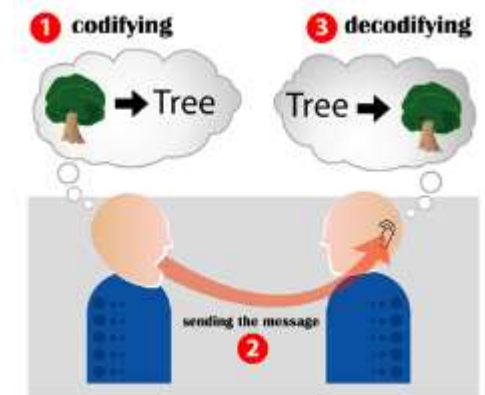
Transfer information between two points A and B separated in

- **time** (info storage) or
- **space** (info communication)
- (or both: e.g. a truck loaded DVCs)

Space-Time diagram



Transferring info in space

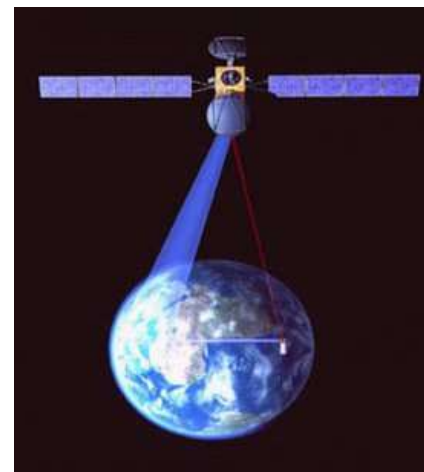


Transferring info in time



Pont-pont összeköttetések

1. Föld-Föld (terrestrial links)
2. Föld-Műhold (satellite links)
3. Műhold-Műhold



A műholdas rádióátvitel előnyei a földihez képest:

- Nagyobb lefedettséget biztosít
- Egy műhold számos földi állomás között képes kapcsolatot létesíteni

Hátrányok:

- Az áthidalandó távolság igen nagy. A Föld-műhold-Föld szakasz együttes késleltetése nagyságrendileg 500 ms. (Egy összeköttetés legfeljebb egy műholdas szakaszt tartalmazhat)

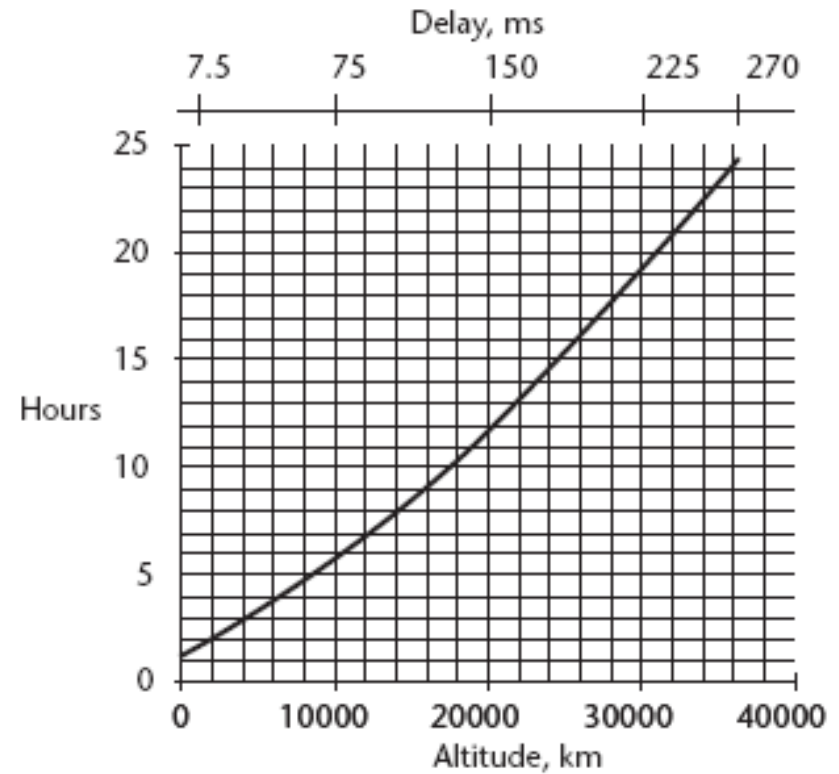
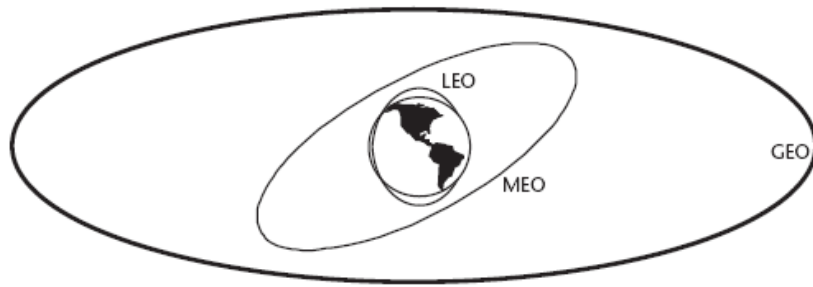
A pont-pont jellegű vezeték nélküli összeköttetések előnyei a vezetékes rendszerekhez képest:

- kiesés (vezetékszakadás)
- olcsóbb, gyors telepítés

Hátrányok:

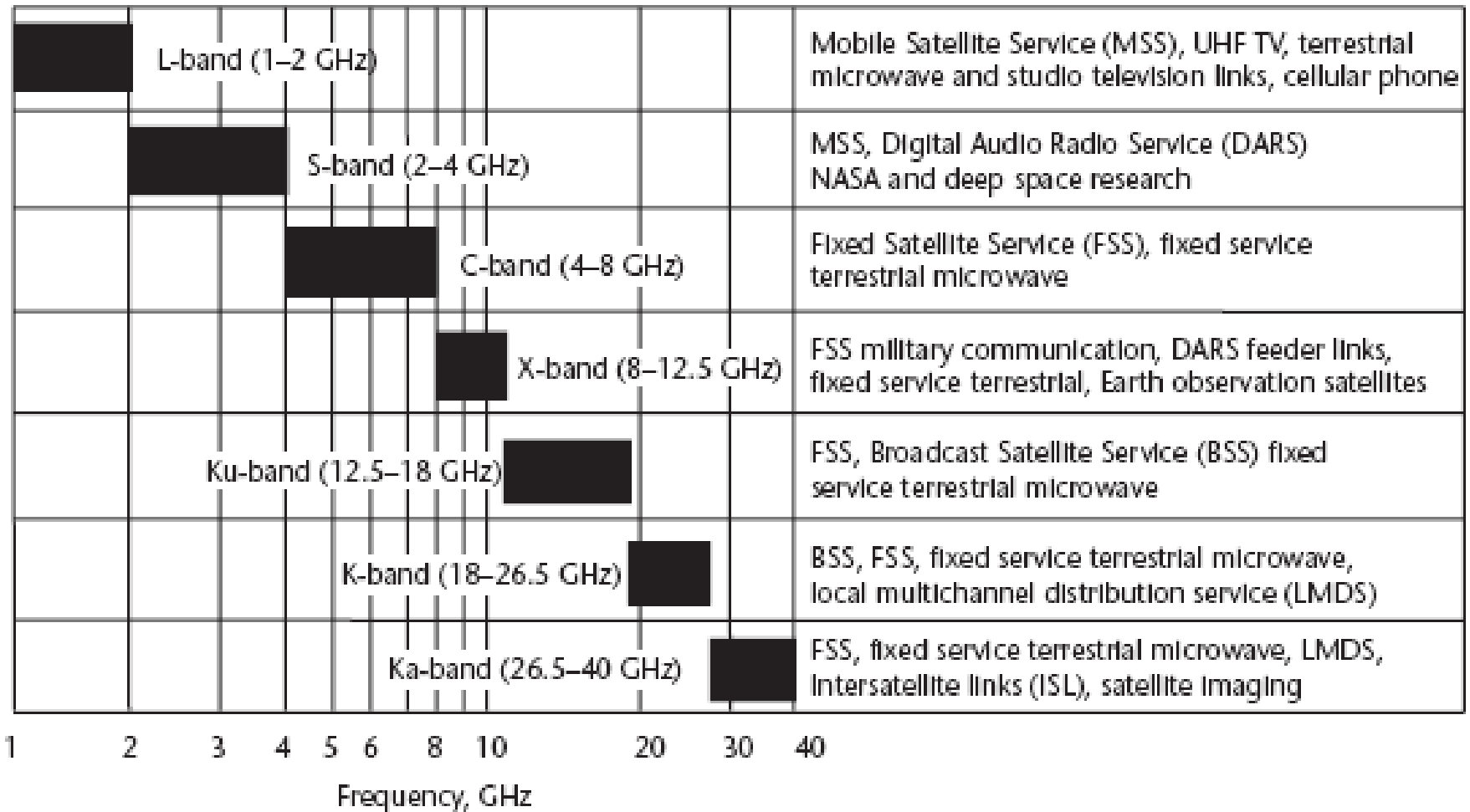
- kapacitásuk korlátozottabb
- kevésbé védettek a meteorológiai jelenségek, vagy más összeköttetések interferenciái ellen

Csatorna - késleltetés



The respective altitude ranges are 500 to 900 km for LEO, 5,000 to 12,000 km for MEO, and 36,000 km for GEO. One-way (single-hop) propagation delay.

Frekvenciasávok (ITU) - Szolgáltatások



Rádiócsatorna - Szabadtér

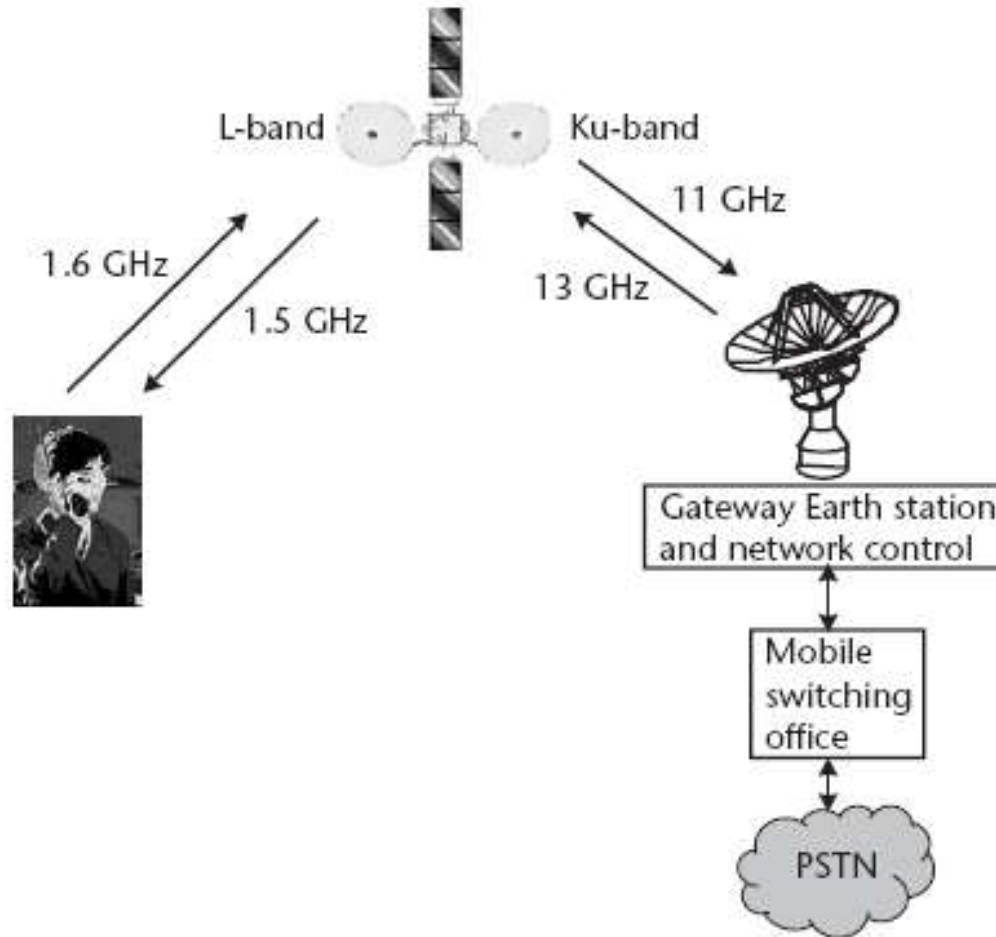


MSS



Thuraya 1 provides high-power mobile satellite links to handheld terminals. (Courtesy of Boeing Satellite Systems.)

The mobile-to-fixed duplex link



VSAT – BSS with one transponder



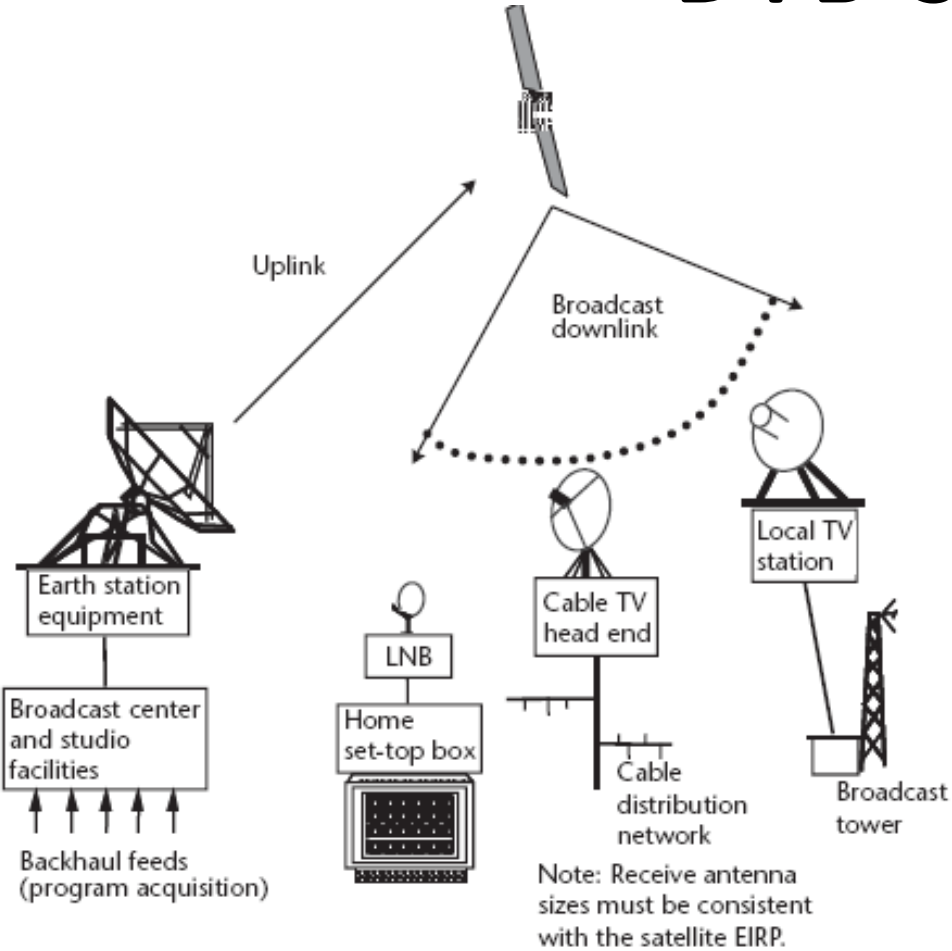
The Measat 1 satellite provides services to Malaysia and throughout Southeast Asia. Total payload power of such satellites reaches 1 kW

VSAT – BSS with 48 transponders



AsiaSat 3C is a hybrid C/Ka satellite with a total of 48 transponders.
Total payload power of such satellites reaches 15 kW

DVB-S



VSAT



Source and Sink of Information

The (almost) **analog signals** of the real (or artificial, therefore **digital**) world around us

Produced – (Source of Info)

- A: by the nature (Waves in the whole spectrum, light, sound, etc...)
- A,D: by humans (speech, music, text (series of symbols - digital), painting, etc...)
- D,A: by artificial sensors (camera, microphone, thermometer, antenna - EM waves, radar, analog voltmeter, etc.).
- D: artificially (Digital devices PC, Machine Type Communication MTC, Vehicle-to-Infrastructure V2I, etc...)

Sensed - (Sink of Info)

- A: by human sensors (we can see, hear, smell, touch – temperature, pressure, taste)
- D: by artificial devices (TV, PC, MTC, etc...)

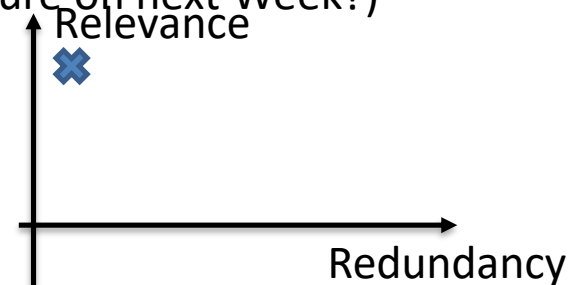
Information bearing signals varying randomly, because constant or deterministic signals we already know.

(Is this a news for you: There will be a Space Communication lecture on next Week?)

Mathematical model: stochastic processes

Relevance and Redundancy - Efficient communication

(songs of whales –not so relevant for human; green background)



Communication Channel

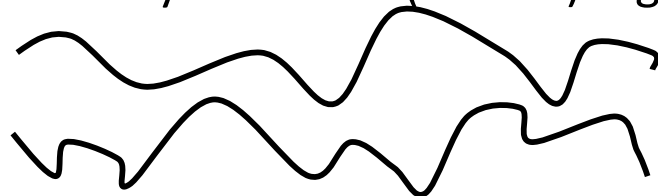
We will deal with technical / engineering aspects of communications and not with the social aspects (like how to introduce ourselves or make a discussion with others).

Physical Channels by different types of information transfer

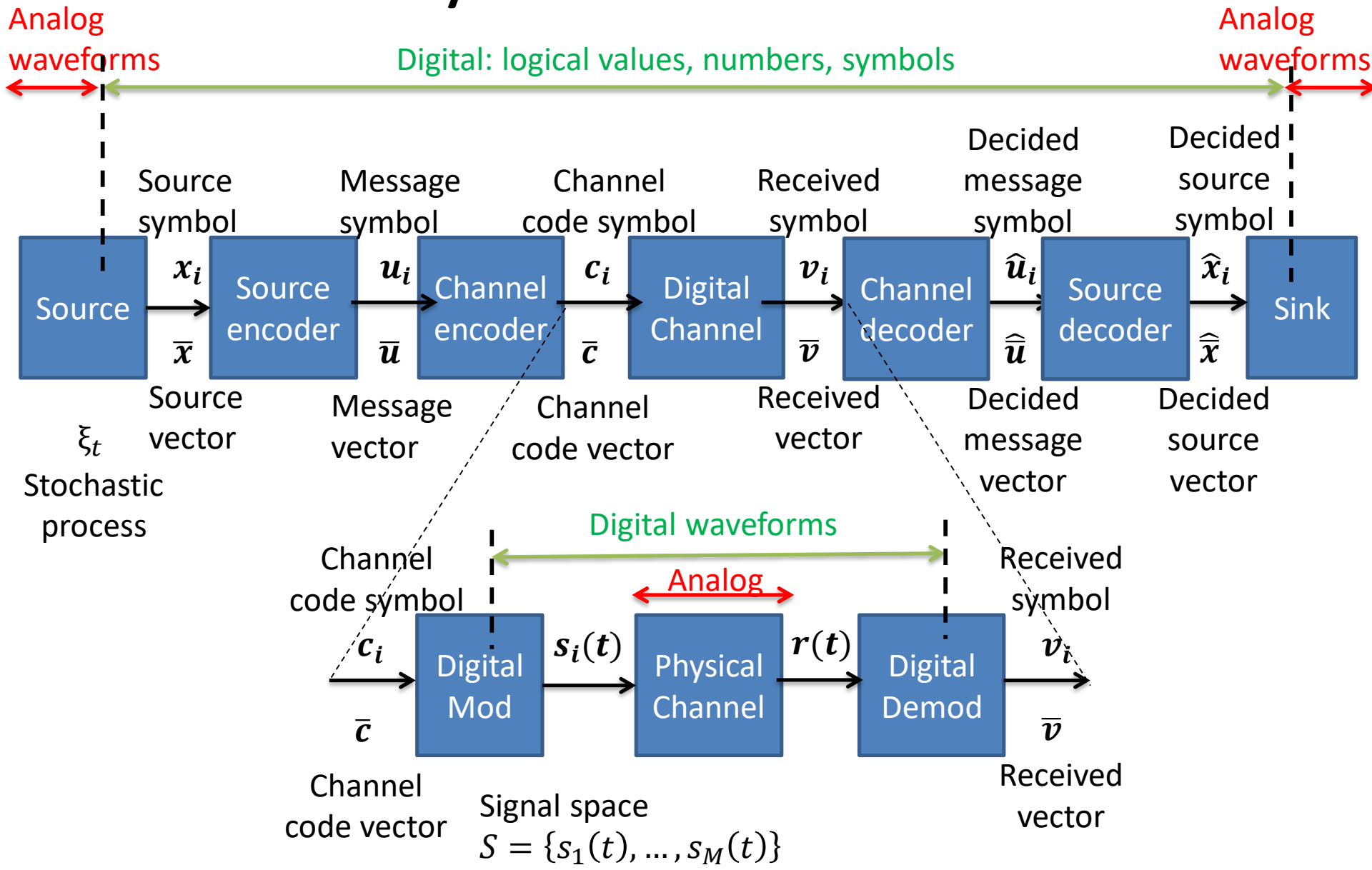
- Information **storage media**: DIN A4 sheet of paper (~1000 Symbols/Letters stored by 8 bit/symbol = 8 kbit), CD, DVD, magnetic tape, papyrus of the ancient Egypt's, etc.
- **Wired communication channels**: twisted pair of copper cable, optical cable, coax cable, wave guide of EM waves, etc.
- **Wireless communication channels**: EM wave propagation, laser, fluctuating air pressure, etc.

We will deal with **digital communication systems** because digital signals/symbols are **better protected / less influenced by the disturbing effects of the communication channels** (we can better differentiate between 0 and 1 as between two analog signal) and because modern systems are (almost) digital.

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System overview



Analog to Digital (A/D) and Digital to Analog (D/A)

A/D: Two steps – **Sampling** and **Quantization**

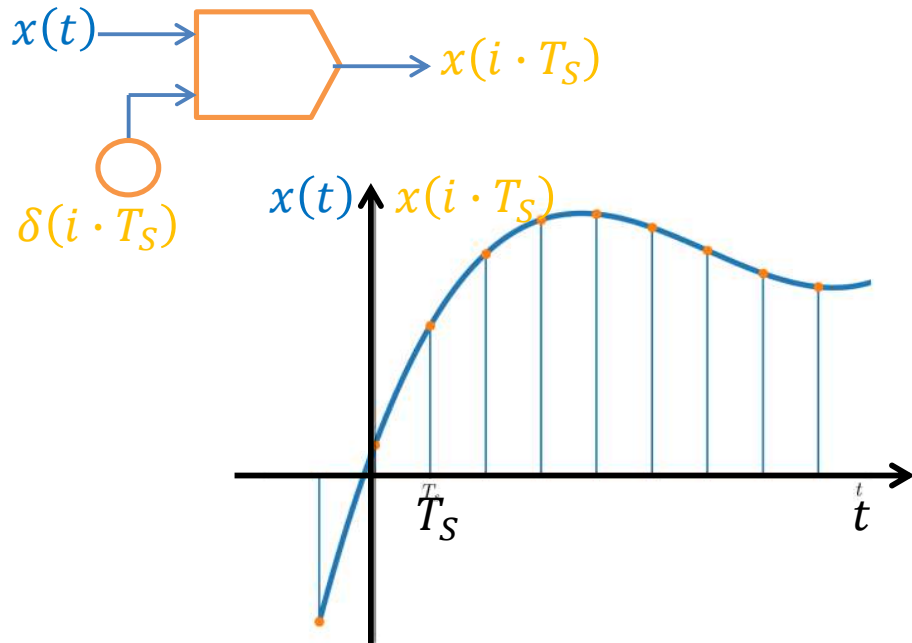
D/A: Time domain – **Interpolation**

Frequency domain – **Low-pass filtering**

Shannon's sampling theorem: If the analog signal $x(t)$ is bandlimited with B and its absolute-valued integral exits, then $x(t)$ is uniquely represented (can be reconstructed free from distortion) by its samples $x(i \cdot T_S)$ taken at a rate $f_S \geq 2 \cdot B$ or equivalently taken at discrete time points separated by $T_S = 1/f_S \leq 1/2B$.

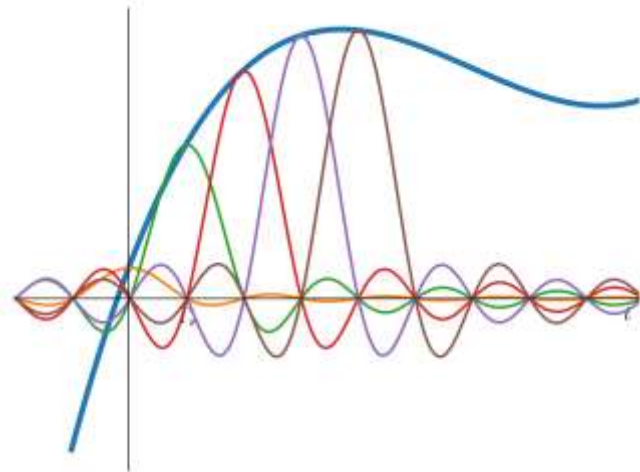
$f_N = 2 \cdot B$ is called Nyquist rate.

Sampling:



Interpolation formula:

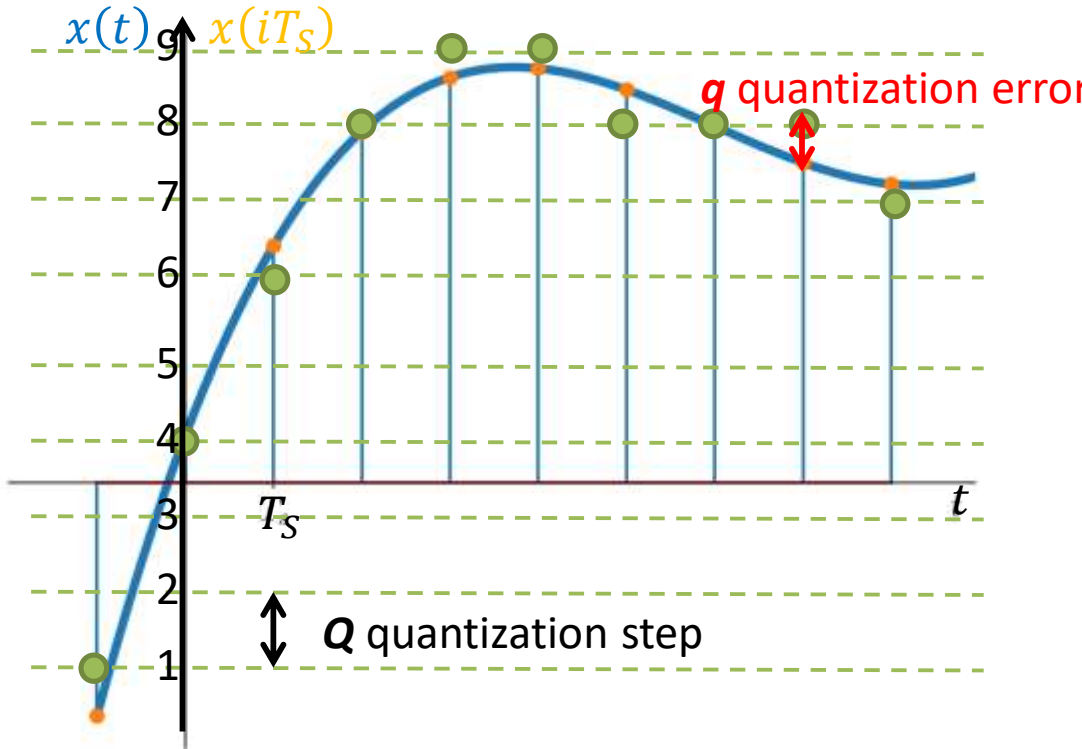
$$x(t) = \sum_{i=-\infty}^{\infty} x(i \cdot T_S) \cdot \frac{\sin 2\pi B(t-i \cdot T_S)}{2\pi B(t-i \cdot T_S)}$$



Analog to Digital (A/D) and Digital to Analog (D/A)

Linear Quantization

$$d(iT_s) = \dots, 1, 4, 6, 8, 9, 9, 8, 8, 8, 7, \dots$$



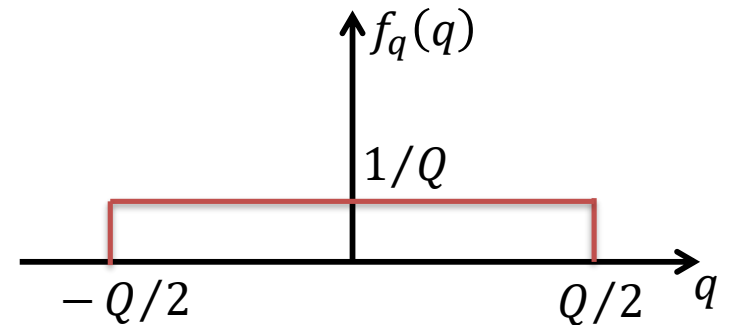
Quantization error is limited for all i
 $-Q/2 \leq q(iT_s) < Q/2$

Random errors causing quantization noise.

$$x(iT_s) = d(iT_s) - q(iT_s)$$

$$x(iT_s) + q(iT_s) = d(iT_s)$$

Assuming uniform distribution:



Power of quantization noise – Relevance? Question of Q:

$$P_q = \int_{-Q/2}^{Q/2} q^2 f_q(q) dq = \frac{1}{Q} \int_{-Q/2}^{Q/2} q^2 dq = \frac{1}{Q} \left[\frac{q^3}{3} \right]_{-Q/2}^{Q/2} = \frac{1}{3Q} \left[\frac{Q^3}{8} - \left(-\frac{Q^3}{8} \right) \right] = \frac{Q^2}{12}$$

Meaning and Measuring of Information

- What is Information?
 - News, new knowledge, surprise, wonder, message, ...?
 - We know what is not: Things, that are constant, not changing, well known by us, if we do not surprise, when the event happens, ...
- How to measure?
 - We need a **quantitative measure** to express the amount of Information for example to be able to compare different systems regarding their efficiency in storage and/or transmission of Information.

The first time to deal quantitatively with the elusive concept of Information was by Ralph Vinton Lyon **Hartley**: “Transmission of Information”, BSTJ, 1928:

- “Reception of a symbol provides information only if there had been other possibilities for its value beside that which was received.”

In other words:

- A symbol can give information only if it the value of a **random variable** (RV).
- Communication systems should be **transmitting random quantities**, not reproduce sinusoidal signals.



Form Wikipedia

Hartley's measure of Information

- Consider single symbol X with D possible values: $X = \{x_1, x_2, \dots, x_D\}$
- How many Information we gather by observing X ?
- The Information conveyed by N such a symbols must be N times as much as that conveyed by just one symbol.

Number of Observation	Number of possibilities	Amount of Information
1 st	D	$\text{Info}(X)$
2 nd	D^2	$2 \cdot \text{Info}(X)$
\vdots	\vdots	\vdots
N^{th}	D^N	$N \cdot \text{Info}(X)$

- Because

$$\log_a(D^N) = N \cdot \log_a(D) \Leftrightarrow N \cdot \text{Info}(X)$$

- This suggests that

$$\text{Info}(X) = \log_a(D)$$

Hartley: "The base selected for the logarithm fix the size of the unit of Information."

- If the base $a=2$, we call Hartley's **unit for Information [bit]**.
- Honoring Hartley, if $a=10$, then the name of the unit [Hartley] (very rarely used).
- If the base $a=e$, we call the **unit for Information [nat]**. $1 [\text{nat}] \approx 1,443 [\text{bit}]$

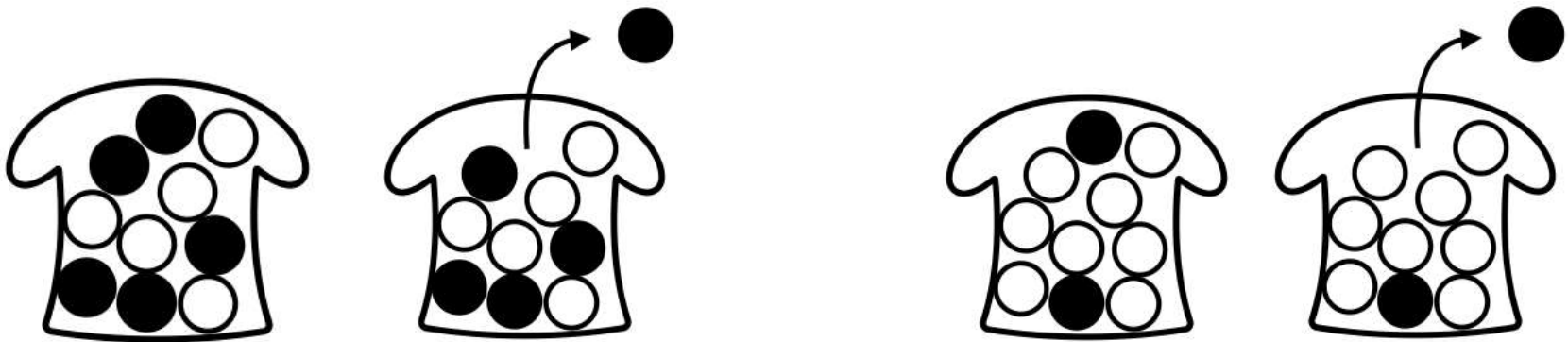
Weakness of Hartley's measure

It ignores the *different probabilities* of the various values of the random variable X.

Consider the random experiment: get out a ball randomly from a hat.

The ball is the symbol X, the number of possibilities is two: $X=\{\text{Black, White}\}$, applying $a=2$ Hartley's Accordingly:

$$Info(X) = \log_{a=2}(D = 2) = ld(2) = 1 [bit]$$



Case A: Same number of Black and White.

Our *“surprise” is the same* by each outcome.

$$Info(X) = \frac{5}{10} ld\left(\frac{10}{5}\right) + \frac{5}{10} ld\left(\frac{10}{5}\right) = 1[bit]$$

Case B: Different number of Black and White.

We are *rather sure in advance, that white* ball will be the result (8 times out of 10), our *“surprise” is higher by black* (from 10 just 2).

$$Info(X) = \frac{8}{10} ld\left(\frac{10}{8}\right) + \frac{2}{10} ld\left(\frac{10}{2}\right) \cong 0,72 [bit]$$

Shannon's measure of Information

Twenty Years After

Claude Elwood **Shannon**: "A Mathematical Theory of Communication", BSTJ, 1948:

The amount of **Information should be reciprocally proportional to the probability** of a random event of a discrete random variable.

Consider a discrete random variable X with n possible values:

$$X = \{x_1, x_2, \dots, x_n\}$$

With a Probability Distribution Function (PDF):

$$p(X) = \{p(x_1), p(x_2), \dots, p(x_n)\}$$

- Definition: **Self Information** of a random event x_i :

$$I(x_i) = \log_2 \frac{1}{p(x_i)} = -\log_2 p(x_i) \text{ [bit, Shannon]}$$

$I(x_i)$ is the amount of Information given in [bit] unit gathered by observing x_i .

- Definition: **Entropy** of a discrete random variable X:

$$H(X) = E\{I(x_i)\} = \sum_{i=1}^n p(x_i) \cdot I(x_i) = \sum_{i=1}^n p(x_i) \cdot \log_2 \frac{1}{p(x_i)} \left[\frac{\text{bit}}{\text{symbol}} \right]$$

$H(X)$ is the Entropy (or uncertainty) is the expected (average) value of the self Information of the events.



Forrás: www.techzibits.com

Some examples

- Self-Information of a **certain event**

$$p(x) = 1 \leftrightarrow I(x) = \text{ld} \frac{1}{p(x)} = \text{ld} 1 = 0$$

- Self Information of an **event that never happens**

$$p(x) = 0 \leftrightarrow I(x) = \text{ld} \frac{1}{p(x)} = ? \text{ div by 0!}$$

However, we can calculate the limit, when $p(x) \rightarrow 0$:

$$\lim_{p(x) \rightarrow 0} p(x) \cdot \text{ld} \frac{1}{p(x)} \iff \lim_{q=1/p(x)} \frac{1}{q} \text{ld} q = \lim_{q \rightarrow \infty} \frac{1 \ln q}{q \ln 2} = \frac{1}{\ln 2} \lim_{q \rightarrow \infty} \frac{\ln q}{q} = \frac{1}{\ln 2} \lim_{q \rightarrow \infty} \frac{1}{q} = 0$$

- We can **imagine the Entropy** as the average number of questions (with binary answer: Yes, or No) needed to find out a character in a word.

Let's try it! I have a word of 4 characters in my mind. You should find it out by questions (e.g. Is the second character A? Answer: Y/N)! The a-priori probabilities of possible characters:

$$p(A)=1/3, p(B)=1/6, p(C)=1/6, p(D)=1/4, p(E)=1/12$$

$$H(X)=1/3 \text{ld}(3)+1/6 \text{ld}(6)+1/6 \text{ld}(6)+1/4 \text{ld}(4)+1/12 \text{ld}(12)= 2,1887... \text{ [bit/symbol]}$$

1 st char.	2 nd char.	3 rd char.	4 th char.	Solution	No. of questions	Average
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1st word

2nd word

3rd word

The Entropy is bounded

Theorem: If the discrete random variable X has n possible values, then

$$0 \leq H(X) \leq \log_2 n = H_0(X)$$

- Proof lower bound:

$$0 \leq p(x_i) \leq 1 \quad \forall i$$

$$\log_2 p(x_i) = \frac{1}{\ln 2} \ln p(x_i) \quad \forall i$$

$$H(X) = -\sum_{i=1}^n p(x_i) \frac{1}{\ln 2} \ln p(x_i) \geq 0 \quad \left[\frac{\text{bit}}{\text{symbol}} \right]$$

- Proof upper bound:

$$H(X) \leq \log_2 n$$

$$H(X) - \log_2 n \leq 0$$

$$\overbrace{\frac{1}{\ln 2} \sum_{i=1}^n p(x_i) \ln \frac{1}{p(x_i)}}^{H(x)} - \overbrace{\frac{1}{\ln 2} \sum_{i=1}^n p(x_i) \ln n}^{\log_2 n} =$$

$$= \frac{1}{\ln 2} \sum_{i=1}^n p(x_i) \ln \frac{1}{\underbrace{n \cdot p(x_i)}_z} \leq \frac{1}{\ln 2} \sum_{i=1}^n p(x_i) [z - 1] = \frac{1}{\ln 2} \sum_{i=1}^n p(x_i) \left[\frac{1}{n \cdot p(x_i)} - 1 \right] =$$

$$= \frac{1}{\ln 2} \left[\underbrace{\sum_{i=1}^n \frac{1}{n}}_1 - \underbrace{\sum_{i=1}^n p(x_i)}_1 \right] = 0 \quad \text{The Entropy } H(X) \text{ has a maximum by } z = 1 = \frac{1}{n \cdot p(x_i)} \quad \forall i$$

$p(x_i) = 1/n, \forall i \rightarrow$ **Uniformly distributed random variable has maximum Entropy.**

