Gyakorló feladatok megoldásai - 1.

MAM112M

1.

(a) $-\dfrac{-18+40\,s-25\,s^2+24\,s^3}{s^4}$ (b) $\dfrac{s}{s^2-25}$

(c) $\dfrac{3}{(s+2)^2+9}$ (d) $5\dfrac{s^2+21}{(s^2+49)(s^2+9)}$

(e) $\dfrac{6}{(s-7)^4}$ (f) $-6\,\dfrac{s-3}{\left((s-3)^2+64\right)^2}+8\,\dfrac{(s-3)^3}{\left((s-3)^2+64\right)^3}$

(g) $\dfrac{1}{s-5\ln 2}$ (h) $\dfrac{5}{s}-\dfrac{1}{s-4i}$

2.

(a) $\frac{3}{4}\sqrt{4}\sin(\sqrt{4}t)$ (b) $2\,t^2e^t$

(c) $2/5\ (e^t-e^{-4\,t})$ (d) $-2/5\,e^{-2\,t}+7/5\,e^{3\,t}$

(e) $2\,e^{-t}\cos(2\,t)$ (f) $2\,e^t\cos(t)+3\,e^t\sin(t)$

(g) $3+5\,\cos(2\,t)-2\,\sin(2\,t)$ (h) $-1/3\ (2\,e^{2\,t}-6e^{4\,t}+e^{-\,t})$

3.

(a) $e^{-t}+1/2\,e^{-2\,t}$ (b) $1/5\,e^{3\,t}+4/5\,e^{-2\,t}$

(c) $e^{2\,t}-e^{2\,t}t$ (d) $2\,e^{-t}\cos(2\,t)+1/2\,e^{-t}\sin(2\,t)$

(e) $2/3\,t^3e^t-t^2e^t+e^tt$ (f) $\frac{1}{5}\,\cos(t)-\frac{2}{5}\,\sin(t)+\frac{4}{5}\,e^t\cos(t)-\frac{2}{5}\,e^t\sin(t)$

(g) $2\,t^2e^{-t}+te^{-t}+2\,e^{-t}$ (h) $-\frac{1}{7}\left(-e^{-3\,t}+\cosh(\sqrt{2}t)-5\,\sqrt{2}\sinh(\sqrt{2}t)\right)e^{2\,t}$

(i) $\mathbf{x}(t)=(2\,e^{2\,t}+e^{-t},\ 2\,e^{-t}+e^{2\,t})$ (j) $\mathbf{x}(t)=(7/2\,e^{4\,t}-3/2\,e^{2\,t},\ -9/2\,e^{2\,t}+7/2\,e^{4\,t})$

4.

(a) $e^{-3s}24/s^5$ (b) $e^{-s}(\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s})+e^{-3s}(-\frac{2}{s^3}-\frac{6}{s^2}-\frac{9}{s})$

(c) $e^{-s}(\frac{6}{s^4}+\frac{6}{s^3}+\frac{2}{s^2}+\frac{2}{s})$ (d) $e^{-s}+2e^{-2s}-4e^{-5s}$

(e) $\frac{2s}{s^3(s^2+4)}$ (f) $\frac{1}{(s+1)(s^2+1)}$

(g) $\frac{1}{s^2(s-1)}$ (h) $\frac{s}{(s^2+1)^2}$

5.

(a) $f(t) = \frac{1}{4}t^3$

(b) $H_2(t)(\frac{1}{3}e^{t-2} - \frac{1}{3}e^{-2t+4})$

(c) $H_2(t)(2\,e^{t-2}\cos(t-2))$

(d) $\frac{1}{2}H_2(t)(e^{2\,t-4} - e^{-2\,t+4})$

(e) $H_1(t) + H_2(t) - H_3(t) - H_4(t)$

(f) $\frac{1}{6}\,t^3 - t + \sin(t)$

(g) $-\frac{1}{5}\,e^{-t} + \frac{1}{5}\,\cos(2\,t) + \frac{2}{5}\,\sin(2\,t)$

(h) $\frac{1}{5}\,te^{-t} + \frac{2}{25}\,e^{-t} - \frac{2}{25}\,\cos(2\,t) - \frac{3}{50}\,\sin(2\,t)$

6.

(a) $1 - \cos t + \sin t - H_{\pi/2}(t)(1 - \sin t)$

(b) $e^{-t}\sin t + \frac{1}{2}H_\pi(t)\left(1 + e^{-(t-\pi)}(\cos t + \sin t)\right) - \frac{1}{2}H_{2\pi}(t)\left(1 - e^{-(t-2\pi)}(\cos t + \sin t)\right)$

(c) $g(t) + H_\pi(t)g(t-\pi), \quad g(t) = \frac{4}{14}(-4\cos t + \sin t + 4e^{-t/2}\cos t + e^{-t/2}\sin t)$

(d) $\frac{1}{6}(1 - H_{2\pi}(t))(2\sin t - \sin 2t)$

(e) $H_1(t)g(t-1) - H_2(t)g(t-2), \quad g(t) = -1 + (\cos t + \operatorname{ch} t)/2$

(f) $e^{-t}\cos t + e^{-t}\sin t - H_\pi(t)e^{-(t-\pi)}\sin t$

(g) $\frac{1}{2}H_\pi(t)\sin 2t - \frac{1}{2}H_{2\pi}(t)\sin 2t$

(h) $\frac{1}{5}\cos t + \frac{2}{5}\sin t - \frac{1}{5}e^{-t}\cos t - \frac{3}{5}e^{-t}\sin t - H_{\pi/2}(t)e^{-(t-\pi/2)}\cos t$

(i) $-\frac{6}{85}\,\cos(3\,t) - \frac{7}{85}\,\sin(3\,t) + \frac{6}{85}\,e^{-t}\cos(t) + \frac{27}{85}\,e^{-t}\sin(t)$

(j) $\frac{1}{5}\,e^{-t/2}\cos(t) - \frac{9}{10}\,e^{-t/2}\sin(t) + \frac{4}{5} - \frac{4}{5}\,H_\pi(t)$
$-\frac{4}{5}\,H_\pi(t)e^{-(t-\pi)/2}\cos(t) - \frac{2}{5}\,H_\pi(t)e^{-(t-\pi)/2}\sin(t)$