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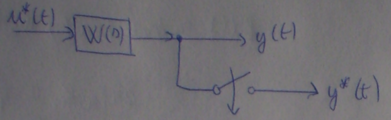
jelterjedés



$$Y(s) = W(s)\,U^*(s) = W(s)\sum_{k=0}^{\infty} u(kT)\,e^{-skT}$$

$$y(t) = \sum_{k=0}^{\infty} w(t-kT)\,u(kT)$$

w: súly fv.

$$Y^*(s) = \sum_{n=0}^{\infty} y(nT)\,e^{-snT} =$$

$$= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{\infty} w([n-k]T)\,u(kT)\right) e^{-snT}$$

$$\Downarrow$$

$$*$$

$$Y^*(s) = \sum_{k=0}^{\infty}\sum_{n=k}^{\infty} w(\underbrace{[n-k]}_{i}T)\,e^{-s\underbrace{(n-k)}_{i}T}\,u(kT)\,e^{-skT} =$$

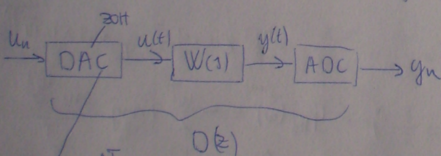
$$= \sum_{k=0}^{\infty}\sum_{i=0}^{\infty} w(iT)\,e^{-siT}\,u(kT)\,e^{-skT} = \left(\sum_{i=0}^{\infty} w(iT)\,e^{-siT}\right)\left(\sum_{k=0}^{\infty} u(kT)\,e^{-skT}\right)$$

$$Y^*(s) = W^*(s)\,U^*(s)$$

$$\sum_{n=0}^{\infty} y(nT)\,e^{-snT} = \left(\sum_{i=0}^{\infty} w(iT)\,e^{-siT}\right)\left(\sum_{k=0}^{\infty} u(kT)\,e^{-skT}\right) \Rightarrow \boxed{z = e^{sT}}$$

Z-transzformáltat kaptunk:

$$Z\{y(nT)\} = Z\{w(nT)\}\,Z\{u(nT)\} \Rightarrow D(z) = Z\{w(nT)\}$$



$$\frac{1-e^{-sT}}{s} \Rightarrow W(s)\,\frac{1-e^{-sT}}{s} = \frac{W(s)}{s}\left(1-e^{-sT}\right)$$

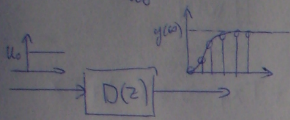
$$\Downarrow$$

$$\boxed{D(z) = (1-z^{-1})\,Z\{v(nT)\}}$$

$$W(s) \xrightarrow[\text{'zoh'}]{\text{c2d}} D(z)$$

Ekvivalens átviteli tényező

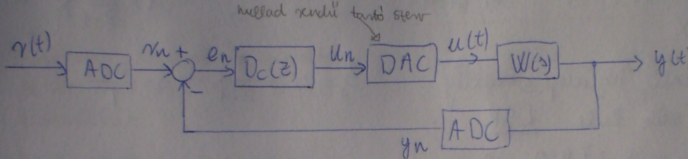
$$A_{ekv} = \frac{y(\infty)}{u_0}$$

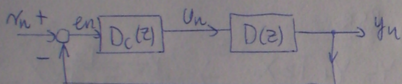


$$Y(z) = D(z)\frac{U_0}{1-z^{-1}}$$

$$y(\infty) = \lim_{z \to 1}(1-z^{-1})Y(z) = D(1)u_0$$

$$\boxed{A_{ekv} = \frac{y(\infty)}{u_0} = D(1)}$$





$$D_{yr} = \frac{D_c D}{1 + \underbrace{D_c D}_{D_0}}$$

$D_0(z) = D_c(z)\,D(z)$ felnyitott kör

$$D_{ur} = \frac{D_c}{1 + \underbrace{D_c D}_{D_0}}$$

Szabályozó tervezés

① Analóg szabályzó mintavételes közelítése

$\omega_c,\ \varphi_t$ (analóg)

$T = ?$

$$\frac{\omega_c T}{2} = 5°\frac{\pi}{180} \Rightarrow \omega_c T = \frac{10\pi}{180} = \frac{\pi}{18} = 0.17$$

$\omega_c T \simeq 0{,}2$

Közelítések: $\frac{d}{dt} \leftrightarrow s$, $\int dt \leftrightarrow \frac{1}{s} = s^{-1}$

① $\frac{dy}{dt} \to sY$ $\frac{dy}{dt} = \frac{y(t) - y(t-T)}{T} \to \frac{1}{T}(Y - z^{-1}Y) = \frac{1-z^{-1}}{T}Y$

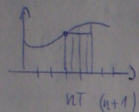
$$s \approx \frac{1-z^{-1}}{T} = \frac{z-1}{Tz}$$ hátratartó differencia BWD

backward diff.

② $\frac{dy}{dt} \approx \frac{y(t+T)-y(t)}{T} \longrightarrow \frac{zY-Y}{T} = \frac{z-1}{T}Y$

$s \approx \frac{z-1}{T_t}$ elöretartó diff. forward diff. $\boxed{FWD}$

③ $y(t)=\int_0^t x(\tau)d\tau$



$y_{n+1} = y_n + Tx_n \rightarrow zY = Y + TX$

$Y = \frac{1}{s}X$ $\qquad$ $Y = \frac{T}{z-1}X$

Baloldali téglalap szabály

Left side Rule $\underline{LSR}$

Megegyezik: FWD-vel

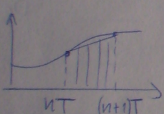
$\frac{1}{s} = s^{-1} = \frac{T}{z-1} \rightarrow s = \frac{z-1}{T}$

④ Jobb oldali téglalap szabály RSR

$y_{n+1} = y_n + Tx_{n+1} \Rightarrow zY = Y + TzX \quad ; \quad Y = \frac{Tz}{z-1}X$

$\frac{1}{s} = s^{-1} = \frac{Tz}{z-1} \longrightarrow s = \frac{z-1}{Tz}$

⑤ Trapéz szabály, Tustin-képlet



$y_{n+1} = y_n + \frac{x_n + x_{n+1}}{2}T$

$zY = Y + \frac{T}{2}(X + zX)$

$Y = \frac{T}{2}\frac{z+1}{z-1}X$

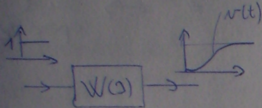
$\frac{1}{s} = s^{-1} = \frac{T}{2}\frac{z+1}{z-1} \longrightarrow s = \frac{2}{T}\frac{z-1}{z+1}$ Tustin képlet

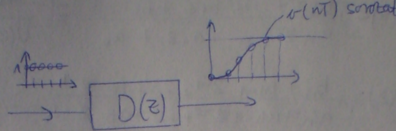
Alkalmazás: $W_c(s) \longrightarrow D_c(z)$

s közelítése

Közelítések helyett alkalmazhatunk ekvivalenciákat is! X.19.p. G.a

Egységugrás ekvivalencia





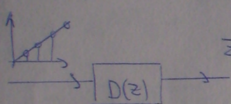
$$Z\{v(nT)\} = D(z)\frac{1}{1-z^{-1}}$$

↑
Ezt már egyszer kiszámoltuk, de épp most is ez a képlet használható.

$$\boxed{D(z) = (1-z^{-1})\ Z\{v(nT)\}}$$

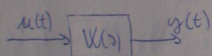
$$W(s) \xrightarrow[\text{'zoh'}]{c2d} D(z)$$

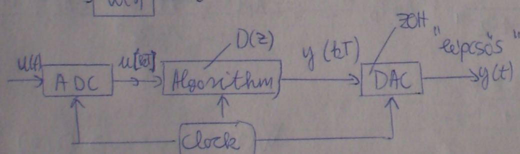
Sebességugrás ekvivalencia



$$Z\mathcal{L}^{-1}\left\{\frac{W(s)}{s^2}\right\}$$

$$Z\mathcal{L}^{-1}\left\{\frac{W(s)}{s^2}\right\} = D(z)\frac{Tz^{-1}}{(1-z^{-1})^2} \longrightarrow D(z) = \frac{(1-z^{-1})^2}{Tz^{-1}}Z\mathcal{L}^{-1}\left\{\frac{W(s)}{s^2}\right\}$$





$$\tilde{W}(j\omega) = \frac{1-e^{-j\omega T}}{j\omega}D(z=e^{j\omega T})$$

Approximáció javul:

Javulási sorrend:

- Tustin
- egységu. ekv.
- sebességu. ekv.

↓ jav.

Algorithm:

$$D(z) = \frac{b_n z^n + \ldots + b_1 z + b_0}{a_n z^n + \ldots + a_1 z + a_0} = \frac{Y(z)}{U(z)}$$

$z^{-k} \Longleftrightarrow$ shift operátor

$$D(z) = \frac{b_n + b_{n-1} z^{-1} + \ldots + b_0 z^{-n}}{a_n + a_{n-1} z^{-1} + \ldots + a_0 z^{-n}} = \frac{Y(z)}{U(z)}$$

$$a_n y_k + \underline{a_{n-1} y_{k-1} + \ldots + a_0 y_{k-n}} = b_n u_k + b_{n-1} u_{k-1} + \ldots + b_n u_{k-n}$$

$$\underline{\underline{y_k}} = -\frac{a_{n-1}}{a_n} y_{k-1} - \ldots - \frac{a_0}{a_n} y_{k-n} + \frac{b_n}{a_n} u_k + \ldots + \frac{b_0}{a_n} u_{k-n}$$

$$\{y_{k-1}, y_{k-2}, \ldots, y_{k-n}, u_k, u_{k-1}, \ldots, u_{k-n}\} \longrightarrow y_k$$

ALGORITHM

Analóg PID mintavételes közelítése

① Ideális PID

$$W_{PID}(s) = A_P\left(1 + \frac{1}{sT_I} + sT_D\right) = A_P + \frac{A_P}{T_I}\frac{1}{s} + A_P T_D s$$

Közelítés: BWD = RSR $\quad s = \frac{z-1}{Tz} = \frac{1-z^{-1}}{T}$

$$D_{PID}(z) = A_P + \frac{A_P}{T_I}\frac{T}{1-z^{-1}} + A_P T_D \frac{1-z^{-1}}{T} = \frac{A_P(1-z^{-1}) + \frac{A_P T}{T_I} + A_P \frac{T_D}{T}(1-z^{-1})^2}{1-z^{-1}}$$

$$= \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1-z^{-1}}$$

$$q_0 = A_P\left\{1 + \frac{T}{T_I} + \frac{T_D}{T}\right\}$$

$$q_1 = -A_P\left\{1 + \frac{2T_D}{T}\right\}$$

$$q_2 = A_P\frac{T_D}{T}$$

$$e_k \longrightarrow \boxed{D_{PID}} \longrightarrow u_k$$

$$W(s) \longrightarrow D(z)$$

$z_i = e^{s_i T}$ helyre képződnek a pólusok

Integrátor esetén:

$$s_i = 0 \longrightarrow z_i = 1$$

② Közelítő PID

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$$W_{PID}(s) = A_P\left(1 + \frac{1}{sT_I} + \frac{sT_D}{1+sT_C}\right) = A_P + \frac{A_P}{T_I}\frac{1}{s} + \frac{A_P T_D}{T_C}\frac{1}{s+\frac{1}{T_C}}$$

Egységugrás ekv.:

$$v_{PID}(t) = \mathcal{L}^{-1}\left\{\frac{A_P}{s} + \frac{A_P}{T_I}\frac{1}{s^2} + \frac{A_P T_D}{T_C}\frac{1}{s+\frac{1}{T_C}}\right\} = A_P + \frac{A_P}{T_I}t + \frac{A_P T_D}{T_C}e^{-\frac{t}{T_C}}, \quad t \geq 0$$

$$D_{PID}(z) = (1-z^{-1})Z\{v_{PID}(nT)\} = \cancel{(1-z^{-1})}\left\{A_P\frac{1}{\cancel{1-z^{-1}}} + \frac{A_P}{T_I}\frac{Tz^{-1}}{(1-z^{-1})^{\cancel{2}}} + \frac{A_P T_D}{T_C}\cdot\frac{1}{1-e^{-T/T_C}z^{-1}}\right\}$$

$1-z^{-1}$

$$e^{s_i t} \to \{1 + e^{s_i T}z^{-1} + e^{2s_i T}z^{-2} + \dots\}$$

$$q = e^{s_i T}z^{-1}$$

$$\frac{1}{1-q} = \frac{1}{1-e^{s_i T}z^{-1}} = \frac{z}{z-e^{s_i T}}$$

Matek

$$D_{PID}(z) = A_P + A_P\frac{T_0}{T_I}\frac{z^{-1}}{1-z^{-1}} + \frac{A_P T_D}{T_C}\frac{1-z^{-1}}{1-e^{-T/T_C}z^{-1}} =$$

$$= \frac{A_P(1-z^{-1})(1-e^{-T/T_C}z^{-1}) + \frac{A_P}{T_I}Tz^{-1}(1-e^{-T/T_C})z^{-1} + \frac{A_P T_D}{T_C}(1-z^{-1})^2}{(1-z^{-1})(1-e^{-T/T_C}z^{-1})}$$

$$D_{PID}(z) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{p_0 + p_1 z^{-1} + p_2 z^{-2}} = \frac{A_P + \frac{A_P}{T_I}\frac{Tz^{-1}}{1-z^{-1}} + \frac{A_P T_D}{T_C}\frac{1-z^{-1}}{1-e^{-T/T_C}z^{-1}}}{1}$$

$p_0 = 1$

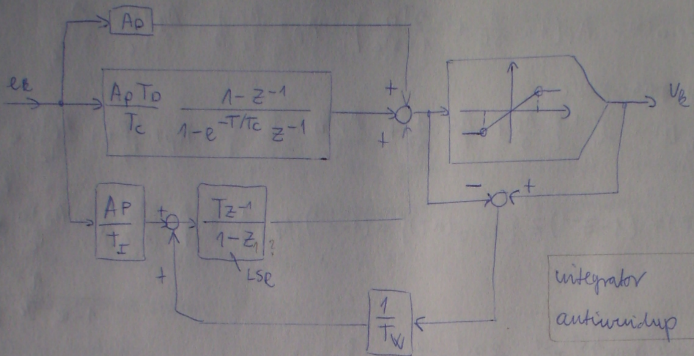
$p_1 = -(1 + e^{T/T_C})$

$p_2 = e^{-T/T_C}$

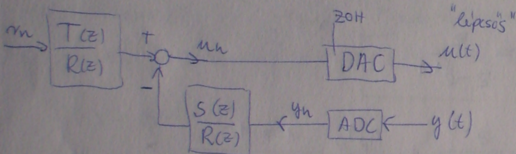
$$q_0 = A_P\left\{1 + \frac{T_D}{T_C}\right\}$$

$$q_1 = -A_P\left\{1 + e^{-T/T_C} - \frac{T}{T_I} + \frac{2T_D}{T_C}\right\}$$

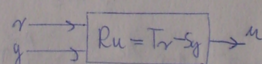
$$q_2 = A_P\left\{e^{-T/T_C}\left(1 - \frac{T}{T_I}\right) + \frac{T_D}{T_C}\right\}$$

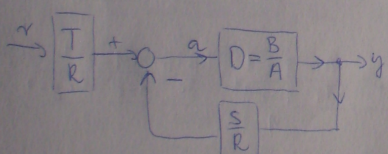


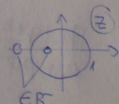
2 szabadságfokú szabályzó : 2 – DOF



$$u = \frac{T}{R} r - \frac{S}{R} y \iff Ru = Tr - Sy$$







$$\frac{\frac{T}{R}\frac{B}{A}}{1+\frac{S}{R}+\frac{B}{A}} = \frac{TB}{AR+BS} = \frac{B_m}{A_m} \cdot \frac{A_o}{A_o} ;$$

$$B = B^+ B^-$$

B^-: • egységkörön kívüli zérushelyek

• valósak, de -1 és 0 között foglalnak helyet

$$R = B^+ R^- = B^+ (z-1)^{l} R'_1$$

l: integrátorok db száma

$$-a = e^{s_i T} \rightarrow s_i = \frac{1}{T} \ln(-a)$$

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$$\frac{T B^+ B^-}{A \cancel{B^+} (z-1)^l R_1' + \cancel{B^+} B^- S} = \frac{B^- B_m'}{A_m} \frac{A_o}{A_o}$$

A_o = megfigyelő polinom

$B_m = B^- B_m'$

$T = B_m' A_o$

$$\boxed{A (z-1)^l R_1' + B^- S = A_m A_o}$$

Ez egy polinom egyenlet, méghozzá diophantoszi! ("diofantoszi")

1-vezetőegyütthatójú: A_m, A_o, A, B^+, R_1'

l: integrátorok száma $(0, 1, 2)$

azaz monik

Nem monik: T, S

gr (fokszám):

① $\frac{T}{R}$ és $\frac{S}{R}$ kauzális $\Longleftrightarrow \overbrace{gr(T)}^{\text{T-nek a fokszáma}} \leq gr(R)$, $gr(S) \leq gr(R)$

② $AX + BY = C$

X_0, Y_0 megoldás $\Rightarrow X_0 + QB$, $Y_0 - QA$

$A(X_0 + QB) + B(Y_0 - QA) = AX_0 + BY_0 + \cancel{AQB} - \cancel{BQA} = C$

Egyetlen megoldás van, ha $gr(X) < gr(B)$ és $gr(Y) < gr(A)$

③ Eredő fokszám feltételek:

$gr(A_m) - gr(B_m) \geq gr(A) - gr(B)$

$gr(S) = gr(A) + l - 1$

$gr(A_o) \geq 2gr(A) + l - 1 - gr(B^+) - gr(A_m)$

$gr(R_1') = gr(A_m) + gr(A_o) - gr(A) - l$

$$AX + BY = C$$

$$A = z^n + a_1 z^{n-1} + \dots + a_n$$

$$B = b_0 z^m + b_1 z^{m-1} + \dots + b_m$$

$$X = z^k + x_1 z^{k-1} + \dots + x_k$$

$$Y = y_0 z^k + y_1 z^{k-1} + \dots + y_k$$

$$C = z^{n+k} + c_1 z^{n+k-1} + \dots + c_{n+k}$$

k db oszlop

$$\left[\begin{array}{cccc|cccc}
1 & 0 & \cdots & 0 & b_0 & 0 & \cdots & 0 \\
a_1 & \ddots & & \vdots & b_1 & \ddots & & 0 \\
a_2 & & \ddots & 0 & b_2 & & \ddots & b_0 \\
\vdots & & & 1 & \vdots & & & b_1 \\
 & & & a_1 & & & & b_2 \\
a_n & & & a_2 & b_m & & & \vdots \\
0 & \ddots & & & 0 & & \ddots & \vdots \\
\vdots & & & & \vdots & & & \\
0 & & & a_n & 0 & & & b_m
\end{array}\right]
\begin{pmatrix} x_1 \\ \vdots \\ x_k \\ y_0 \\ \vdots \\ \\ y_k \end{pmatrix}
=
\begin{pmatrix} c_1 - a_1 \\ c_2 - a_2 \\ c_3 - a_3 \\ \vdots \\ c_n - a_n \\ c_{n+1} \\ \vdots \\ c_{n+k} \end{pmatrix}$$

Ez egy Toeplitz mátrix

Nálunk:

$$A := A(z-1)^{\ell}$$

$$B := B^{-}$$

$$X := R_1'$$

$$Y := S$$

$$C := A_m A_0$$