1/a, $\lambda^4+8\lambda^2+16=(\lambda^2+4)^2=(\lambda+2i)^2(\lambda-2i)^2=0$ (3)

$\boxed{6}$ $y_{H,\acute{a}}(x)=A\sin(2x)+B\cos(2x)+Cx\sin(2x)+Dx\cos(2x)$ (3)

b, Nincs külső rezonancia!

$\boxed{5}$ $y_{I,p}(x)=Ae^{2x}$ (2) ; $y^{(4)}_{I,p}(x)=2^4Ae^{2x}=16Ae^{2x}$

$y^{(2)}_{I,p}(x)=2^2Ae^{2x}=4Ae^{2x}$

$16Ae^{2x}+8\cdot 4Ae^{2x}+16Ae^{2x}=e^{2x}$ (2) $\Rightarrow A=\dfrac{1}{16+32+16}=\underline{\underline{\dfrac{1}{64}}}$ (1)

c, Van külső rezonancia.

$\boxed{4}$ $y_{I,p}(x)=x^2\left(A\sin(2x)+B\cos(2x)\right)$

2/a, $(1+x)^\alpha=\sum_{n=0}^{\infty}\binom{\alpha}{n}x^n$ (2); $R=1$ (1); $\binom{\alpha}{n}=\dfrac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}$ (3)

$\boxed{6}$

b, $f(x)=(8+3x^3)^{-1/3}=\dfrac{1}{2}\left(1+3\left(\dfrac{x}{2}\right)^3\right)^{-1/3}$ (2) $=\underline{\underline{\dfrac{1}{2}\sum_{n=0}^{\infty}\binom{-1/3}{n}\left(\dfrac{x}{2}\right)^{3n}\cdot 3^n}}$ (3),

$\boxed{9}$

ha $\left|3\left(\dfrac{x}{2}\right)^3\right|<1$, $\underline{\underline{R=2\sqrt[3]{\dfrac{1}{3}}}}$ (2)

$T_5(x)=\dfrac{1}{2}\cdot\underbrace{\binom{-1/3}{0}}_{1}\cdot\underbrace{3^0}_{1}\underbrace{\left(\dfrac{x}{2}\right)^0}_{1}+\dfrac{1}{2}\underbrace{\binom{-1/3}{1}}_{-\frac{1}{3}}\underbrace{3^1}_{3}\underbrace{\left(\dfrac{x}{2}\right)^3}_{\frac{x^3}{8}}=\underline{\underline{\dfrac{1}{2}+\dfrac{-x^3}{16}}}$ (2)

3/, $f(x)=\dfrac{1}{1+x^2}=\dfrac{1}{1-(-x^2)}$ (2) $=\sum_{n=0}^{\infty}(-x^2)^n=\sum_{n=0}^{\infty}(-1)^nx^{2n}$ (3), ha

$\boxed{7}$

$|-x^2|<1$, azaz $|x|<1$ (2)

$(R=1)$

$\boxed{8}$ $g(x)=\operatorname{arctg}x=\int_{t=0}^{x}\dfrac{1}{1+t^2}\,dt$ (2) $=\int_{t=0}^{x}\left(\sum_{n=0}^{\infty}(-1)^nt^{2n}\right)dt=\sum_{n=0}^{\infty}(-1)^n\dfrac{x^{2n+1}}{2n+1}$ (4); $R=1$ (2)

4, a, $\boxed{5}$ $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)=A$, ha $\forall\,\varepsilon>0$ esetén $\exists\,\delta(\varepsilon)$,

hogy $|f(x,y)-A|<\varepsilon$, ha $\sqrt{(x-x_0)^2+(y-y_0^2)}<\delta(\varepsilon)$

b, $\boxed{5}$ $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)=A \iff \forall\,(x_n,y_n)\to(x_0,y_0)$ esetén

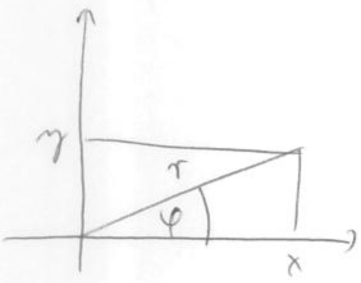
$(x_n,y_n)\neq(x_0,y_0)$ $\quad \lim\limits_{n\to\infty} f(x_n,y_n)=A$

$(x_n,y_n)\in D_f$

c, $\boxed{5}$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2y)}{x^2\cos(y^2)} = \lim_{(x,y)\to(0,0)} \underbrace{\frac{\sin(x^2y)}{x^2y}}_{\downarrow \atop 1} \underbrace{\frac{y}{\cos(y^2)}}_{\downarrow \atop \frac{0}{1}=0} = 0$$

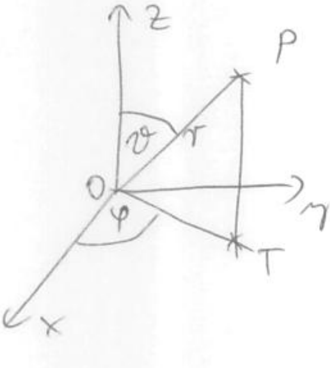
5, a,



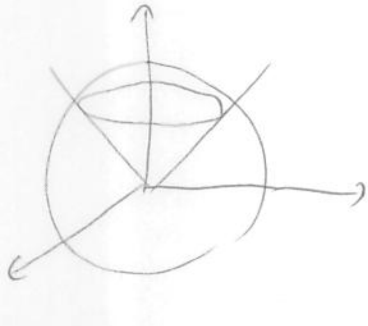
$$\left.\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \end{aligned}\right\}$$

$$J = \begin{vmatrix} x'_r & x'_\varphi \\ y'_r & y'_\varphi \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} = \underline{\underline{r}}$$

b,



c,

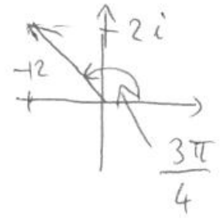


$$V = \int_{\vartheta=0}^{\pi/4} \int_{\varphi=0}^{2\pi} \int_{r=0}^{1} 1\cdot r^2 \sin\vartheta \, dr\, d\varphi\, d\vartheta =$$

$$= 2\pi \int_{r=0}^{1} r^2\,dr \int_{\vartheta=0}^{\pi/4} \sin\vartheta\, d\vartheta = 2\pi\cdot\frac{1}{3}\cdot\Big[-\cos\vartheta\Big]_0^{\pi/4} =$$

$$= \underline{\frac{2\pi}{3}\left(1-\frac{1}{\sqrt{2}}\right)}$$

6,
$$e^{3}\cdot e^{2z}=2(i-1)$$

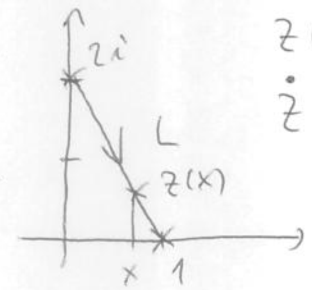


$$e^{2z+3}=-2+2i \qquad |-2+2i|=\sqrt{(-2)^2+2^2}=2\sqrt{2}$$

$$2z+3=\mathrm{Ln}(-2+2i)=\ln(2\sqrt{2})+i\left(\frac{3\pi}{4}+2k\pi\right)\ ;\quad k\in\mathbb{Z}$$

$$\underline{\underline{z_k=\frac{\ln(2\sqrt{2})-3}{2}+i\left(\frac{3\pi}{8}+k\pi\right)}}\ ;\ k\in\mathbb{Z}$$

7, a,



$$z(x)=x+i(2-2x)$$
$$\dot{z}(x)=1-2i$$

$\boxed{7}$
$$\int_L \mathrm{Re}(z)\,dz=\int_{x=0}^{1}\underbrace{\mathrm{Re}\,z(x)}_{x}\cdot\underbrace{\dot{z}(x)}_{1-2i}\,dx=$$

$$=(1-2i)\int_0^1 x\,dx=(1-2i)\left[\frac{x^2}{2}\right]_0^1=\frac{1-2i}{2}=\underline{\underline{\frac{1}{2}-i}}$$

b,

$\boxed{6}$
$$\int_L \sin(iz)\,dz=i\int_L \mathrm{sh}(z)\,dz=i\left[\mathrm{ch}\,z\right]_{2i}^{1}=i\left(\mathrm{ch}\,1-\mathrm{ch}(2i)\right)=$$
$$\underline{\underline{=i\left(\mathrm{ch}\,1-\cos 2\right)}}$$

8,
$$\frac{dy}{dx}=y^{3}\cdot\cos(3x)$$
$$\int y^{-3}\,dy=\int\cos(3x)\,dx$$
$$\frac{-1}{2y^{2}}=\frac{\sin(3x)}{3}+C$$

Kezdeti feltétel: $y_0=4,\ x_0=\frac{\pi}{4}$

$$-\frac{1}{2\cdot4^{2}}=\frac{1}{3\sqrt{2}}+C$$
$$C=-\frac{1}{32}-\frac{1}{3\sqrt{2}}$$

$$y=\sqrt{\frac{1}{-\frac{2}{3}\sin(3x)-2C}}=\underline{\underline{\sqrt{\frac{-3}{2\sin(3x)-\frac{3}{16}-\sqrt{2}}}}}$$

9,
$$\sum_{n=1}^{\infty}\frac{(-2)^n}{n^2}x^n\ ;\qquad a_n=\frac{(-2)^n}{n^2}\ ;\quad \sqrt[n]{|a_n|}=\sqrt[n]{\left|\frac{(-2)^n}{n^2}\right|}=\frac{2}{\left(\sqrt[n]{n}\right)^2}\longrightarrow 2$$

$\Longrightarrow R=\frac{1}{2}$; $x=+\frac{1}{2}$ esetén $\sum_{n=1}^{\infty}\frac{(-2)^n}{n^2}\left(\frac{1}{2}\right)^n=\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ konvergens (Leibniz-sor)

$x=-\frac{1}{2}$ esetén $\sum_{n=1}^{\infty}\frac{(-2)^n}{n^2}\left(-\frac{1}{2}\right)^n=\sum_{n=1}^{\infty}\frac{1}{n^2}<\infty$

$\Longrightarrow$ Konvergenciatartomány: $\left[-\frac{1}{2},+\frac{1}{2}\right]$