Gyakorló feladatok megoldásai - 5.

MAM112m

1. (a) $$\sum_{k=1}^{\infty} -2\frac{(-1)^k \sin(kx)}{k} = \begin{cases} x, & x \in (-\pi, \pi) \\ 0, & x = -\pi, \pi \end{cases}$$

(b) $$\frac{1}{3}\pi^2 + \sum_{k=1}^{\infty} 4\frac{(-1)^k \cos(kx)}{k^2} = x^2, \qquad x \in [-\pi, \pi]$$

(c) $$3 + \sum_{k=1}^{\infty} 36\frac{(-1)^k \cos(\frac{1}{3}k\pi x)}{k^2\pi^2} + \sum_{k=1}^{\infty} 6\frac{(-1)^k \sin(\frac{1}{3}k\pi x)}{k\pi} = \begin{cases} x^2 - x, & x \in (-3, 3) \\ 9, & x = -3, 3 \end{cases}$$

(d) $$1 + \sum_{k=1}^{\infty} -12\frac{(-1)^k \sin(\frac{1}{3}k\pi x)}{k\pi} = \begin{cases} 2x + 1 & x \in (-3, 3) \\ 1 & x = -3, 3 \end{cases}$$

(e) $$\frac{1}{6}\sin(6) + \sum_{k=1}^{\infty} -12\frac{\sin(6)\,(-1)^k \cos(\frac{1}{2}k\pi x)}{k^2\pi^2 - 36} = \begin{cases} \cos 3x, & x \in (-2, 2) \\ \cos 6, & x = -2, 2 \end{cases}$$

(f) $$-\frac{1}{8}e^{-4} + \frac{1}{8}e^{4} + \sum_{k=1}^{\infty} 4\frac{(-1)^k \left(e^4 - e^{-4}\right) \cos(\frac{1}{2}k\pi x)}{k^2\pi^2 + 16}$$
$$+\sum_{k=1}^{\infty} \frac{k\pi\,(-1)^k \left(e^4 - e^{-4}\right) \sin(\frac{1}{2}k\pi x)}{k^2\pi^2 + 16} = \begin{cases} e^{-2x}, & x \in (-2, 2) \\ \frac{1}{2}(e^4 + e^{-4}), & x = -2, 2 \end{cases}$$

(g) $$\frac{1}{2} + \sum_{k=1}^{\infty} \frac{\left(-1 + (-1)^k\right) \sin(kx)}{\pi k} = \begin{cases} \frac{1}{2}, & x = -\pi, 0, \pi \\ 1, & x \in (-\pi, 0) \\ 0, & x \in (0, \pi) \end{cases}$$

(h) $$\frac{1}{4}\pi + \sum_{k=1}^{\infty} \frac{\left((-1)^k - 1\right) \cos(kx)}{\pi k^2} + \sum_{k=1}^{\infty} -\frac{(-1)^k \sin(kx)}{k} = \begin{cases} 0, & x \in (-\pi, 0] \\ x, & x \in (0, \pi) \\ \frac{\pi}{2}, & x = -\pi, \pi \end{cases}$$

(i) $$\frac{1}{4} + \sum_{k=1}^{\infty} \frac{\left((-1)^k - 1\right) \cos(k\pi x)}{k^2\pi^2} + \sum_{k=1}^{\infty} -3\frac{(-1)^k \sin(k\pi x)}{k\pi} = \begin{cases} x, & x \in (-1, 0] \\ 2x, & x \in (0, 1) \\ \frac{1}{2}, & x = -1, 1 \end{cases}$$

(j) $$\frac{1}{2} + \sum_{k=1}^{\infty} -2\frac{\sin(\frac{1}{2}k\pi) \cos(\frac{1}{2}k\pi x)}{k\pi} + \sum_{k=1}^{\infty} 4\frac{\left(-(-1)^k + \cos(\frac{1}{2}k\pi)\right) \sin(\frac{1}{2}k\pi x)}{k\pi}$$
$$= \begin{cases} -1, & x \in (-2, -1) \\ 0, & x \in (-1, 1) \\ 3, & x \in (1, 2) \\ 1, & x = -2, 2 \\ -\frac{1}{2}, & x = -1 \\ \frac{3}{2}, & x = 1 \end{cases}$$

2. (a) koszinuszos: $1, \qquad x \in [0,\pi]$

szinuszos: $\displaystyle\sum_{k=1}^{\infty} -2\frac{\Big((-1)^k - 1\Big)\sin(kx)}{\pi k} = \begin{cases} 1, & x \in (0,\pi) \\ 0, & x = 0,\pi \end{cases}$

(b) koszinuszos: $\displaystyle\frac{1}{3} + \sum_{k=1}^{\infty} 4\frac{(-1)^k \cos(k\pi x)}{k^2\pi^2} = x^2, \qquad x \in [0,1]$

szinuszos: $\displaystyle\sum_{k=1}^{\infty} -2\frac{\Big(k^2\pi^2\,(-1)^k - 2\,(-1)^k + 2\Big)\sin(k\pi x)}{k^3\pi^3} = \begin{cases} x^2, & x \in [0,1) \\ 0, & x = 1 \end{cases}$

(c) koszinuszos: $\displaystyle\frac{2}{\pi} + \sum_{k=2}^{\infty} -2\frac{\Big((-1)^k + 1\Big)\cos(kx)}{\pi(k^2-1)} = \sin x, \qquad x \in [0,\pi]$

szinuszos: $\sin x, \qquad x \in [0,\pi]$

(d) koszinuszos: $\displaystyle\frac{3}{2} + \sum_{k=1}^{\infty} -2\frac{\sin(\frac{1}{2}k\pi)\cos(\frac{1}{2}k\pi x)}{k\pi} = \begin{cases} 1, & x \in [0,1) \\ 2, & x \in (1,2] \\ \frac{3}{2}, & x = 1 \end{cases}$

szinuszos: $\displaystyle\sum_{k=1}^{\infty} -2\frac{\Big(2\,(-1)^k - \cos(\frac{1}{2}k\pi) - 1\Big)\sin(\frac{1}{2}k\pi x)}{k\pi} = \begin{cases} 1, & x \in (0,1) \\ 2, & x \in (1,2) \\ \frac{3}{2}, & x = 1 \\ 0, & x = 0,2 \end{cases}$