Elektromágneses terek

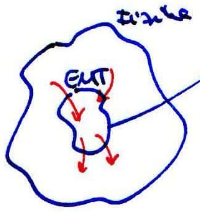
2015. ősz

készítette: Vándor Ádám

Terek E
15. 09. 09.

Zombory László: Elektromágneses terek
Bilics: A matematika villamosmérnöki alkalmazásairól

Fizikai összefoglaló



- elektromos és mágneses jelenségek + mérnöki megközelítés
- matematikai modell
- fizikai mennyiségek és egyenletek

 └→ (mérőszám) · (mértékegység)

Az elektromágneses tér közvetlen forrásai (töltés, áram)

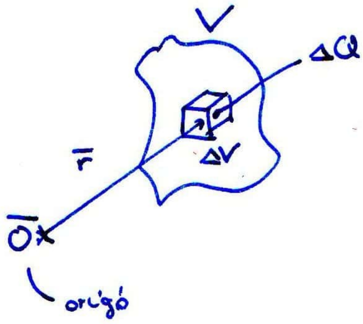
- töltés: elemi egysége: $e = -1{,}6 \cdot 10^{-19}$ C

  kvantált: ⟶

  jele: Q

  egység: $[Q] = C = As$

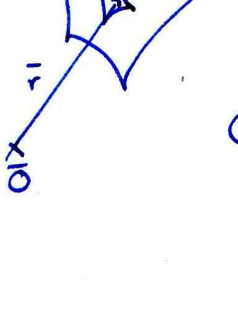
a) Térfogati töltéssűrűség:



$$\lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} \triangleq \rho(\bar{r})$$

$$[\rho] = \frac{C}{m^3} \qquad Q_V = \int_V \rho(\bar{r})\, dV$$

b) Felületi töltéssűrűség:

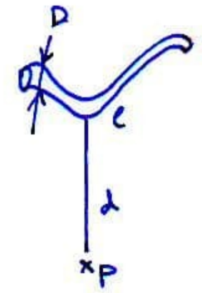


$$\lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta S} \triangleq \sigma(\bar{r})$$

$$[\sigma] = \frac{C}{m^2}$$

$$Q_S = \int_S \sigma(\bar{r}) \cdot dS$$

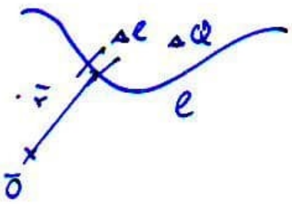
c) Vonalmenti töltéssűrűség



$e \gg D$

$d \gg D$

$\longrightarrow$

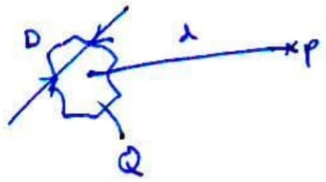


$$\lim_{\Delta e \to 0} \frac{\Delta Q}{\Delta e} \triangleq q(\bar{r})$$

$$[q] = \frac{C}{m}$$

d) Ponttöltés

$d \gg D$

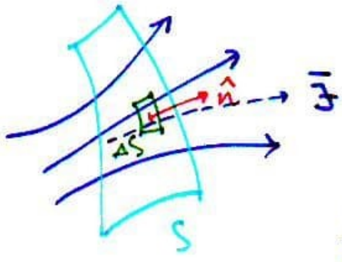


$\longrightarrow \; \bullet \; Q$

Áram

- töltéshordozók rendezett mozgása $- \; [I] = A$

a) Térfogati áramsűrűség



normális komponens:

$$J_n = \bar{J} \cdot \hat{n}$$

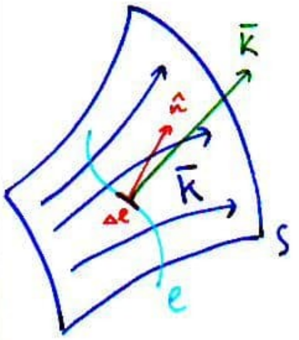
$$\lim_{\substack{\Delta S \to 0 \\ \Delta t \to 0}} \frac{\Delta Q}{\Delta S \cdot \Delta t} \triangleq J_n$$

felületelem vektor

$$\text{összáram: } I_S = \int_S J_n \cdot dS = \int_S \bar{J} \cdot d\bar{S}$$

$$d\bar{S} = dS \cdot \hat{n}$$

b) Felületi áramsűrűség



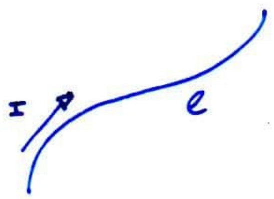
$$I_e = \int_e \bar{K} \cdot (\hat{n} \; de) = \int_e K_n \, de$$

$$K_n = \bar{K} \cdot \hat{n}$$

$$[K] = \frac{A}{m}$$

$$\lim_{\substack{\Delta e \to 0 \\ \Delta t \to 0}} \frac{\Delta Q}{\Delta e \cdot \Delta t} = K_n$$

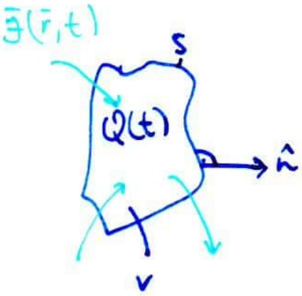
c) Vonaláram



$[I] = A$

Töltésmegmaradás elve:

↳ folytonossági egyenlet



$$\frac{dQ(t)}{dt} = -I(t)$$

$$\boxed{\frac{d}{dt}\int_V \rho(\bar{r},t)\,dV = -\oint_S \bar{J}(\bar{r},t)\,d\bar{S}}$$

Folytonossági egyenlet integrális alakja

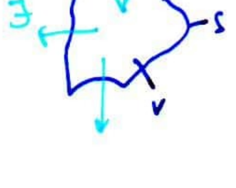
Gauss-tétel

$$\rightarrow \frac{d}{dt}\int_V \rho(\bar{r},t)\,dV = -\int_V \operatorname{div}\bar{J}(\bar{r},t)\,dV$$

$$\boxed{\int_V \underbrace{\left(\frac{\partial}{\partial t}\rho(\bar{r},t) + \operatorname{div}\bar{J}(\bar{r},t)\right)}_{\equiv 0}dV = 0}$$

Folytonossági egyenlet (differenciális alak)

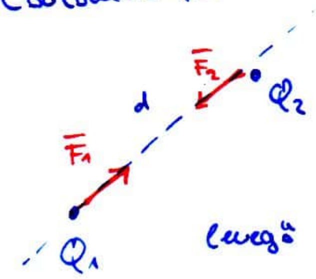
Megj.:



$$\operatorname{div}\bar{J} \triangleq \lim_{V\to 0}\frac{\oint \bar{J}\cdot d\bar{S}}{V}$$

Erőhatások

a) Coulomb-tr.

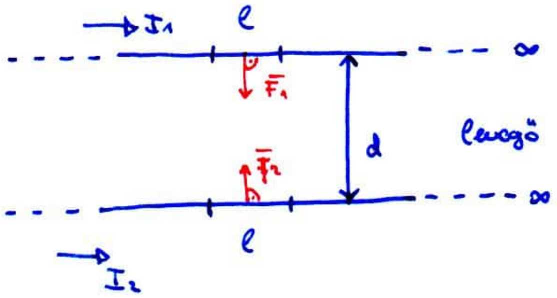


$$\bar{F}_2 = -\bar{F}_1$$

$$|\bar{F}_1| = |\bar{F}_2| = F_{1,2} = \frac{1}{4\pi\varepsilon_0}\frac{|Q_1 Q_2|}{d^2}$$

ε_0 : vákuum permittivitása $\approx 8{,}854 \cdot 10^{-12} \frac{As}{Vm}$

b) Amper-tr.



$$|\bar{F}_1| = |\bar{F}_2| = F_{12} = \frac{\mu_0}{2\pi}\frac{|I_1 \cdot I_2|\, l}{l}$$

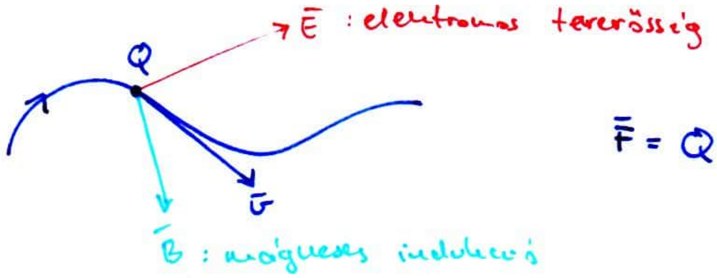
μ_0: vákuum permeabilitása

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$$

$I_1 \cdot I_2 > 0 \Rightarrow$ vonzó erő

$I_1 \cdot I_2 < 0 \Rightarrow$ taszító erő

c) Lorentz-tr.



$$\bar{F} = Q\left(\bar{E} + \bar{v} \times \bar{B}\right)$$

$\bar{E}, \bar{B}$: „álló" rendszerben

$\bar{v}$: elemi helyzeti sebesség

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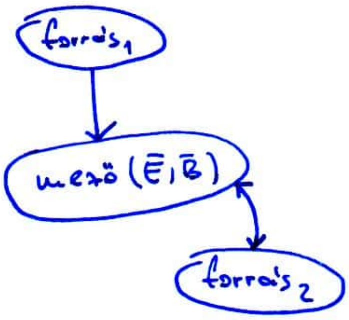
Erőhatások

a) Coulomb-erő tv. } távolhatás
b) Amper-tv. }
c) Lorentz-erő → közelhatás



• $\bar E$ definíciója:

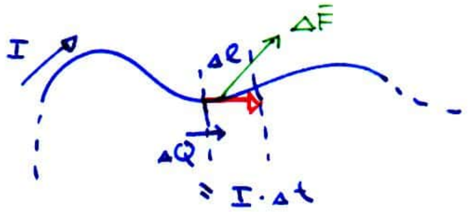
$\bar v \equiv \bar 0$: $\boxed{\bar E \triangleq \frac{\bar F}{Q}}$

$[E] = \dots = \frac{V}{m}$

• $\bar B$ definíciója:

$\bar E \equiv \bar 0$: $\bar F = q\bar v \times \bar B \rightarrow \bar B = \dots$

alternatív definíció:



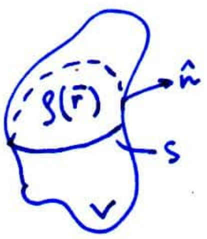
$\Delta Q \cdot \bar v = I \cdot \underbrace{\Delta t \cdot \bar v}$

Δl → vonalelem

$\Delta \bar F = \Delta Q \cdot (\bar v \times \bar B) = I \cdot (\Delta \bar l \times \bar B)$

Az elektromos és mágneses mező gerjesztése

a) Elektrosztatika Gauss-tétele

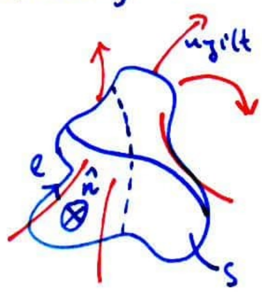


$$\int_V \rho(\bar r)\, dV = \oint_S \bar D\, d\bar S$$

elektromos eltolás vektora

(levegő: $\bar D = \varepsilon_0 \cdot \bar E$)

b) Gerjesztési törvény:



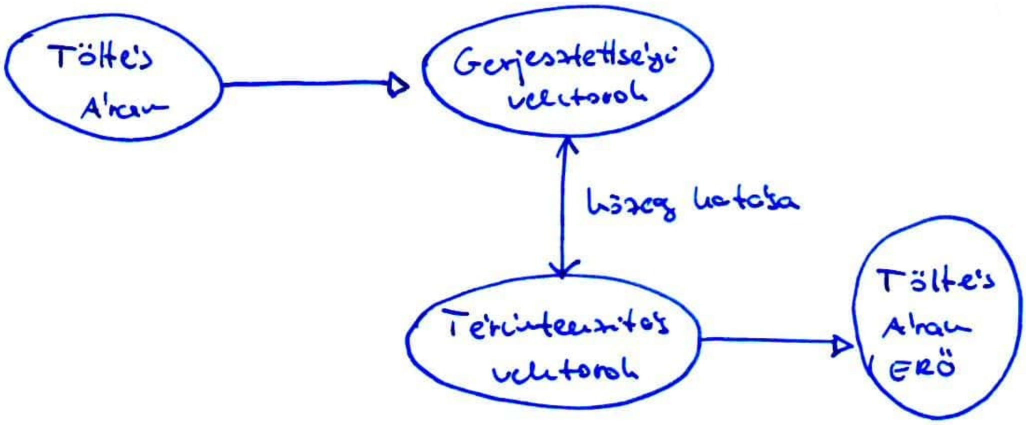
$$\int_{s} \bar{J}\, d\bar{S} = \oint_{e} \bar{H}\, d\bar{l}$$

mágneses térerősség vektora

(levegő + ∀, ami nem ferromágneses közeg : $\bar{B} = \mu_0 \cdot \bar{H}$)

$\bar{E}, \bar{B}$: térintenzitás vektorok

$\bar{D}, \bar{H}$: gerjesztettségi vektorok

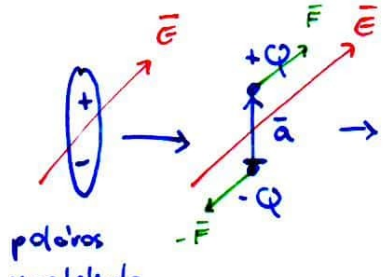


A közegek hatása

- Vákuumban, levegőben: $\bar{D} = \varepsilon_0 \cdot \bar{E}$, $\varepsilon_0 = 8{,}854 \cdot 10^{-12} \frac{As}{Vm}$

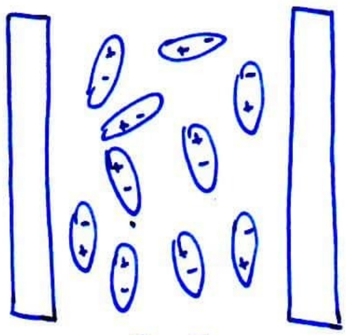
  $\bar{B} = \mu_0 \cdot \bar{H}$, $\mu_0 = 4\pi \cdot 10^{-7} \cdot \frac{Vs}{Am}$

- Anyagban: Polarizáció léphet fel

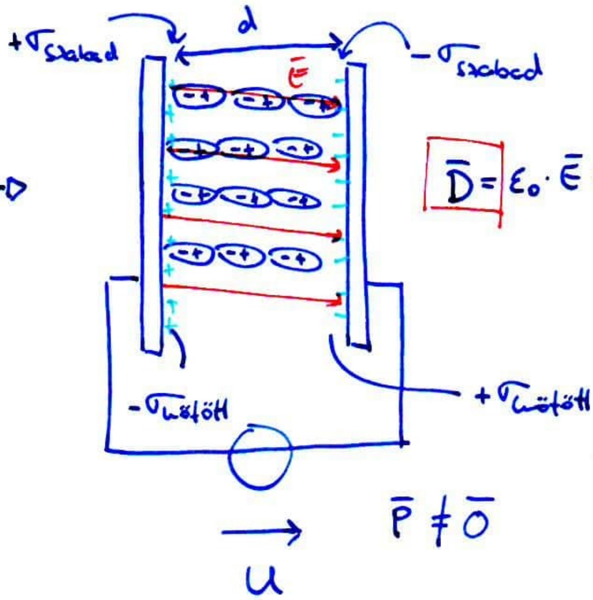


$\rightarrow \bar{p} \approx \bar{a} \cdot Q$

nyomaték



$$\bar P = \bar 0$$



$$\bar P \neq \bar 0$$

$$\boxed{\bar D} = \varepsilon_0 \cdot \bar E + \bar P = \varepsilon_0 \underbrace{(1+\chi_e)}_{\varepsilon_r} \bar E = \boxed{\varepsilon_0 \varepsilon_r \bar E}$$

polarizáció vektora

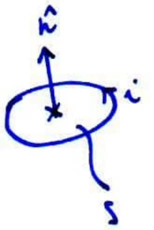
$$\boxed{\bar P = \sum_V \bar p \Big/ V}$$

$$\boxed{\bar P = \varepsilon_0 \cdot \chi_e \cdot \bar E} \qquad (1+\chi_e = \varepsilon_r : \text{relatív permittivitás})$$

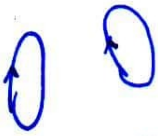
elektromos szuszceptibilitás

- Mágneses polarizáció

elfordulásra képes elemek



$$\boxed{\bar m = \hat n \cdot S \cdot i}$$



mágnesezettség vektor : $\boxed{\bar M = \dfrac{\sum_V \bar m}{V}}$

$$\bar B = \mu_0 \cdot \bar H + \mu_0 \cdot \bar M \qquad \left(\bar H = \frac{1}{\mu_0}\bar B - \bar M\right)$$

$$\boxed{\bar M = \chi_m \cdot \bar H} \Rightarrow \boxed{\bar B = \mu_0 (1+\chi_m) \bar H = \mu_0 \mu_r \bar H}$$

lineáris közegben

mágneses szuszceptibilitás

relatív permeabilitás

$\chi_m \gtrapprox 0$: paramágnes

$\chi_m \lessapprox 0$: diamágnes

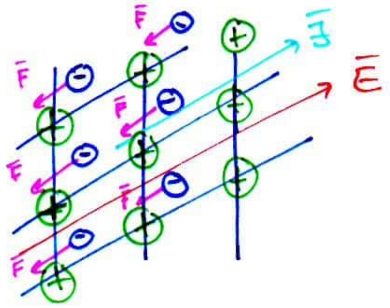
$\chi_m \gg 0$: ferromágnes

Egységek:

$$[D] = [P] = \frac{C}{m^2}$$

$$[H] = [M] = \frac{A}{m}$$

- Elektromos vezetés

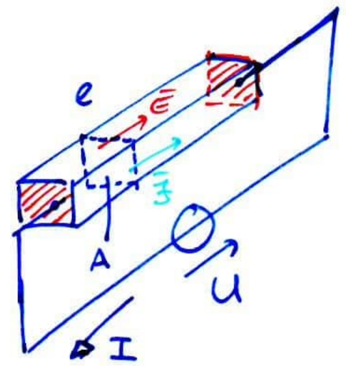


$\bar{J} = \sigma \cdot \bar{E}$: differenciális Ohm-tv.

σ: fajlagos vezetőképesség

$$[\sigma] = \frac{[J]}{[E]} = \frac{A/m^2}{V/m} = \frac{S}{m}$$

pl.:



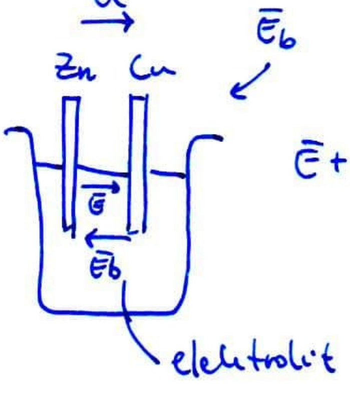
$$\left.\begin{aligned} E &= \frac{U}{l} \\ J = \sigma \cdot E &\longrightarrow I = J \cdot A \end{aligned}\right\} \quad \frac{U}{I} = \frac{E \cdot l}{\sigma E A} = \frac{l}{\sigma A}$$

Beiktatott mennyiségek:

$$\bar{J} = \sigma \cdot (\bar{E} + \bar{E}_b) + \bar{J}_b$$

$\bar{E}_b$

J_b : beiktatott áramsűrűség/~~[illegible]~~ (Van de Graaf generátor)



$$\bar{E} + \bar{E}_b = \bar{0} \Rightarrow \bar{J} = \bar{0}$$

A Maxwell-egyenletek teljes rendszere

Differenciális alak | Integrális alak

I. $\text{rot}\,\bar{H} = \bar{J} + \frac{\partial D}{\partial t}$ $\xrightarrow{\text{Stokes}}$ $\oint_{l} \bar{H}\, d\bar{l} = \int_{S} \left(\bar{J} + \frac{\partial D}{\partial t}\right) d\bar{S}$

II. $\text{rot}\,\bar{E} = -\frac{\partial B}{\partial t}$ $\xrightarrow{\text{Stokes}}$ $\oint_{l} \bar{E}\, d\bar{l} = -\int_{S} \frac{\partial B}{\partial t}\, d\bar{S}$

III. $\text{div}\,\bar{B} = \phi$ $\xrightarrow{\text{Gauss}}$ $\oint_{S} \bar{B}\, d\bar{S} = \phi \left(\equiv \int_{V} \text{div}\,\bar{B}\, d\bar{V}\right)$

IV. $\text{div}\,\bar{D} = \rho$ $\xrightarrow{\text{Gauss}}$ $\oint_{S} \bar{D}\, d\bar{S} = \int_{V} \rho\, dV$

V. Konstitúciós egyenletek

lineáris, izotróp

$\bar{D} = \varepsilon_0 \varepsilon_r \bar{E}$

$\bar{B} = \mu_0 \mu_r \bar{H}$

$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$

(nemlineáris, anizotróp)

$\bar{D} = \mathcal{D}\{\bar{E}\}$

$\bar{B} = \mathcal{B}\{\bar{H}\}$

$\bar{J} = \mathcal{J}\{\bar{E}\}$

VI.

$w = \frac{1}{2}\bar{E}\cdot\bar{D} + \frac{1}{2}\bar{H}\cdot\bar{B}$: Energia sűrűség

VII.

$\bar{F} = Q(\bar{E} + \bar{v}\times\bar{B})$: Erőhatás

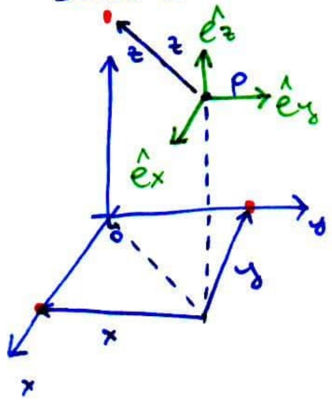
Terek Gy

15. 09. 17.

Koordináta-rendszerek

3D-s pont ↔ 3 „érték"

- Descartes:

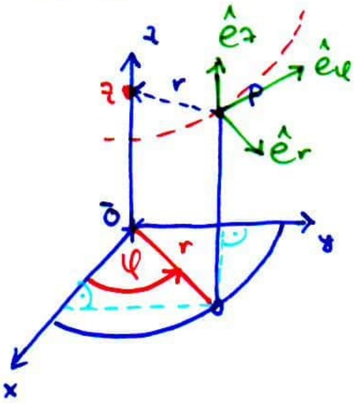


$|\hat{e}_x| = |\hat{e}_y| = |\hat{e}_z| \equiv 1$

$\hat{e}_x \perp \hat{e}_y \perp \hat{e}_z$

$\left[\hat{e}_x \times \hat{e}_y = \hat{e}_z\right]$: jobbsodrású koord. rendszer

- Hengerkoord.-rendszer



$\hat{e}_z \perp \hat{e}_r \perp \hat{e}_\varphi$

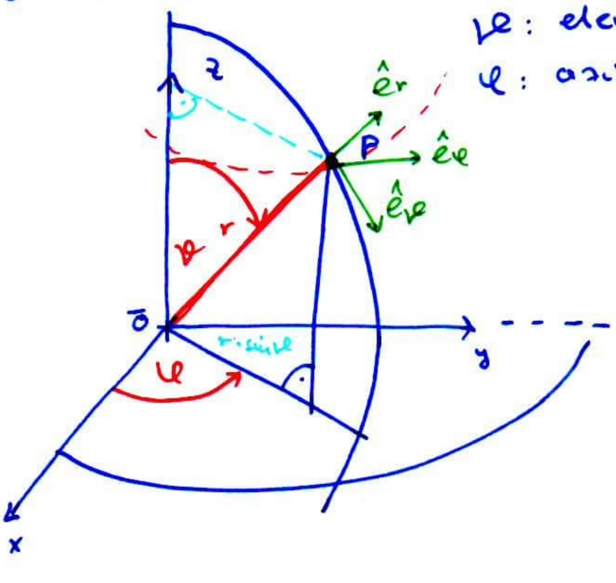
$\hat{e}_z \times \hat{e}_r = \hat{e}_\varphi$: jobbsodrású koord. r.

$z = z$

$x = r \cdot \cos\varphi$

$y = r \cdot \sin\varphi$

- Gömbi koord.-rendszer



ϑ : elevációs szög

φ : azimutális szög

$\hat{e}_r \perp \hat{e}_\vartheta \perp \hat{e}_\varphi$

$\hat{e}_r \times \hat{e}_\vartheta = \hat{e}_\varphi$

$x = (r \cdot \sin\vartheta) \cos\varphi$

$y = (r \cdot \sin\vartheta) \sin\varphi$

$z = r \cdot \cos\vartheta$

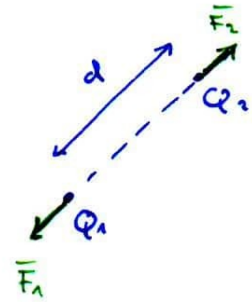
Elektrosztatika   példatár: 2.1 – 2.7

sztatika: nyugvó töltések : $\frac{\partial}{\partial t} \equiv 0$

$\downarrow$

elektromos teret keltenek $\quad \bar{E} \triangleq \frac{\bar{F}}{Q} \leftarrow$ próbatöltés $\quad [E] = \frac{V}{m}$

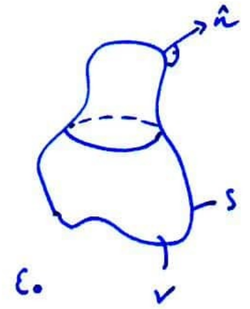
Coulomb-tv :



$$\varepsilon_0 = 8{,}854 \cdot 10^{-12} \frac{As}{Vm}$$

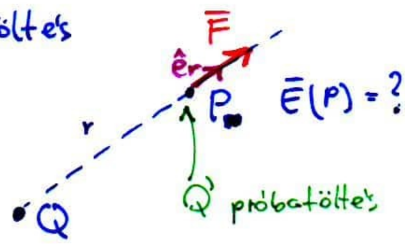
$$\bar{F}_{1,2} = \frac{1}{4\pi\varepsilon_0} \frac{|Q_1 \cdot Q_2|}{d^2}$$

Gauss-tv. :



$$\oint_S \bar{E}\, d\bar{S} = \frac{1}{\varepsilon_0} \sum_V Q \quad \text{(homogén közegben)}$$

$\boxed{1}$ Ponttöltés



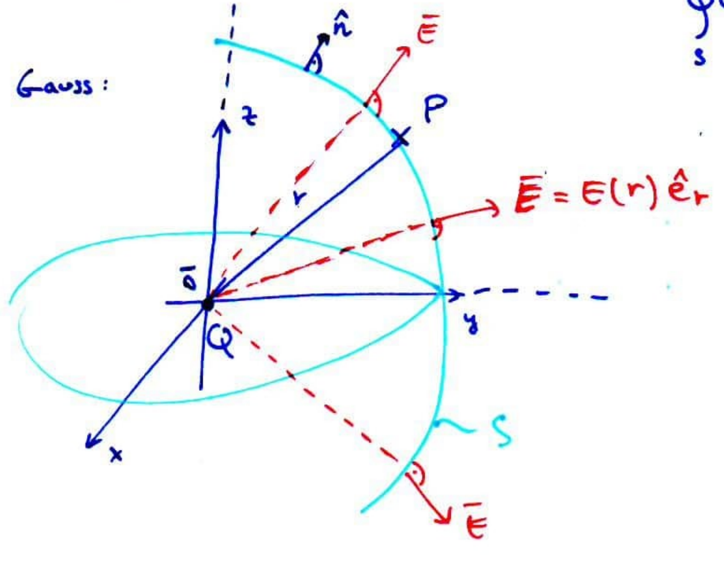
a) $\bar{F} = \frac{1}{4\pi\varepsilon_0} \frac{|Q \cdot Q'|}{r^2} \cdot \hat{e}_r$ (Coulombbal)

$$\bar{E} = \frac{\bar{F}}{Q'} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^2} \cdot \hat{e}_r$$

b)

levegő ε_0

Gauss :



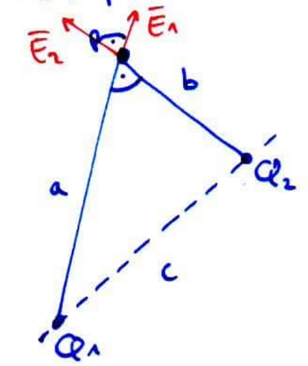
$$\oint_S \bar{E}\, d\bar{S} = \oint_S (E(r)\,\hat{e}_r) \cdot (dS \cdot \hat{n}) =$$

$$= \oint_S E(r) \underbrace{\boxed{\hat{e}_r \cdot \hat{n}}}_{=1} \cdot dS = E(r) \underbrace{\oint_S dS}_{\text{gömbfelszín}} =$$

$$= E(r) \cdot 4 r^2 \pi =$$

$$= \frac{1}{\varepsilon_0} Q \quad \Rightarrow \quad E(r) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^2}$$

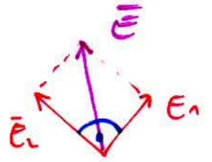
2 Két ponttöltés



$a = 4m \qquad Q_1 = 3\mu C$

$b = 3m \qquad Q_2 = 8\mu C$

$c = 5m$

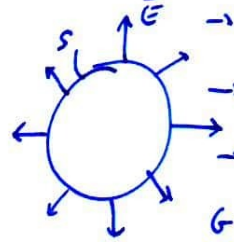


$$|\bar{E}_1| = \frac{Q_1}{4\pi\varepsilon_0}\cdot\frac{1}{a^2} = \dots$$

$$|\bar{E}_2| = \frac{Q_2}{4\pi\varepsilon_0}\cdot\frac{1}{b^2} = \dots$$

$$|\bar{E}|^2 = |\bar{E}_1|^2 + |\bar{E}_2|^2 \rightarrow |\bar{E}| = 8{,}16\ \frac{kV}{m}$$

S-en:



→ $|E|$ = állandó

→ $\bar{E} \parallel \hat{n}$

→ S zárt

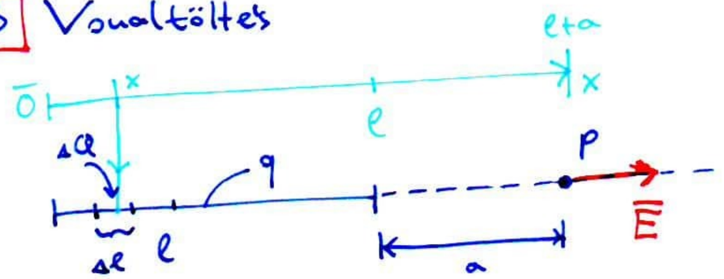
Gauss-felületek

ε_0 (levegő)

$$[E] = \frac{[Q]}{[\varepsilon_0]\cdot[a]^2} = \frac{C^{=As}}{\frac{As}{Vm}\cdot m^2} = \frac{V}{m}$$

szuperpozíció elve ⟺ lineáris közeg

3 Vonaltöltés



a) $\bar{E}(P) = ?$

$$\Delta E = \frac{\Delta Q}{4\pi\varepsilon_0}\cdot\frac{1}{[(\ell+a)-x]^2} \xrightarrow{\to\infty} dE = \frac{dQ}{4\pi\varepsilon_0}\cdot\frac{1}{(\ell+a-x)^2}$$

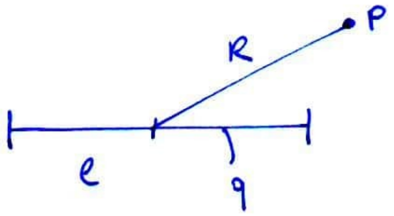
$$E = \int dE = \int \frac{dQ}{4\pi\varepsilon_0}\cdot\frac{1}{(\ell+a-x)^2} = \int_{x=0}^{x=\ell}\frac{q}{4\pi\varepsilon_0}\cdot\frac{dx}{(\ell+a-x)^2} = \frac{q}{4\pi\varepsilon_0}\int_{x=0}^{x=\ell}\frac{dx}{(\ell+a-x)^2} =$$

$(dx\cdot q) = dQ$

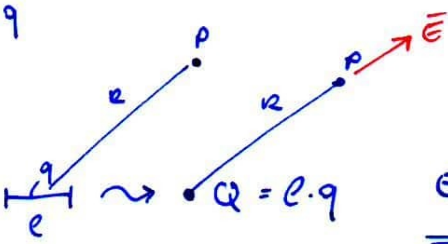
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\ell+a-x}\right]_0^{\ell} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{\ell+a}\right)$$

7

b)



$\bar{E}(P) = ?$

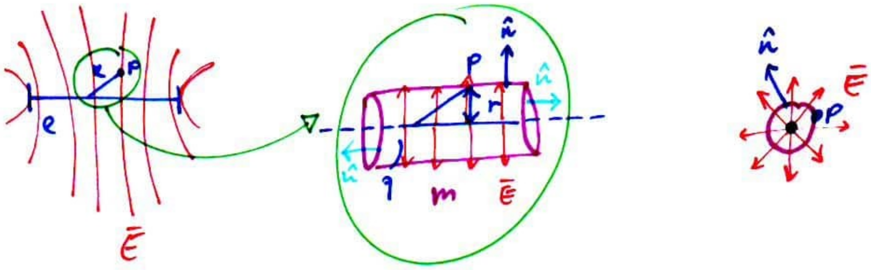


Közelítések:

i) $R \gg l$: $\rightsquigarrow$ $Q = l \cdot q$ $\qquad \underline{\underline{E \simeq \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{R^2}}}$

Ell.: $\lim\limits_{a\to\infty} \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{l+a}\right) = \lim\limits_{a\to\infty} \frac{q}{4\pi\varepsilon_0} \frac{l}{a(l+a)} \approx \frac{q \cdot l}{4\pi\varepsilon_0} \frac{1}{a^2+al} \approx \frac{ql}{4\pi\varepsilon_0} \cdot \frac{1}{a^2}$

ii) $R \approx 0$, $R \ll l$:
(középpontjában)

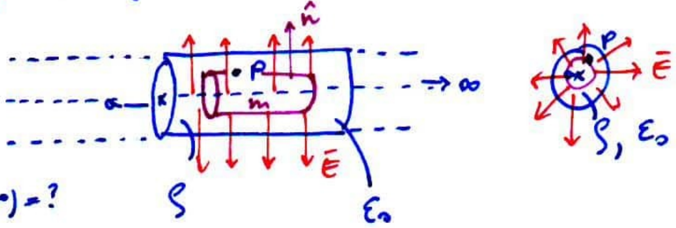


Gauss:

$$\oint\limits_{S_{henger}} \bar{E}\,d\bar{s} = \underbrace{\int\limits_{S_{palást}} \bar{E}\,d\bar{s}}_{\hookrightarrow E(r)\cdot m\cdot 2r\pi} + \underbrace{\int\limits_{S_{alap}} \bar{E}\,d\bar{s} + \int\limits_{S_{fedlap}} \bar{E}\,d\bar{s}}_{\phi} = \frac{1}{\varepsilon_0}\cdot m \cdot q$$

$$\Rightarrow \underline{\underline{E(r) = \frac{q}{2\pi\varepsilon_0} \cdot \frac{1}{r}}}$$

$\boxed{4}$ Térfogati töltéseloszlás



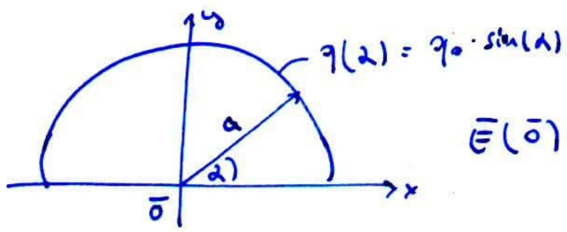
$\bar{E}(r) = ?$ $\quad \rho$

$E(r) \cdot 2r\pi \cdot m = \frac{1}{\varepsilon_0}\sum\limits_{V} Q = \rho \cdot V \cdot \frac{1}{\varepsilon_0} =$

$= \rho \cdot r^2\pi \cdot m \cdot \frac{1}{\varepsilon_0}$

$$\Rightarrow \underline{\underline{E(r) = \frac{\rho \cdot r}{2\varepsilon_0}}}$$

HF:



$\bar{E}(\bar{0}) = ?$

Terek E
15.09.23.

A Maxwell-egyenletek

$$\mathrm{rot}\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\mathrm{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\mathrm{div}\,\bar{B} = 0$$

$$\mathrm{div}\,\bar{D} = \rho$$

beiktatott elektromos tér

$$\bar{D} = \varepsilon_0 \bar{E}, \quad \bar{B} = \mu \bar{H}, \quad \bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$$

$$w = \frac{1}{2}\bar{E}\bar{D} + \frac{1}{2}\bar{H}\bar{B} = \frac{1}{2}\varepsilon|\bar{E}|^2 + \frac{1}{2}\mu|\bar{H}|^2$$

$\Downarrow$

1. asszimetria:

   mágneses áram, mágneses töltés $\Big\}\ \nexists$

2. differenciális alak => közelhatási szemlélet
3. integrális alak => távolhatási szemlélet

   ↳ általánosabb (diszkontinuitásoknál nem jó a diff. alak)

4. ellentmondásmentes, teljes
5. evolúciós egyenletek: jelenbeli állapotból lehet következtetni a jövőbelire

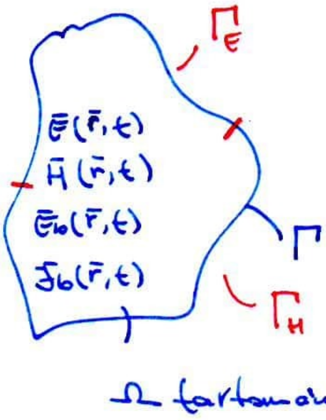
   ↳ kauzalitás

6. határa: statisztikus sokaság legyen (kvantumos hatást nem írnak le az egyenletek)
7. posztulátumok (nem vezethetők le a fizika más törvényeiből)

Az egyenletek egyértelmű megoldhatósága

$$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$$

forrás



t_0 : kezdeti pillanat

$t_1 > t_0$

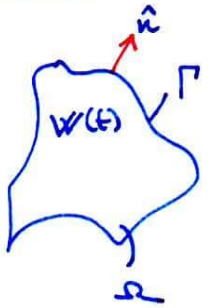
Adott:

- $\bar{E}(\bar{r},t_0)$, $\bar{H}(\bar{r},t_0)$, $\bar{r} \in \Omega$: KEZDETI FELTÉTELEK
- források: $\bar{E}_b(\bar{r},t)$, $\bar{J}_b(\bar{r},t)$, $\bar{r} \in \Omega$, $t \in [t_0,t_1]$: GERJESZTÉSEK
- $\boxed{\Gamma_E}$: $\bar{E}_t\Big|_{\Gamma_E}(t)$
  $\boxed{\Gamma_H}$: $\bar{H}_t\Big|_{\Gamma_H}(t)$

  tangenciális komponensek időfüggése : PEREMFELTÉTELEK

$\Rightarrow$ $\bar{E}(\bar{r},t_1)$, $\bar{H}(\bar{r},t_1)$ a teljes tartományon egyértelműen meghatározható.

Az energiamérleg



$W(t)$: elektromágneses energia

$$W(t) = \int_\Omega w \, d\Omega$$

$$[w] = \frac{J}{m^3}, \quad [W] = J$$

$$\text{I.} \quad \operatorname{rot}\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t} \quad / \cdot \bar{E} \qquad \Rightarrow \text{I.} - \text{II.}$$

$$\text{II.} \quad \operatorname{rot}\bar{E} = -\frac{\partial\bar{B}}{\partial t} \quad / \cdot \bar{H}$$

$$\underbrace{\bar{E}\cdot\operatorname{rot}\bar{H} - \bar{H}\cdot\operatorname{rot}\bar{E}}_{} = \bar{J}\cdot\bar{E} + \bar{E}\cdot\frac{\partial\bar{D}}{\partial t} + \bar{H}\cdot\frac{\partial\bar{B}}{\partial t}$$

azonosság: $-\operatorname{div}(\bar{E}\times\bar{H}) =$

$$E\cdot \mathrm{rot}H - H\cdot \mathrm{rot}E = J\cdot E + E\cdot\frac{\partial D}{\partial t} + H\cdot\frac{\partial B}{\partial t} \qquad \left(\bar E = \frac{1}{\sigma}(J - J_b) - \bar E_b\right)$$

$$(D = \varepsilon E,\ B = \mu H)$$

$$\cancel{-\mathrm{div}(E\times H) = \frac{J\cdot J}{\sigma} - \frac{J\cdot J_b}{\sigma} - J\cdot E_b}$$

$$-\mathrm{div}(\bar E\times\bar H) = \frac{\bar J\cdot\bar J}{\sigma} - \frac{\bar J\cdot\bar J_b}{\sigma} - \bar J\cdot\bar E_b + \underbrace{\varepsilon\bar E\cdot\frac{\partial\bar E}{\partial t} + \mu\bar H\cdot\frac{\partial\bar H}{\partial t}}_{=\frac{\partial w}{\partial t}}$$

$$\left(\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon|E|^2 + \frac{1}{2}\mu|H|^2\right) = \varepsilon|\bar E|\cdot\frac{\partial|\bar E|}{\partial t} + \mu|\bar H|\cdot\frac{\partial|\bar H|}{\partial t}\right)$$

$$\Rightarrow\ -\frac{\partial w}{\partial t} = \mathrm{div}(\bar E\times\bar H) + \frac{|\bar J|^2}{\sigma} - \left(\frac{\bar J\cdot\bar J_b}{\sigma} + \bar J\bar E_b\right) \Big/ \int_\Omega d\Omega$$

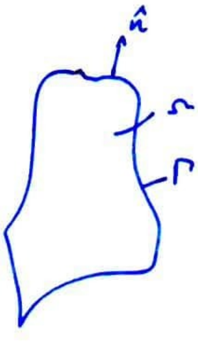
$$\Rightarrow\ \underbrace{-\frac{d}{dt}\int_\Omega w\, d\Omega}_{\text{E.M. energia változása}} = \underbrace{\int_\Omega \mathrm{div}(\bar E\times\bar H)\, d\Omega}_{\overset{\text{Gauss}}{=}\ \oint_\Gamma(\bar E\times\bar H)\,d\bar\Gamma} + \underbrace{\int_\Omega \frac{|\bar J|^2}{\sigma}\, d\Omega}_{>0} - \underbrace{\int_\Omega\left(\frac{\bar J\bar J_b}{\sigma} + \bar J\bar E_b\right) d\Omega}_{\text{külső hatások teljesítménye}}$$

$\oint_\Gamma(\bar E\times\bar H)\,d\bar\Gamma$: Γ-n átsugárzó E.M. teljesítmény

>0: vektor hosszát integrálja (>0), Joule-hő

külső hatások teljesítménye (+ és − is lehet)

$$\boxed{-\frac{d}{dt}\int_\Omega w\, d\Omega = \int_\Omega \mathrm{div}(\bar E\times\bar H)\, d\Omega + \int_\Omega \frac{|\bar J|^2}{\sigma}\, d\Omega - \int_\Omega\left(\left(\frac{\bar J\cdot\bar J_b}{\sigma}\right) + \bar J\bar E_b\right) d\Omega}$$



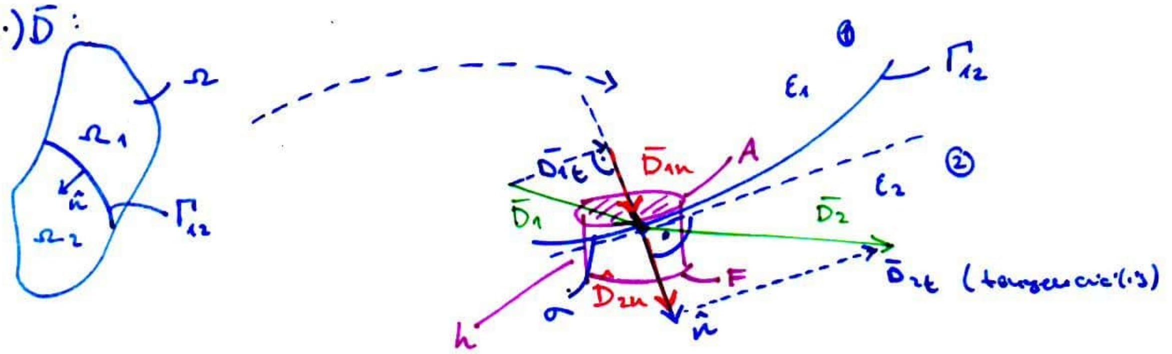
$\oint_\Gamma(\bar E\times\bar H)\,d\bar\Gamma$: Γ-n átsugárzó teljesítmény

↳ $\bar S$: Poynting-vektor : teljesítmény sűrűség vektor

$[S] = \frac{W}{m^2}$

Folytonossági feltételek közeghatáron

1.) $\bar{D}$:



$$\oint_F \bar{D}\,d\bar{F} = \int_{\Gamma_{12}^A} \sigma\, d\Gamma$$

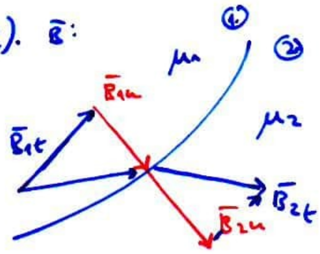
$$\left.\begin{matrix} h \to 0 \\ A \to 0 \end{matrix}\right\} \quad \underbrace{\int_{palást} \bar{D}\,d\bar{F}}_{\phi,\ \text{ha } h\to 0} + A \cdot D_{2n} - A D_{1n} = A \cdot \sigma$$

$$\Rightarrow \boxed{D_{2n} - D_{1n} = \sigma}$$

$$\bar{D}_2 \cdot \hat{n} - \bar{D}_1 \cdot \hat{n} = \sigma$$

$\boxed{(\bar{D}_2 - \bar{D}_1)\hat{n} = \sigma}$: vektoriális alak

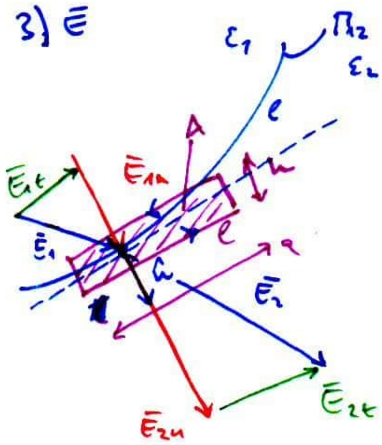
2). $\bar{B}$:



$$B_{2n} - B_{1n} = 0$$

$$\boxed{(\bar{B}_2 - \bar{B}_1) \cdot \hat{n} = 0}$$

3) $\bar{E}$



$\bar{E}_{2t} - \bar{E}_{1t} = 0$

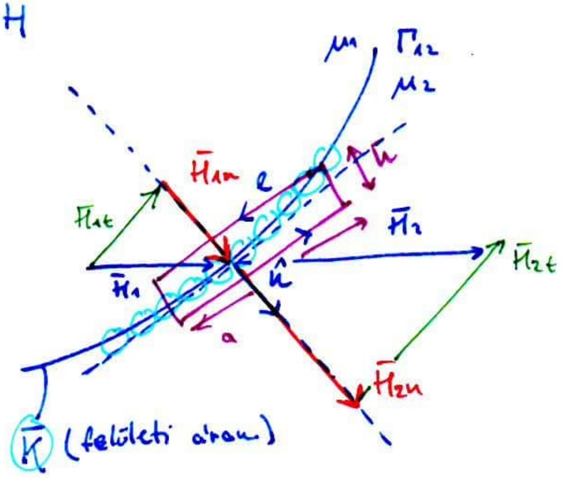
$$\oint_l \bar{E}dl = \int_A \left(-\frac{\partial B}{\partial t}\right) d\bar{A}$$

$\left.\begin{matrix} h \to 0 \\ a \to 0 \end{matrix}\right\} \; aE_{2t} - aE_{1t} = 0$

$E_{2t} - E_{1t} = 0$: tetszőleges irányban

$$\underbrace{\bar{E}_2 \times \hat{n}}_{\bar{E}_{2t}} - \underbrace{\bar{E}_1 \times \hat{n}}_{\bar{E}_{1t}} = 0 \Rightarrow \boxed{(\bar{E}_2 - \bar{E}_1) \times \hat{n} = 0}$$

4.) $\bar{H}$



$$\oint_l H dl = \int_A \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right) d\bar{A}$$

$\bar{J} \to K$

$h \to 0$

$a \to 0$

$aH_{2t} - aH_{1t} = aK$

$H_{2t} - H_{1t} = K$

$$\boxed{(\bar{H}_2 - \bar{H}_1) \times \bar{n} = \bar{K}}$$

Az elektrodinamika felosztása

- Sztatikus terek: elektrosztatika, magnetosztatika : töltésmozdulások nyugalomban vannak

időfüggetlenség: $\frac{\partial}{\partial t} = 0$ és $\bar{J} \equiv \bar{0}$

- Stacionárius terek: stacionárius áramlási tér, stacionárius áramlások mágneses tere — egyirányú kapcsolat

$\frac{\partial}{\partial t} = 0, \quad \bar{J} \neq \bar{0}$

- ~~Kvázistacionárius tér~~
- Kvázistacionárius tér : $\frac{\partial}{\partial t} \neq 0$
- Hullámtan

Terek E.
15.09.24.

Az elektrodinamika filozófiája

Cél: Maxwell-egy. m.o.-ának megkönnyítése (közelítésekkel)

$$\mathrm{rot}\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \qquad \bar{D} = \varepsilon_0 \bar{E} + \bar{P}$$

$$\mathrm{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \bar{B} = \mu_0(\bar{H} + \bar{M})$$

$\Big\}$ nem vákuumban

$$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$$

$$\mathrm{div}\,\bar{B} = 0$$

$$\mathrm{div}\,\bar{D} = \varrho$$

I. Sztatikus terek $\left(\frac{\partial}{\partial t} = 0,\ \bar{J} \equiv \bar{0}\right)$

- Elektrosztatika egyenletei:

$$(*)\ \begin{cases} \mathrm{rot}\,\bar{E} = \bar{0} \\ \mathrm{Div}\,\bar{D} = \varrho \\ \bar{D} = \varepsilon_0\bar{E} + \bar{P} \end{cases}$$

forrás

szétcsatolódnak

- Magnetosztatika:

$$(\Box)\ \begin{cases} \mathrm{rot}\,\bar{H} = 0 \\ \mathrm{div}\,\bar{B} = 0 \\ \bar{B} = \mu_0(\bar{H} + \bar{M}) \end{cases}$$

forrás

(*) Nagyfesz. technika, kapacitások

(□) Állandó mágnesek, villamos gépek

II. Stacionárius terek $\left(\frac{\partial}{\partial t} = 0,\ \bar{J} \neq \bar{0}\right)$

vezető

- Stacionárius áramlás

$$\mathrm{rot}\,\bar{E} = \bar{0}$$
$$(*)\ \mathrm{div}\,\bar{J} = \bar{0}$$
$$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$$

forrás

$\bar{J}$ forrás

Egyirányú csatolás

- Stac. áramlás mágneses tere

$$\mathrm{rot}\,\bar{H} = \bar{J}$$
$$\mathrm{div}\,\bar{B} = \bar{0}$$
$$\bar{B} = \mu_0(\bar{H} + \bar{M})$$

forrás

(○) (□)

(*) $\mathrm{rot}\,\bar{H} = \bar{J} \Rightarrow \mathrm{div}\,\mathrm{rot}\,\bar{H} = \mathrm{div}\,\bar{J} \equiv 0$ | (○) Ellenállás, melegedés (□) Villamos gépek

III. Kvázistacionárius terek $\left(\underbrace{\frac{\partial}{\partial t}}_{\text{„lassú"}} \neq 0,\ \bar{J} \neq \bar{0}\right)$

$$\operatorname{rot} \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

elhanyagolható

$$|\bar{J}| \gg \left|\frac{\partial \bar{D}}{\partial t}\right|$$

$$\operatorname{rot} \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\operatorname{div} \bar{B} = \bar{0}$$

$$\operatorname{div} \bar{J} = \bar{0}$$

$$\bar{B} = \mu_0 (\bar{H} + \bar{M})$$

$$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$$

$$\bar{J} = \sigma \bar{E}, \quad \bar{D} = \varepsilon \bar{E}$$

$$|\sigma \bar{E}| \gg \left|\varepsilon \frac{\partial \bar{E}}{\partial t}\right|$$

„$\sigma \gg \varepsilon \frac{\partial}{\partial t} (\ldots)$"

jó vezetőkben (forgás) E.M. tér

ÁRAMKISZORÍTÁS $\Leftarrow$

IV. Elektromágneses hullámok $\left(\frac{\partial}{\partial t} \neq 0,\ \bar{J} \neq \bar{0}\right)$

- Egyenletek:

Maxwell-egy. teljes rendszere

$\frac{\partial \bar{D}}{\partial t}$ szerepe nagyon jelentős

pl.: szigetelőben ($\bar{J} = \bar{0}$)

$$\operatorname{rot} \bar{H} = -\frac{\partial \bar{D}}{\partial t}$$

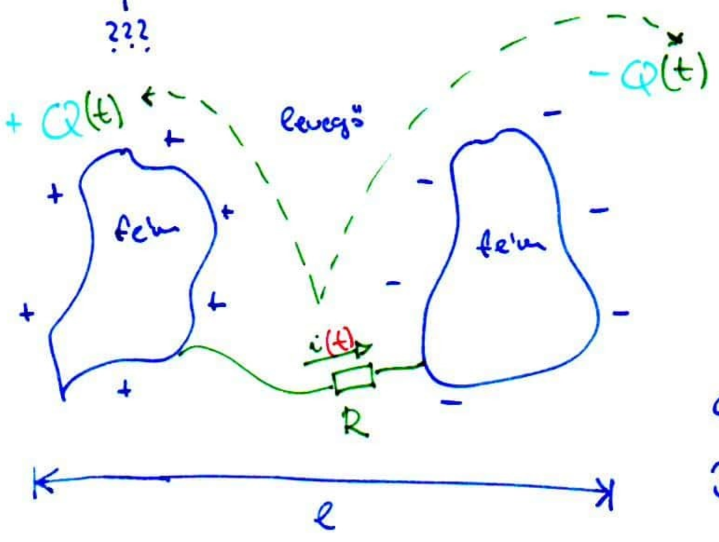
$$\operatorname{rot} \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

μH $\quad$ $\bar{D} = \varepsilon \bar{E}$ $\quad$: egymást keltő hullámok

antennák, hullámvezetők

$\frac{\partial}{\partial t} = 0$ fizikai tartalma?:

???



„kvázistatikus méret"

(t) : Q(t) mvatt

$c = 3\cdot 10^8 \frac{m}{s}$

$\tau = \frac{\ell}{c}$: kvázistatikus idő

$|Q(t+\tau) - Q(t)| \ll |Q(t)| \Rightarrow$ „t" környékében $Q(t) \approx$ konstans

$\Rightarrow$ Elektrosztatikai feladat : $\frac{\partial}{\partial t} = 0$

Elektrosztatika

Nyugvó töltések $\rightarrow \frac{\partial}{\partial t} = 0, \quad \bar{J} \equiv \bar{0}$

- $\text{rot}\,\bar{E} = \bar{0}$
- $\text{div}\,\bar{D} = \varrho$
- $\bar{D} = \varepsilon\bar{E}$, levegő : $\varepsilon = \varepsilon_0$

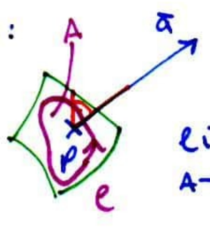
$\oint_\ell \bar{E}\,d\bar{\ell} = 0$; örvénymentes

$\oint_F \bar{D}\,d\bar{F} = \int_V \varrho\,dV$; forrásos

Megj.: (ROT)

$\bar{v}(x,y,z) = \bar{v}_x(x,y,z)\,\hat{e}_x + \bar{v}_y(x,y,z)\,\hat{e}_y + \bar{v}_z(x,y,z)\,\hat{e}_z$

rot $\bar{v}$:



$$\lim_{A\to 0} \frac{\oint_\ell \bar{v}\,d\bar{\ell}}{A} = (\text{rot}\,\bar{v})\cdot\underbrace{\frac{\bar{a}}{|\bar{a}|}}$$

„a" irányú, egység hossz.

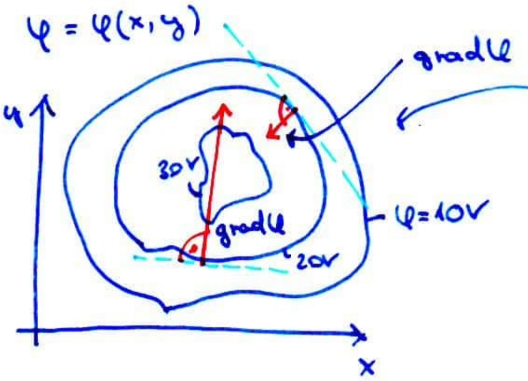
$$\mathrm{rot}\,\bar{V} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$\mathrm{rot}\,\bar{E} = \bar{0} \longrightarrow \boxed{\bar{E} \hat{=} -\mathrm{grad}\,\varphi} \quad (\text{skalárpotenciál})$$

Nyereség: Vektor (3 vektor) $\longleftrightarrow$ 1 skalár

$$[\varphi] = V, \quad [E] = \frac{V}{m}$$

Megj.: GRAD

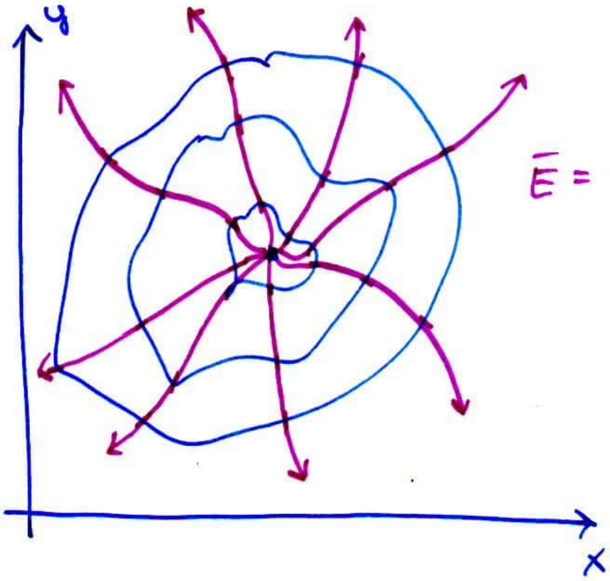


ekvipotenciális görbék

$\mapsto$: merre a leggyorsabb a változás

$$(\mathrm{grad}\,\varphi) \cdot \bar{a} = \frac{\partial \varphi}{\partial a} |\bar{a}|$$

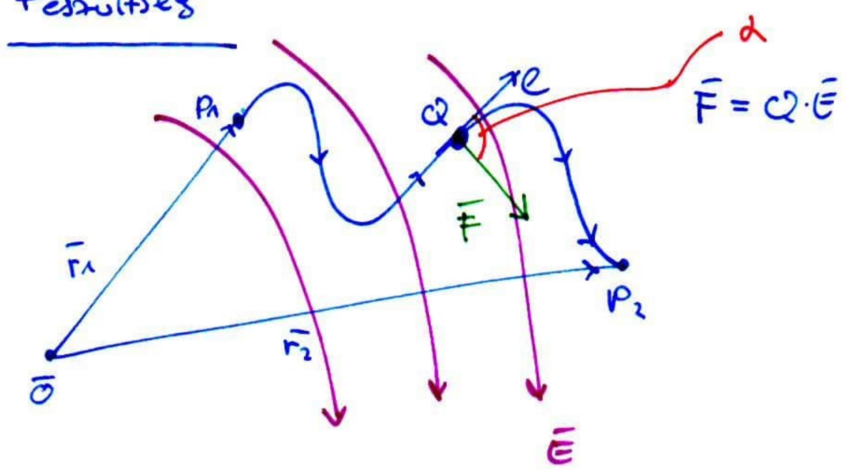
$$\lim_{h \to 0} \frac{\varphi(\bar{r} + h\bar{a}) - \varphi(\bar{r})}{h|\bar{a}|} \hat{=} \frac{\partial \varphi}{\partial a}$$



$$\bar{E} = -\mathrm{grad}\,\varphi$$

$$\varphi(x,y,z) \Rightarrow \mathrm{grad}\,\varphi = \frac{\partial \varphi}{\partial x}\hat{e}_x + \frac{\partial \varphi}{\partial y}\hat{e}_y + \frac{\partial \varphi}{\partial z}\hat{e}_z$$

Feszültség



A mező által végzett munka:

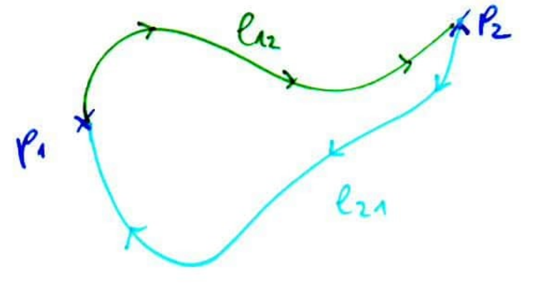
$$W_{12} = \int_{l_1=P_1}^{P_2} \overbrace{|\bar F|\cdot dl\cdot\cos\alpha}^{\bar F d\bar l} = \int_l Q\bar E\, d\bar l = Q\int_l \bar E\, d\bar l = Q\int_l \underbrace{(-\mathrm{grad}\,\varphi)\, d\bar l}_{-\frac{\partial\varphi}{\partial l}|d\bar l|} =$$

$$= Q\cdot\int_l -d\varphi = Q\underbrace{(\varphi_1-\varphi_2)}_{\triangleq U_{12}\ \text{feszültség}}$$

volt definíciója: egységnyi töltésen végzett munka:

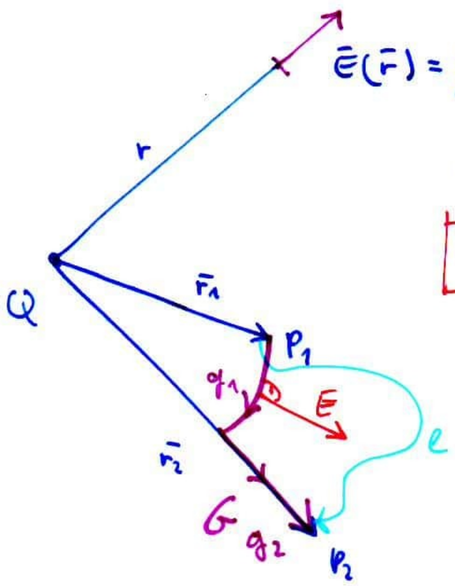
$$V = \frac{J}{Q}$$

Feszültség úttól való függetlensége:



$$\oint_l \bar E\, d\bar l = \underbrace{\oint_{l_1} \bar E\, d\bar l + \oint_{l_2} \bar E\, d\bar l}_{U_{12} \equiv -U_{21}} = 0$$

Ponttöltés potenciáltere



$$\bar{E}(\bar{r}) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^2}\hat{e}_r$$

$U_{12} = ?$

$$\boxed{U_{12} =} \int_{l=P_1}^{l=P_2} \bar{E}\,d\bar{l} = \int_{G=P_1}^{P_2} \bar{E}\,d\bar{l} = \underbrace{\int_{g_1} \bar{E}\,d\bar{l}}_{=\phi} + \int_{g_2} \bar{E}\,d\bar{l} =$$

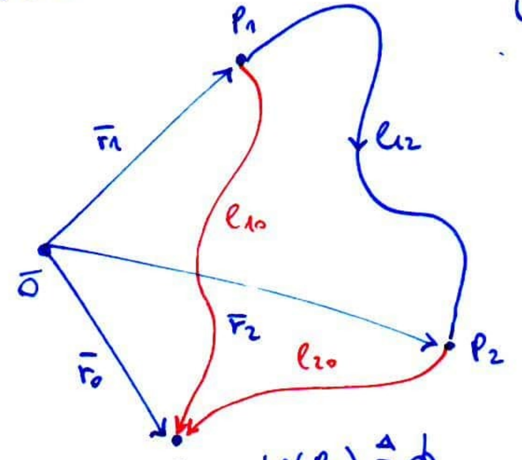
$$= \int_{g_2} \underbrace{\hat{e}_r \cdot \frac{Q}{4\pi\varepsilon} \cdot \frac{1}{r^2}}_{\bar{E}} \cdot \underbrace{\hat{e}_r dr}_{d\bar{l}} \qquad (\hat{e}_r \cdot \hat{e}_r \equiv 1)$$

$$\Rightarrow \int_{g_2} \frac{Q}{4\pi\varepsilon} \frac{1}{r^2} dr = \frac{Q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_{r_1}^{r_2} \boxed{= \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Terek E.
15.03.30.

Referenciapont választása

$\varphi(\bar{r})$



$P_0 : \varphi(P_0) \triangleq 0$

$$U_{12} = \int_{l_{12}} \bar{E}\,d\bar{l} = \varphi(P_1) - \varphi(P_2)$$

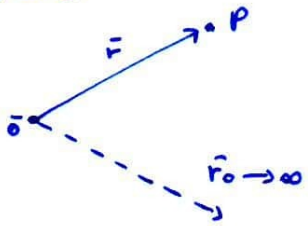
$\bar{E}$: $-\text{grad}\,\varphi$

$$\left.\begin{aligned} \varphi(P_1) &= \int_{l_{10}} \bar{E}\,d\bar{l} + \underbrace{\varphi(P_0)}_{\equiv 0} \\ \varphi(P_2) &= \int_{l_{20}} \bar{E}\,d\bar{l} + \underbrace{\varphi(P_0)}_{\equiv 0} \end{aligned}\right\}$$ Potenciálok a referenciaponthoz képest.

Pl.: ponttöltés esetén

$$U_{12} = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right); \quad P_0 \to \infty, \quad \bar{r}_0 \triangleq \infty$$

$$\boxed{\varphi(P) = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{Q}{4\pi\varepsilon} \cdot \frac{1}{r}}$$



Laplace - Poisson egyenlet

$\text{rot}\,\bar{E} = \bar{0} \Rightarrow \bar{E} = -\text{grad}\,\varphi$

$\text{div}\,\bar{D} = \rho$

$\bar{D} = \varepsilon\bar{E}$

$$\Rightarrow \text{div}\,\varepsilon\bar{E} = \rho$$

$$\text{div}\,\varepsilon(-\text{grad}\,\varphi) = \rho$$

$$\boxed{-\text{div}\,\varepsilon\,\text{grad}\,\varphi = \rho}$$: L-P egyenlet

Ha ε konstans (homogén közeg) $\Rightarrow$ $\underbrace{\text{div}\,\text{grad}}_{\Delta}\varphi = -\frac{\rho}{\varepsilon}$

„divgrad" ≡ „Δ" Laplace operátor

$$\Delta\varphi = -\frac{\rho}{\varepsilon}$$

Megj.: Δ kiszámítása Descartes-koord. r.-ben

$$\varphi(x,y,z) \longrightarrow \Delta\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}$$

Töltésmentes térben: $\rho \equiv \phi$

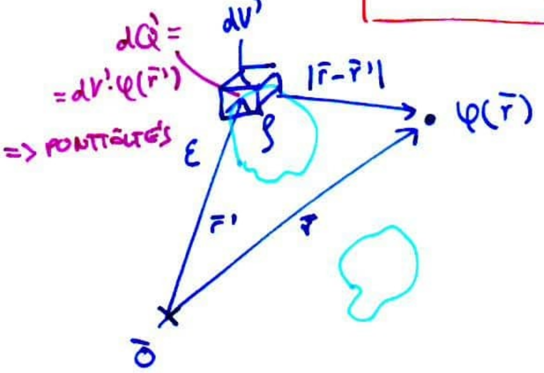
$$\left.\begin{aligned} -\operatorname{div}\operatorname{grad}\varphi &= 0 \\ \Delta\varphi &= 0 \end{aligned}\right\}$$ Laplace egyenletek: másodrendű parciális differenciálegyenlet (PDE)

A Laplace-Poisson szabadtéri megoldása

↳ ε = konstans ∈ teljes végtelen tér

$$\Delta\varphi = -\frac{\rho}{\varepsilon} \longrightarrow \boxed{\varphi(\bar r) = \int_{V(\infty)} \underbrace{\frac{1}{4\pi\varepsilon|\bar r - \bar r'|}}_{(*)} \cdot \underbrace{\varphi(\bar r')\, dV'}_{dQ'}}$$: konvolúciós integrál

dQ' ← gerjesztés



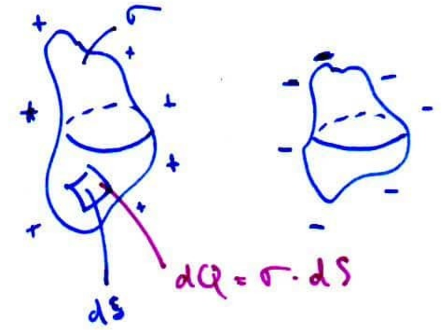
Magyarázat: lineáris közegben igaz a szuperpozíció elve.

$$\Rightarrow d\varphi = \frac{dQ}{4\pi\varepsilon} \frac{1}{|\bar r - \bar r'|} \longrightarrow \varphi = \int_{V(\infty)} d\varphi$$

jelek: $y(t) = \int_{t(\infty)} h(t-\tau)u(\tau)\,d\tau$

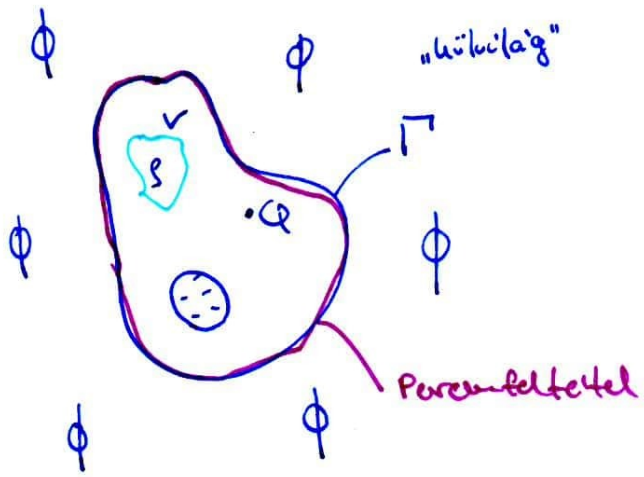
(*) : GREEN-függvény (impulzus válasz)

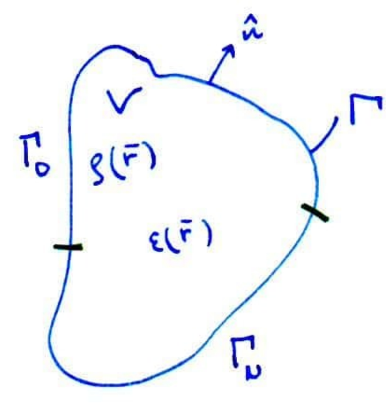
Gyakorlatban: $\rho = 0$, de $\sigma \neq 0$



$$\varphi(\bar r) = \int_S \frac{1}{4\pi\varepsilon|\bar r - \bar r'|} \cdot \underbrace{\sigma(\bar r) \cdot dS'}_{dQ'}$$

Az elektrosztatika peremérték-feladata





V: $-\mathrm{div}\,\underbrace{\varepsilon(\bar{r})}_{\text{adott}}\,\mathrm{grad}\,\underbrace{\varphi(\bar{r})}_{?} = \underbrace{\rho(\bar{r})}_{\text{adott}}$: parciális diff. e.

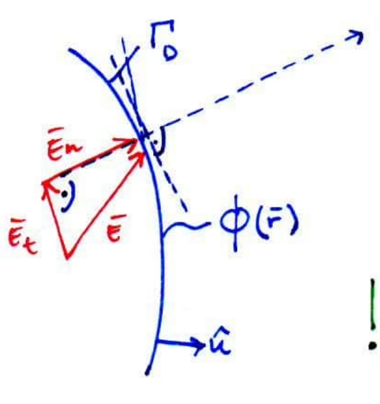
Γ_D: $\varphi\Big|_{\Gamma_D} = \underbrace{\Phi(\bar{r})}_{\text{adott}}$: Dirichlet-peremfeltétel

Γ_N: $\varepsilon(\bar{r}) \cdot \dfrac{\partial \varphi}{\partial n}\Big|_{\Gamma_N} = \underbrace{\sigma(\bar{r})}_{\text{adott}}$: Neumann-peremfelt.

Állítás: PDE megoldása egyértelmű D. és N. peremfeltételek mellett.

A peremfeltételek fizikai jelentése

Dirichlet:



$\bar{E} = -\mathrm{grad}\,\varphi$

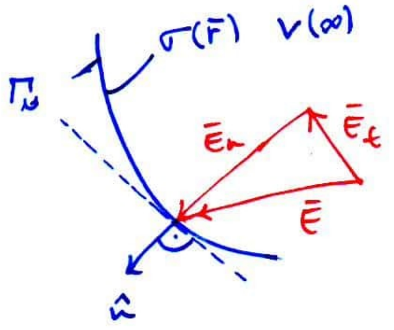
$$\boxed{\bar{E}_t\Big|_{\Gamma_D} = -\mathrm{grad}_t\,\Phi(\bar{r})}$$

! [fémelektródák felszínén : Φ = konstans
$\Rightarrow \bar{E}_t = \bar{0}$

Megj.: $\mathrm{grad}_t\,u(\bar{r})$

$$\mathrm{grad}_t\,u(\bar{r}) = \mathrm{grad}\,u(\bar{r}) - \hat{n}\left(\hat{n}\cdot\mathrm{grad}\,u(\bar{r})\right)$$

Neumann:



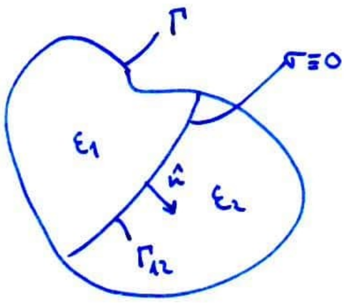
$$\bar E = -\mathrm{grad}\,\varphi$$

$$\bar E_n = -\hat n \cdot \mathrm{grad}\,\varphi = -\frac{\partial \varphi}{\partial n} \quad \Big/ \cdot\, \varepsilon(\bar r)$$

$$\boxed{\bar D_n\Big|_{\Gamma_N} = -\varepsilon(\bar r)\frac{\partial \varphi}{\partial n}\Big|_{\Gamma_N} = (-)\sigma(\bar r)}$$; felületi töltéssűrűség

$\hat n$ kifele mutat

Belső peremek:

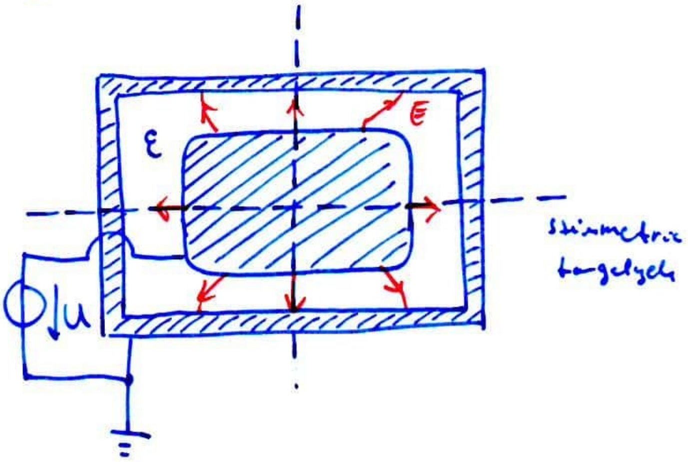


$$\Gamma_{12}:\ E_t^{(1)}\Big|_{\Gamma_{12}} = E_t^{(2)}\Big|_{\Gamma_{12}} \rightarrow \boxed{\varphi^{(1)}\Big|_{\Gamma_{12}} = \varphi^{(2)}\Big|_{\Gamma_{12}}}$$

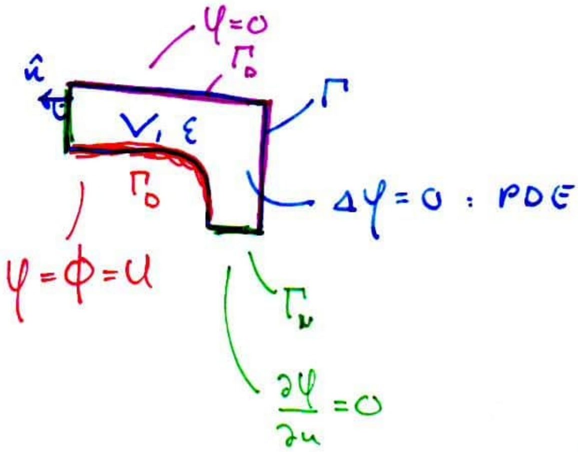
perem mentén megegyeznek a potenciálok

$$D_n^{(1)}\Big|_{\Gamma_{12}} = D_n^{(2)}\Big|_{\Gamma_{12}} \rightarrow \boxed{\varepsilon_1 \frac{\partial \varphi^{(1)}}{\partial n}\Big|_{\Gamma_{12}} = \varepsilon_2 \frac{\partial \varphi^{(2)}}{\partial n}\Big|_{\Gamma_{12}}}$$

Pl. Kábelben kialakuló tér



$\rightarrow$



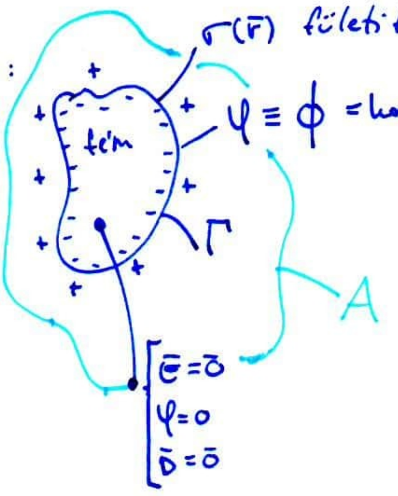
$\Rightarrow$ egyértelműen megoldható

Elektródarendszerek

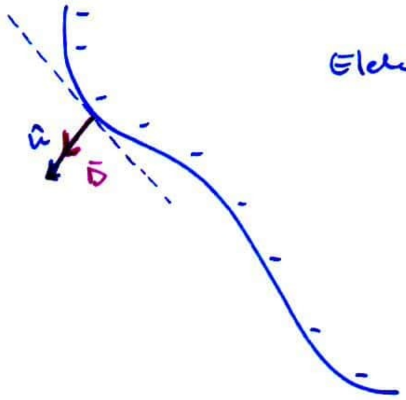
elektróda:

$\sigma(\bar{r})$ felületi töltéssűrűség

$\varphi \equiv \Phi$ = konst. (ekvipotenciális felület)



$$\begin{cases} \bar{E} = \bar{0} \\ \varphi = 0 \\ \bar{D} = \bar{0} \end{cases}$$



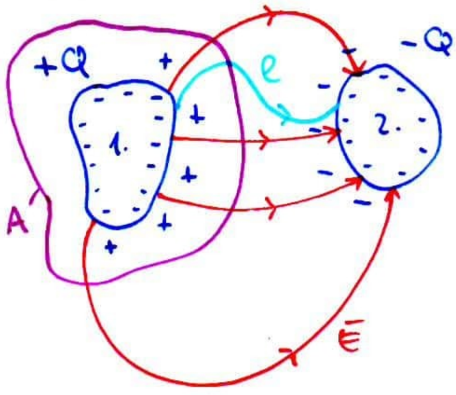
Elektróda felszínén:

$$\boxed{\begin{aligned} \bar{E}_t &= \bar{0} \\ \bar{D}_t &= \bar{0} \\ \bar{D}_n &= \sigma \end{aligned}}$$

Össztöltés:

$$Q = \oint_{\Gamma} \sigma \, d\Gamma \overset{\text{Gauss}}{=} \oint_{A} \bar{D} \, d\bar{A}$$

a) Elektródapár:



$\sum Q = 0$ (zárt rendszer)

$$\left.\begin{aligned} U_{12} &= \int_{l} \bar{E} \, d\bar{l} \\ Q &= \oint_{A} \bar{D} \, d\bar{A} \end{aligned}\right\}$$

lineáris közegben:

↓

KAPACITÁS

$$C \hat{=} \frac{Q}{U_{12}} = \text{konstans}$$

$C = 1F$ (farad)

Terek Gy.
15.10.01.

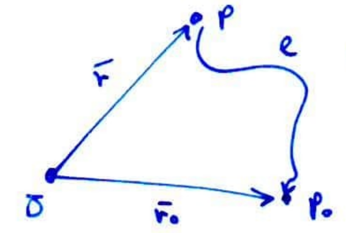
példatárból: 2.1, 2.2, 2.11, 2.15 – 2.21

Elektrosztatika

$$\mathrm{rot}\,\bar{E} = \bar{0} \iff \oint_l \bar{E}\,d\bar{l} = 0$$

$\downarrow$

$$\bar{E} = -\mathrm{grad}\,\varphi$$

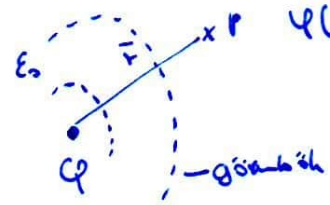


$$\varphi(P) = \int_{l=P}^{l=P_0} \bar{E}\,d\bar{l} + \underbrace{\varphi(P_0)}_{\equiv 0}$$

kapacitás:

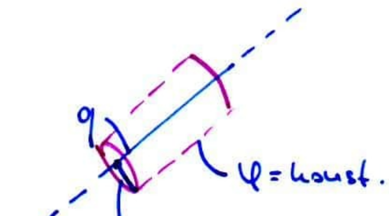
$$„C = \frac{Q}{U}"$$

Pl.: 1) Ponttöltés



$$\varphi(\bar{r}) = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{r} \qquad (\varphi(\infty) \triangleq 0)$$

2) Végtelen hosszú egyenes vonaltöltés



$$\varphi(r) = \frac{q}{2\pi\varepsilon_0}\cdot\ln\frac{r_0}{r}$$

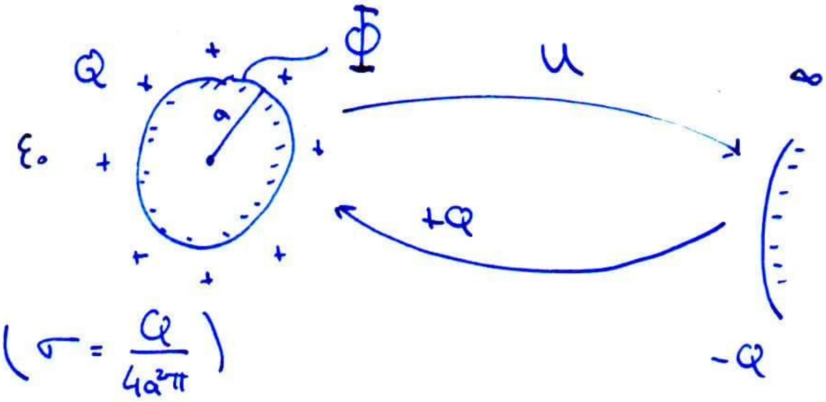
$$\varphi(r_0) \triangleq 0$$

$r_0 := k r_0 \ (k>0)$

$$\Rightarrow \varphi'(r) = \frac{q}{2\pi\varepsilon_0}\ln\frac{k r_0}{r} = \underbrace{\frac{q}{2\pi\varepsilon_0}\ln\frac{r_0}{r}}_{\varphi(r)} + \underbrace{\frac{q}{2\pi\varepsilon_0}\ln k}_{\text{konst.}}$$

$\Rightarrow$ r_0 megválasztása lényegtelen

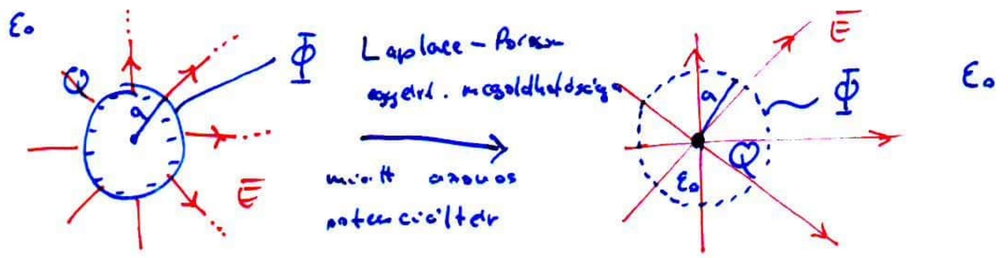
1 Magában álló fémgömb kapacitása



$$\left(\sigma = \frac{Q}{4a^2\pi}\right)$$

$$C = \frac{Q}{U} = ? \quad \Rightarrow \quad U = ?$$

Helyettesítő töltések módszere:



$\varphi(\infty) = \emptyset$

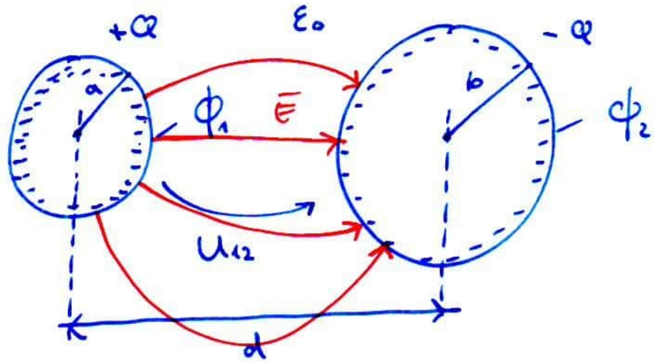
$\varphi(\infty) = 0$

$$\varphi(r) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r} \longrightarrow \boxed{\Phi = \varphi(r=a) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{a}}$$: gömbfelszín potenciálja

$$U = \Phi - 0 = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{a}$$

$$\Rightarrow \boxed{C = 4\pi\varepsilon_0 \cdot a}$$: ∞-re vonatkoztatott kapacitás

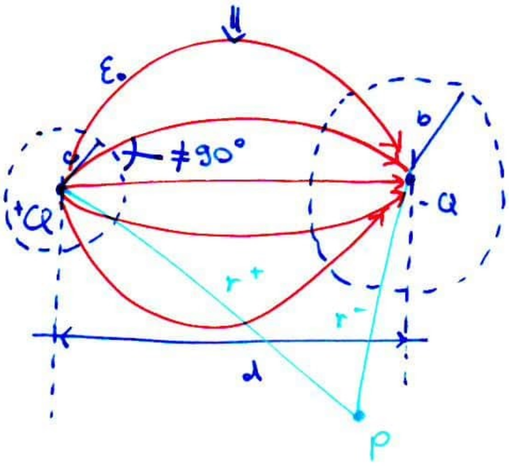
$\boxed{2}$ Két gömb közötti kapacitás



$$C_{12} = \frac{Q}{U_{12}}$$

Q: átvitt töltések

→ : merőlegesen lép ki és be



→ : NEM merőlegesen lép ki és be

⇒ feltételezés: $a, b \ll d$

$$\varphi(P) = \frac{+Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^+} + \frac{-Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^-}$$

$$\phi: \left.\begin{matrix} r^+ \equiv a \\ r^- = [d-a,\ d+a] \end{matrix}\right\} \phi_1 \cong \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{a} + \frac{-Q}{4\pi\varepsilon_0} \cdot \frac{1}{d}$$

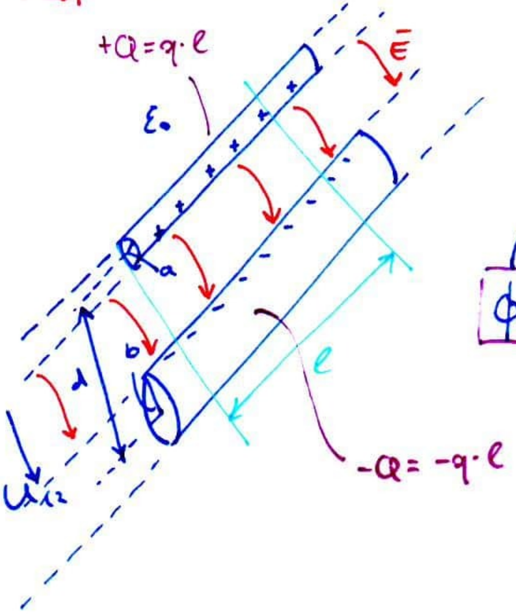
$$\phi_2 \cong \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{d} + \frac{-Q}{4\pi\varepsilon_0} \cdot \frac{1}{b}$$

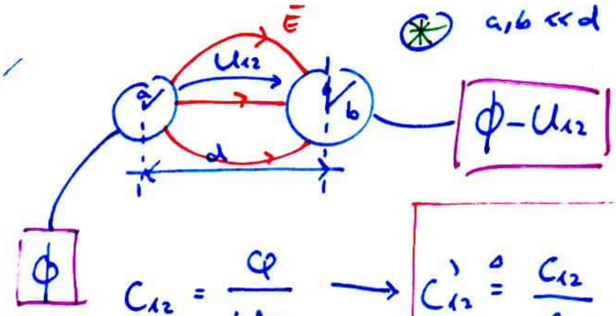
$$U = \phi_1 - \phi_2 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)$$

$$\boxed{C_{12} = \frac{Q}{U_{12}} = \frac{4\pi\varepsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}}$$

Ha $\begin{matrix} a \to \infty \\ b \to \infty \end{matrix}$ ⇒ $\boxed{1}$ feladat ⇒ $C_{12} \to 4\pi\varepsilon_0 a$

$\boxed{3}$ Végtelen hosszú vezetékpár hosszegységre eső kapacitása



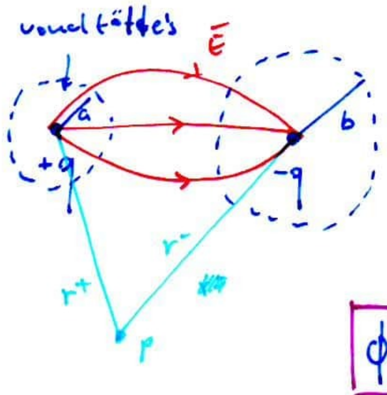


(*) $a, b \ll d$

$$C_{12} = \frac{Q}{U_{12}} \longrightarrow \boxed{C'_{12} \doteq \frac{C_{12}}{l} = \frac{q}{U_{12}}}$$

(*) →

Ha $a, b \ll d$



$$\varphi(P) = \frac{+q}{2\pi\varepsilon_0} \cdot \ln\frac{r_0^+}{r} + \frac{-q^-}{2\pi\varepsilon_0} \cdot \ln\frac{r_0^-}{r}$$

$$\boxed{\Phi} = \frac{q}{2\pi\varepsilon_0} \cdot \left(\ln\frac{r_0^+}{a} - \ln\frac{r_0^-}{d}\right) \quad : \mathrm{I.}$$

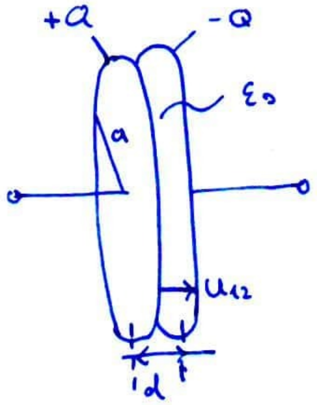
$$\boxed{\Phi - U_{12}} = \frac{q}{2\pi\varepsilon_0}\left(\ln\frac{r_0^+}{d} - \ln\frac{r_0^-}{b}\right) \quad : \mathrm{II.}$$

$$\mathrm{I.} - \mathrm{II.}: \quad U_{12} = \frac{q}{2\pi\varepsilon_0}\underbrace{\left(\ln\frac{r_0^+}{a} - \ln\frac{r_0^-}{d} - \ln\frac{r_0^+}{d} + \ln\frac{r_0^-}{b}\right)}_{\ln\frac{r_0^+ \cdot d \cdot d \cdot r_0^-}{a \cdot r_0^- \cdot r_0^+ \cdot b} = \ln\frac{d^2}{ab}} \quad ⊜$$

$$⊜ \quad \frac{q}{2\pi\varepsilon_0} \cdot \ln\frac{d^2}{ab}$$

$$\Rightarrow \boxed{C'_{12} = \frac{2\pi\varepsilon_0}{\ln\frac{d^2}{ab}}}$$

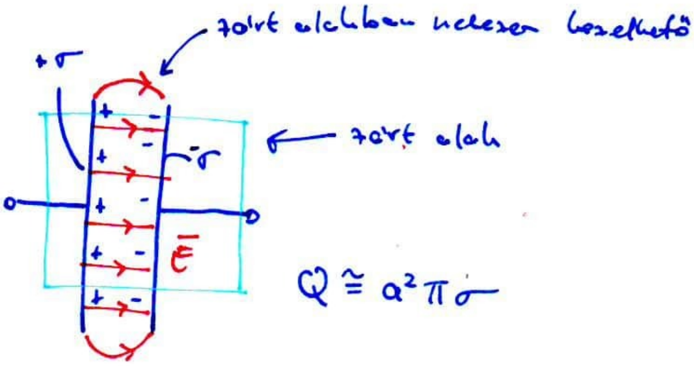
4 Síkkondenzátor



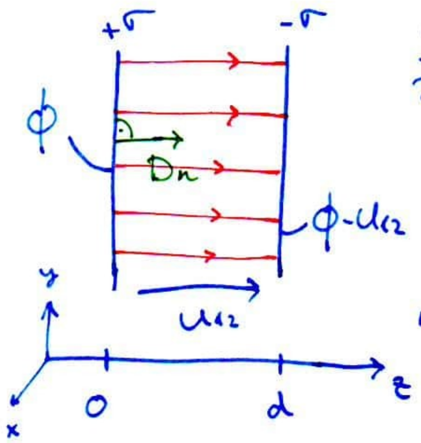
$C_{12} = \dfrac{Q}{U_{12}} = ?$

$d \ll a$

$\longrightarrow$



$Q \cong a^2 \pi \sigma$



$$\frac{\partial}{\partial x} \equiv \frac{\partial}{\partial y} \equiv 0$$

$$\Delta \varphi = 0$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = \frac{d^2 \varphi}{dz^2} = 0$$

$$\left.\begin{array}{lll} [0, d]: & \dfrac{d^2\varphi}{dz^2} = 0 & : \text{diff. e.} \\ z = 0: & \varphi = 0 & \\ z = d: & \varphi = \phi - U_{12} & \end{array}\right\}$$

1 dimenziós peremérték feladat

$$\rightarrow \int dz: \quad \frac{d\varphi}{dz} + A = 0$$

$$\iint dz: \quad \boxed{\varphi(z) + Az + B = 0}$$

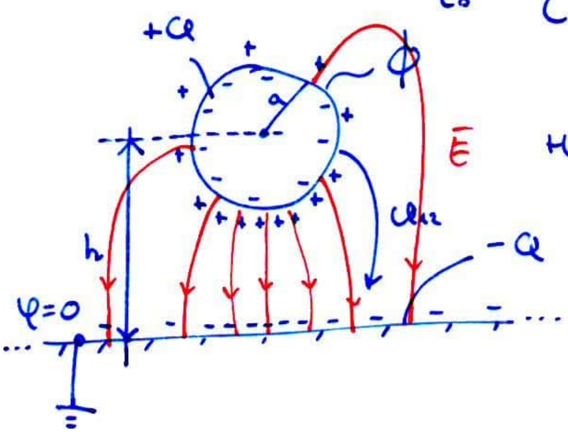
$$\left.\begin{array}{ll} z = 0: & \phi + A \cdot 0 + B = 0 \\ z = d: & \phi - U_{12} + A \cdot d + B = 0 \end{array}\right\} \quad \text{I.} - \text{II.} : \quad U_{12} - A \cdot d = 0 \Rightarrow A = \frac{U_{12}}{d}$$

$$\sigma = D_n = E_n \cdot \varepsilon_0 = \varepsilon_0 \left(- \frac{d\varphi}{dz} \right) \quad \leftarrow \quad -\frac{d\varphi}{dz} = A \quad \Rightarrow \sigma = + \varepsilon_0 \cdot \frac{U_{12}}{d}$$

↑ grad-ból

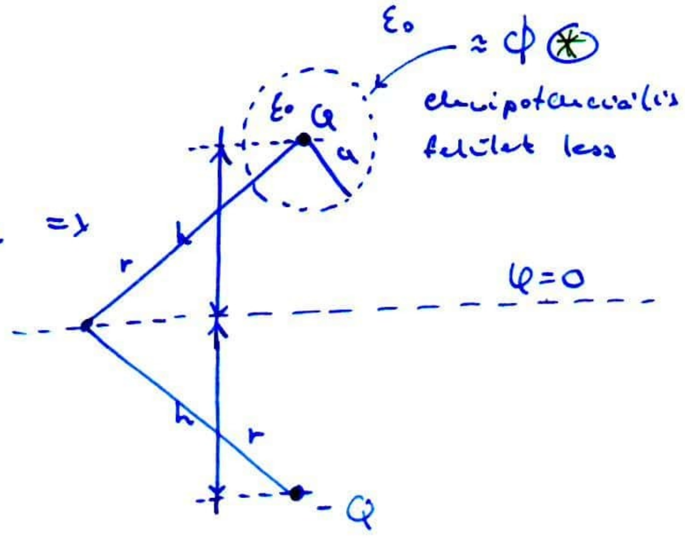
$$\Rightarrow \boxed{Q = a^2 \pi \sigma = \underbrace{\varepsilon_0 \frac{a^2 \pi}{d}}_{C_{12}} \cdot U_{12}}$$

5 Töltéstükrözés módszere



ε_0 $\quad C_{12} = \dfrac{Q}{U_{12}}$

Ha: $a \ll h$ =>

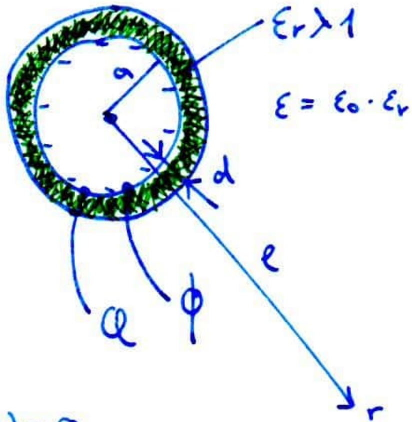


$$⊛ : \phi = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{a} + \frac{-Q}{4\pi\varepsilon_0}\cdot\frac{1}{2h} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{2h}\right)$$

$$U_{12} = \phi - 0$$

$$\boxed{C_{12} = \frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{2h}}}$$

6 Közegek hatása



$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$

$$C = \frac{Q}{\phi - 0} = ?$$

$$\varphi(P) = \int_{\ell=P}^{P_0} \bar{E}\,d\bar{\ell} + \underbrace{\varphi(P_0)}_{\equiv 0}$$

$$\phi = \int_a^{a+d} E^{sz}(r)\,dr + \int_{d+a}^{\infty} E^{\ell}(r)\,dr = \ldots$$

(szigetelőben) (levegőben)

$\varphi(\infty) = 0$

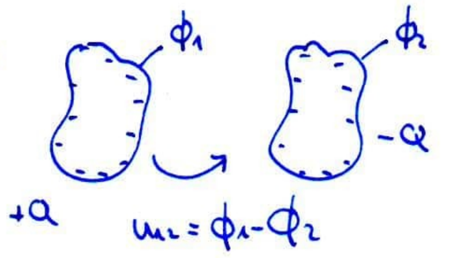
$r = a+d$: $D^{sz} = D^{\ell}$

$\varepsilon_0\varepsilon_r E^{sz} \neq \varepsilon_0 E^{\ell}$

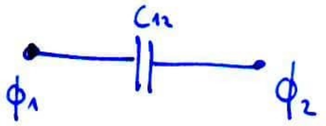
$D(r)\cdot 4r^2\pi = Q$ (Gaussból) ...

Terek E.
15.10.07.

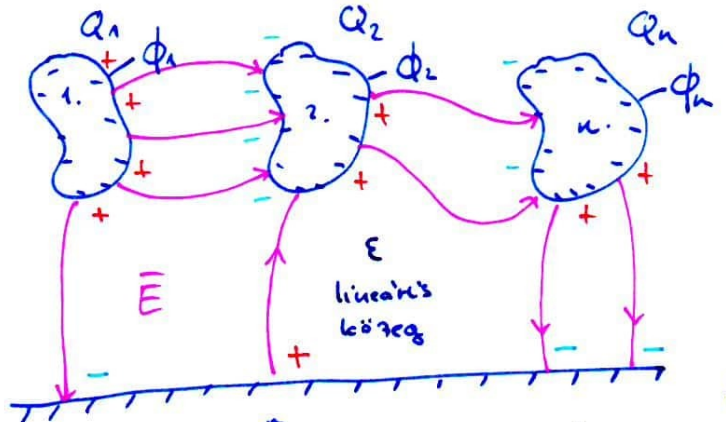
Eml.:



$$C_{12} \triangleq \frac{Q}{U}$$



Többelektródás rendszer



$$\phi_j = \sum_{i=1}^{n} p_{ij} Q_i$$

p_{ij}: potenciálegyüttható

$j = 1, \dots, n$

$$Q_0 = -\sum_{i=1}^{n} Q_i$$

$$\phi_0 \triangleq 0$$

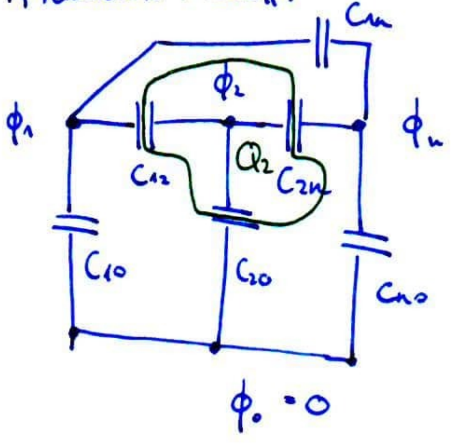
$$\underline{\phi} = \underline{\underline{p}} \cdot \underline{Q}$$

$$\underline{Q} = \underline{\underline{c}} \cdot \underline{\phi}$$

$\underline{\underline{p}} = [p_{ij}]_{ij}$, $p_{ij} \equiv p_{ji}$ reciprocitás miatt

$\underline{\underline{c}} = \underline{\underline{p}}^{-1}$: kapacitás együttható mátrix

Áramköri modell:



$$Q_j = C_{j0}(\phi_j - 0) + \sum_{\substack{i=1 \\ i \neq j}}^{n} C_{ji}(\phi_j - \phi_i)$$

$$\underline{Q} = \underline{\underline{c}} \cdot \underline{\phi}$$

$$\begin{bmatrix} Q_1 \\ \vdots \\ Q_n \end{bmatrix} = \begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & & \vdots \\ C_{n1} & \cdots & C_{nn} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_n \end{bmatrix}$$

re

$$Q_j = C_{j0}(\phi_j - 0) + \sum_{\substack{i=1 \\ i\neq j}}^{n} C_{ji}(\phi_j - \phi_i), \quad j = 1, \dots, n$$

$$c_{jj} = \left(C_{j0} + \sum_{\substack{i=1 \\ i\neq j}}^{n} C_{ji} \right)$$

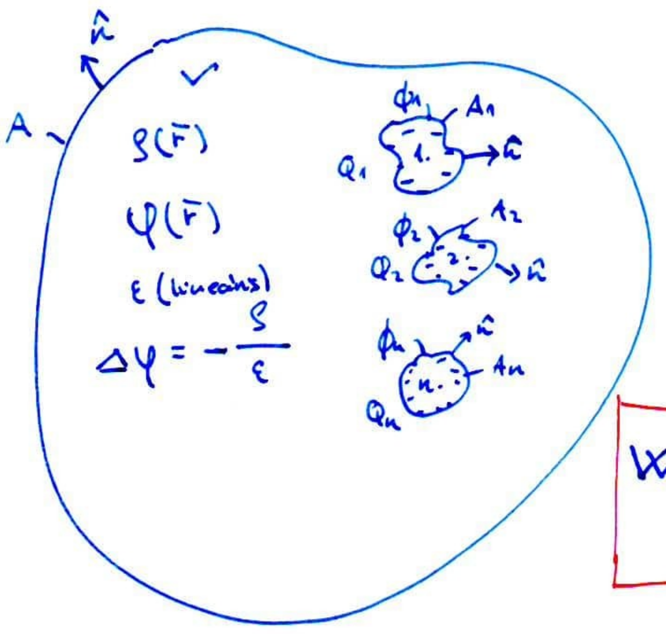
$$c_{ij} \equiv c_{ji} = -C_{ji}$$

Részkapacitások:

- földkapacitás: C_{j0} $j = 1, \dots, n$
- főkapacitások: C_{ij} $i\neq j$, $i \neq 0$, $j \neq 0$

Gyakorlatban: kondenzátor

Az elektrosztatikus mező energiája



$W_E = ?$ V-ben

1.) $w = \frac{1}{2}\bar{E}\cdot\bar{D} + \cancel{\frac{1}{2}\bar{B}\cdot\bar{H}} = \frac{1}{2}\varepsilon|\bar{E}|^2$

$\bar{D} = \varepsilon\cdot\bar{E}$

$$\boxed{W_E = \int_V w \, dV}$$

2.) $w = \frac{1}{2}\varepsilon|\operatorname{grad}\varphi|^2$

azonosság: $\operatorname{div}(\varphi \operatorname{grad}\varphi) = \underbrace{\operatorname{grad}\varphi\cdot\operatorname{grad}\varphi}_{|\operatorname{grad}\varphi|^2} + \varphi\cdot\underbrace{\operatorname{div}\operatorname{grad}\varphi}_{\Delta\varphi}$

$$W_E = \int_V \frac{1}{2}\varepsilon\left[\operatorname{div}(\varphi\cdot\operatorname{grad}\varphi) - \varphi\Delta\varphi\right]dV \underset{\text{Gauss}}{=} \qquad \Delta\varphi = -\frac{\rho}{\varepsilon}$$

$$= \oint_A \frac{1}{2}\varepsilon(\varphi\operatorname{grad}\varphi)\,d\bar{A} - \sum_{i=1}^{n}\oint_{A_i}\frac{1}{2}\varepsilon(\varphi\operatorname{grad}\varphi)\,d\bar{A} + \int_V \frac{1}{2}\varphi\rho\,dV$$

$$\boxed{W_E} = \oint_A \frac{1}{2}\varepsilon(\varphi \operatorname{grad}\varphi)\,d\bar{A} - \sum_{i=1}^{n} \oint_{A_i} \frac{1}{2}\varepsilon(\underbrace{\varphi \operatorname{grad}\varphi}_{-\bar{E}})\,d\bar{A} + \int_V \frac{1}{2}\varphi\rho\,dV \;\ominus$$

$$-\bar{D} \longrightarrow -D_n\,dA = -\sigma\,dA$$

$$\ominus \underbrace{\oint_A \frac{1}{2}\varepsilon(\varphi \operatorname{grad}\varphi)\,d\bar{A}}_{\substack{\varphi(\infty)=0 \\ A\to\infty \\ \downarrow \\ 0}} + \underbrace{\sum_{i=1}^{n}\frac{1}{2}\oint_{A_i}\varphi\,\sigma\,dA}_{\sum_{i=1}^{n}\frac{1}{2}\phi_i \cdot Q_i} + \int_V \frac{1}{2}\varphi\rho\,dV \;\ominus$$

$$\ominus\;\boxed{\sum_{i=1}^{n}\frac{1}{2}\phi_i Q_i + \int_V \frac{1}{2}\varphi\rho\,dV}$$

gyakorlatban: $\rho = 0$

Alkalmazások:

1. kondenzátor:

$$\left.\begin{array}{l} Q_1 = -Q_2 \\ U_{12} = \phi_1 - \phi_2 \end{array}\right\} W_E = \frac{1}{2}Q\phi_1 + \frac{1}{2}(-Q\phi_2) = \frac{1}{2}Q\cdot U_{12} = \frac{1}{2}\frac{Q^2}{C_{12}} = \frac{1}{2}C_{12}U_{12}^2$$

2. Többelektródás rendszer:

$$W_E = \sum_{i=1}^{n}\frac{1}{2}\left(\sum_{j=1}^{n} c_{ij}\cdot Q_{ij}\right)Q_i = \frac{1}{2}\underline{Q}^T\cdot \underline{\underline{c}}\cdot Q \geq 0$$

pozitív definit

$$W_E = \sum_{\substack{\text{(összes} \\ \text{részkapacitás)}}} \frac{1}{2}c_{ij}(\phi_i - \phi_j)^2$$

A helyettesítő töltések módszere

Cél: $\Delta\varphi = 0$ megoldása, ha elektródák potenciálja vagy töltés adott

Eszköz: $\underbrace{\text{elektróda}}_{\text{peremfeltétel}}$ + felületi töltés $\Rightarrow$ szabad térben egyszerű töltés

peremfeltétel $\longrightarrow$ azonos peremfeltétel

$\downarrow$

$\Delta\varphi = 0$ egyértelmű megoldása

Egyszerű töltéselrendezések potenciálja

1.) Ponttöltés: $\boxed{\varphi(r) = \frac{Q}{4\pi\varepsilon} \cdot \frac{1}{r}}$ r: gömbsugár

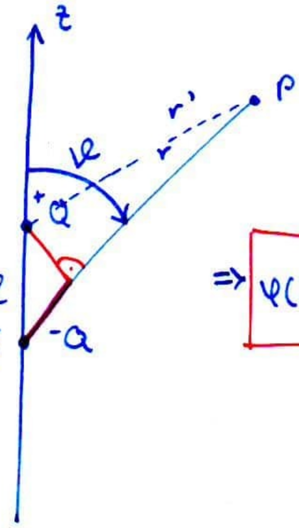
↳ ekvipotenciális koncentrikus gömbfelületek

2.) Végtelen hosszú egyenes vonaltöltés: $\boxed{\varphi(r) = \frac{q}{2\pi\varepsilon} \ln \frac{r_0}{r}}$

r: hengersugár, r_0: referencia pont

ekvipotenciális koaxiális hengerpalástok

3.) Elektrosztatikus dipólus:



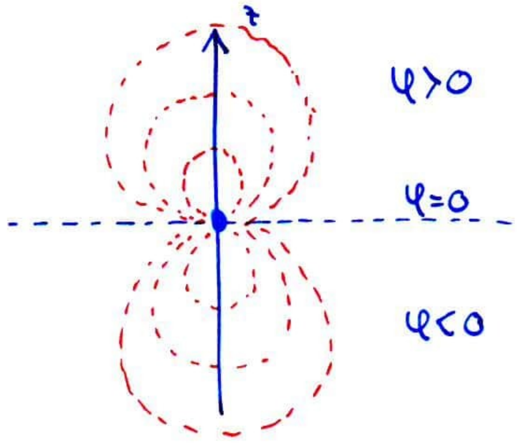
$$\varphi(P) = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r'} - \frac{1}{r}\right)$$

Ha $l \ll r$: $r' = r - \underbrace{l \cdot \cos\vartheta}$

$$\Rightarrow \boxed{\varphi(P) = \frac{Q}{4\pi\varepsilon}\,\frac{l\cdot\cos\vartheta}{r(r - l\cos\vartheta)}} = \boxed{\frac{p}{4\pi\varepsilon} \cdot \frac{\cos\vartheta}{r^2}}$$

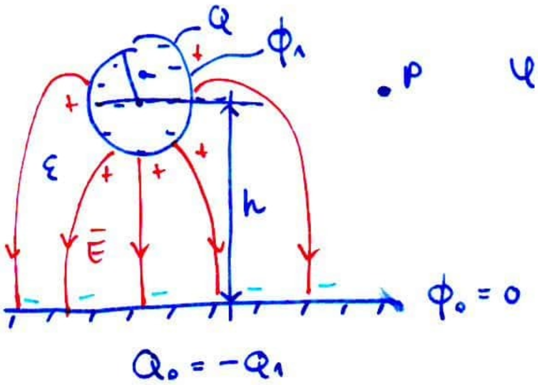
$\left.\begin{matrix} l \to 0 \\ Q \to \infty \end{matrix}\right\}$ $Q \cdot l \triangleq \text{const.} = p$

$\bar{p} = Q \cdot l \cdot \hat{e}_z$: dipólus nyomaték vektor

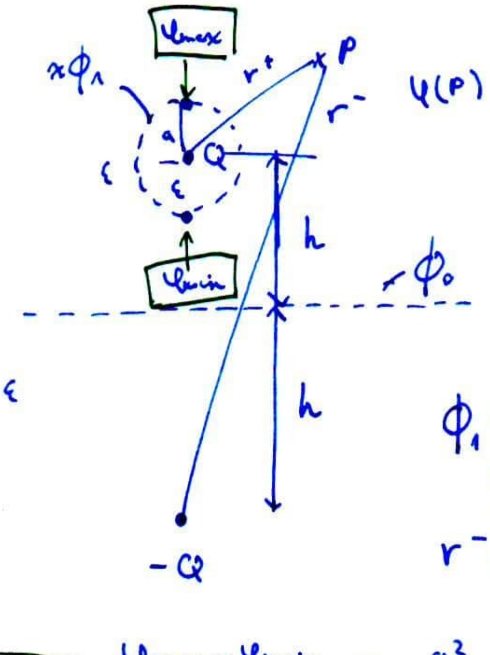


• dipólus

1. példa: gömb + ∞ félsík



• P $\qquad \varphi(P) = ?$



$$\varphi(P) = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r^+} - \frac{1}{r^-}\right)$$

közelítés: $a \ll h$

$\Rightarrow$ kissugarú közelítés

$$\phi_1 \approx \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{2h}\right)$$

$\uparrow$

$$r^- \in [2h-a,\ 2h+a]$$

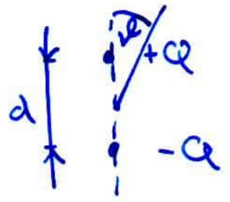
$\boxed{\text{hiba:}}$ $\dfrac{\varphi_{max} - \varphi_{min}}{\phi_1} \approx \dfrac{a^2}{2h^2}$

alkalmazás:

$$\boxed{C_{10} = \frac{Q}{\phi_1 - \phi_0} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{2h}}}$$

Tereh E.
15.10.14.

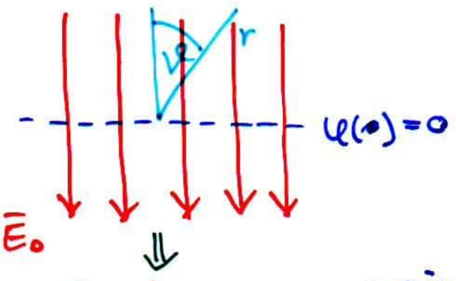
Ismétlés:



$$p = Q \cdot d$$

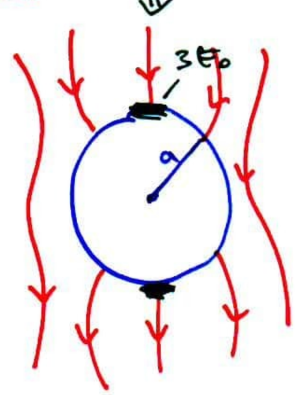
$$\varphi(r,\vartheta) = \frac{p}{4\pi\varepsilon} \cdot \frac{\cos\vartheta}{r^2}$$

példa: Homogén térbe helyezett fémgömb



$$\bar{E} = -\hat{e}_z \cdot E_0$$

$$\varphi_0(z) = zE_0$$



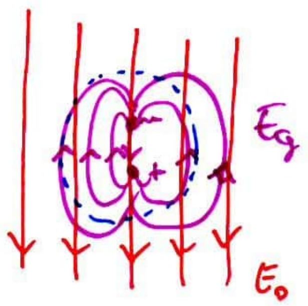
- gömb torzítja a teret
- $\bar{E} = \bar{E}_0 + \bar{E}_g$
- $\varphi = \varphi_0 + \varphi_g$

$\varphi(r=a, \vartheta) = \text{const.} = 0$ (polárkoordináta)

- $\varphi_0(r,\vartheta) = E_0 \cdot r \cdot \cos\vartheta$

$$0 = \varphi(a,\vartheta) = E_0 \cdot a \cdot \cos\vartheta + \varphi_g$$

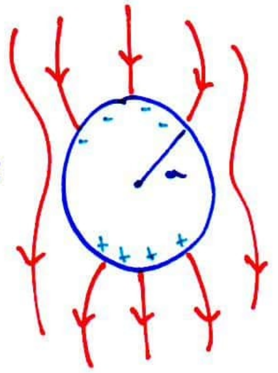
$$\varphi_g(a,\vartheta) = -E_0 \cdot a \cdot \cos\vartheta$$



$\Rightarrow$ helyettesítő töltés: dipólus

$$\frac{-p}{4\pi\varepsilon} \cdot \frac{\cos\vartheta}{a^2} = E_0 \cdot a \cdot \cos\vartheta \Rightarrow p = -E_0 \cdot a^3 \cdot 4\pi\varepsilon$$

A két tér összege lesz a gömb által torzított tér

Megj:  töltésszétválasztás (esőcseppeknél pl.)

Stacionárius áramlási tér

I. Alapösszefüggések

1) Maxwell: $\frac{\partial}{\partial t} = 0$, $\bar{J} \neq \bar{0}$

- $\text{rot}\,\bar{H} = \bar{J}$

$\text{div}\,\text{rot}\,\bar{H} = \text{div}\,\bar{J} = 0$ => mágneses egyenlet kiküszöbölése

- $\text{div}\,\bar{J} = \bar{0}$
- $\text{rot}\,\bar{E} = \bar{0}$
- $\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$

beiktatott mezők

2) Laplace - Poisson:

$\text{rot}\,\bar{E} = \bar{0} \Leftarrow \bar{E} = -\text{grad}\,\varphi$

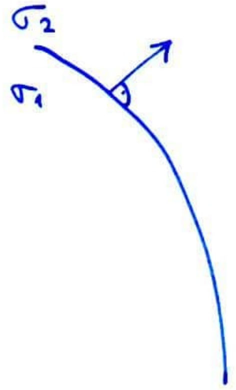
$\text{div}\left(\sigma(-\text{grad}\,\varphi + \bar{E}_b) + \bar{J}_b\right) = 0$

$\boxed{\text{div}(\sigma \cdot \text{grad}\,\varphi) = \text{div}(\sigma \bar{E}_b + \bar{J}_b)}$: általános alak

Ha σ állandó:

=> $\boxed{\Delta\varphi = \text{div}\left(\bar{E}_b + \frac{\bar{J}_b}{\sigma}\right)}$: homogén térben

3) Határfeltételek: σ_1, σ_2: különböző vezetőképességek



a) $E_{2t} = E_{1t}$: tangenciális komponensek

$\Updownarrow$

$\phi_1 = \phi_2$

b) $J_{2n} = J_{1n}$: normális komponensek

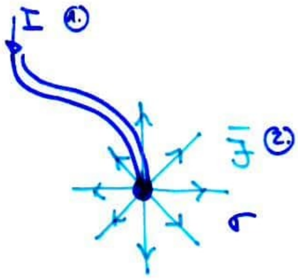
Ha nem lenne egyenlő $\Rightarrow$ töltés halmozódna fel $\Rightarrow$ lenne időbeli változás.

$\Updownarrow$

$$\sigma_2 \cdot \frac{\partial \varphi}{\partial n} = \sigma_1 \cdot \frac{\partial \varphi}{\partial n} \qquad (\bar{J} = \sigma \cdot \bar{E})$$

II. Egyszerű konfigurációk

1) Áram-pontforrás:



① → ②

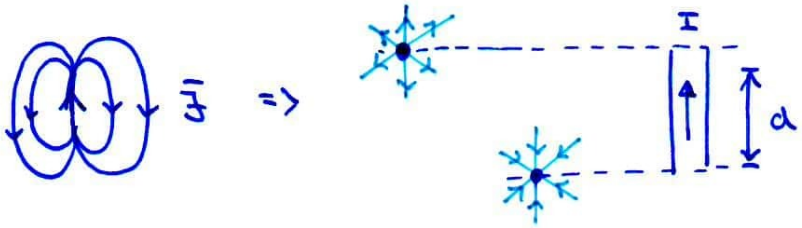
$$\bar{J}(r) = \frac{I}{4\pi r^2} \cdot \hat{e}_r$$

$$\bar{E}_r(r) = \frac{I}{4\pi \sigma r^2}$$

σ

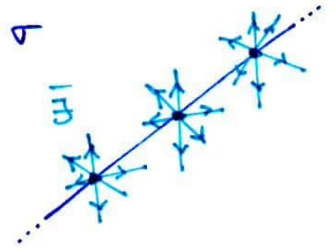
$$\varphi(r) = \frac{I}{4\pi \sigma r}$$

2) Áramdipólus:



dipólusmomentum: $I \cdot d$

3) Áram-vonalforrás:



$i\left[\frac{A}{m}\right]$

$$E_r(r) = \frac{i}{2\pi\sigma r}$$

$$\varphi(r) = \frac{i}{2\pi\sigma}\cdot \ln\frac{r_0}{r}$$

III. Elektródarendszerek

1) Az elektróda fogalma:

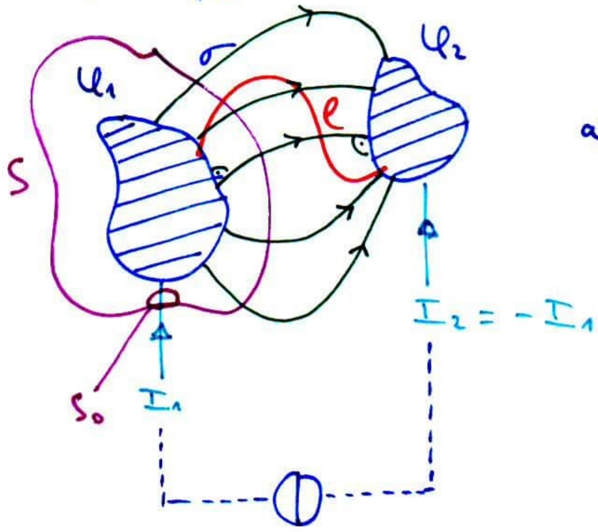
$$\sigma_{Cu} = 6{,}3\cdot 10^{7}\ \frac{S}{m}$$

$$\sigma_{teflon} \cong 10^{-25}\ \frac{S}{m}$$

$$\frac{\sigma_{elektróda}}{\sigma_{közeg}} > \underbrace{1{,}2}_{\text{(egy-kettő)}} \text{ nagyságrend}$$

2) Elektródapár:

$\longrightarrow$: áramvonalak



a) Lineáris közegre:

$$U_{12} = \varphi_1 - \varphi_2 \sim I.$$

$$R = \frac{U}{I}\ [\Omega]$$

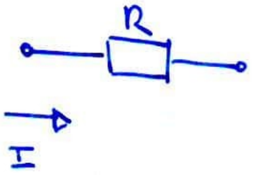
$$G = \frac{I}{U}\ [S]$$

$$U_{12} = \int_l \bar{E}\,d\bar{l}$$

$$I = \int_{S\setminus S_0} \bar{J}\,d\bar{s} \rightarrow \boxed{R = \frac{\int_l \bar{E}\,d\bar{l}}{\int_{S\setminus S_0} \bar{E}\sigma\,d\bar{s}}}$$

$\curvearrowright$

b) Ellenállás Joule-veszteséggel:



$$P = I^2 \cdot R \Rightarrow \boxed{R \triangleq \frac{P}{I^2}}$$

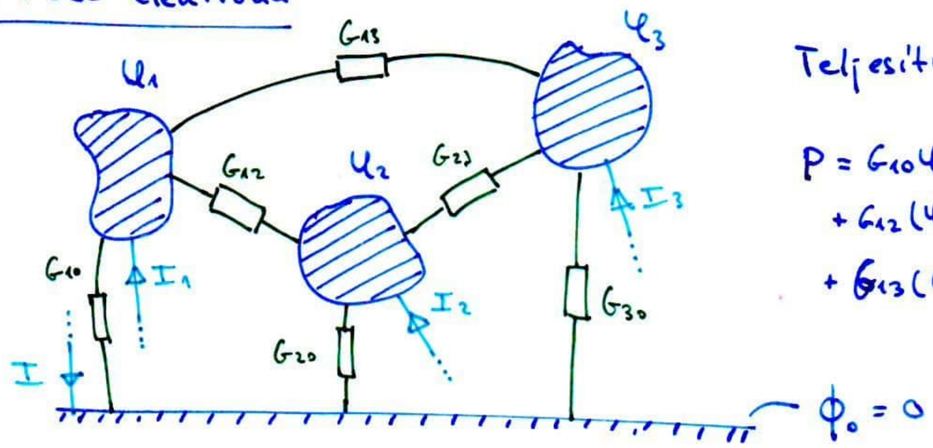
disszipált teljesítmény

$$p = \bar{J} \cdot \bar{E} \quad \left[\frac{W}{m^3}\right]$$

$$p = \frac{J^2}{\sigma} \quad * \; (E_b = 0,\ J_b = 0)$$

$$\boxed{R = \frac{\int_V \frac{J^2}{\sigma}\, dV}{I^2}}$$

3) Több elektróda



Teljesítmény:

$$P = G_{10}\varphi_1^2 + G_{20}\varphi_2^2 + G_{30}\varphi_3^2 + G_{12}(\varphi_1-\varphi_2)^2 + G_{23}(\varphi_2-\varphi_3)^2 + G_{13}(\varphi_1-\varphi_3)^2$$

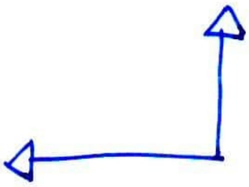
$$I = -(I_1 + I_2 + I_3)$$

részkonduktanciák:

$$I_1 = G_{10}\varphi_1 + G_{12}(\varphi_1-\varphi_2) + G_{13}(\varphi_1-\varphi_3)$$

$$I_2 = G_{20}\varphi_2 + G_{\underset{12}{21}}(\varphi_2-\varphi_1) + G_{23}(\varphi_2-\varphi_3)$$

$$I_3 = G_{30}\varphi_3 + G_{\underset{13}{31}}(\varphi_3-\varphi_1) + G_{23}(\varphi_3-\varphi_2)$$



$$\varphi_3 = \varphi_3 - \varphi_0 = \varphi_3 - 0$$

IV. Elektrosztatika analógiája

$$\begin{cases} \operatorname{div}(\varepsilon \operatorname{grad} \varphi) = -\rho & \longleftrightarrow \quad \Delta\varphi = -\dfrac{\rho}{\varepsilon} \\ \operatorname{div}(\sigma \operatorname{grad} \varphi) = \operatorname{div}(\sigma \bar{E}_b + \bar{J}_b) \end{cases}$$

elektrosz.	$(\varphi, \bar{E})$	ε	$\bar{D}$	ρ	C	részkapacitások	Q
	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
áramlás	$(\varphi, \bar{E})$	σ	$\bar{J}$	$\operatorname{div}(\sigma \bar{E}_b + \bar{J}_b)$	G	részkonduktanciák	I

...	Qd	q	
	$\updownarrow$	$\updownarrow$	
	Id	i	

spec.: homogén lineáris közeg:

$$\boxed{G = C\Big|_{\varepsilon \leftarrow \sigma} = C \cdot \frac{\sigma}{\varepsilon}}$$

: feladatmegoldásnál

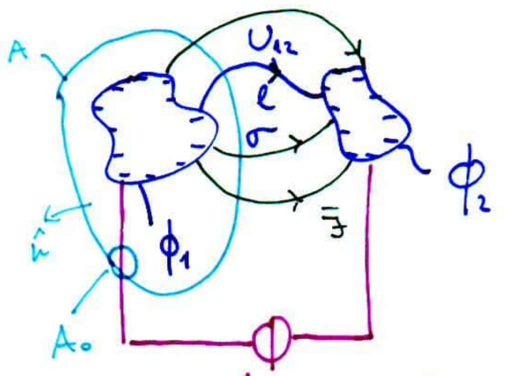
Terek Gy.
15. 10. 15.

Stacionárius áramlás

$\frac{\partial}{\partial t} = 0$, $\bar{J} \neq 0$, $\bar{J} = \sigma \cdot \bar{E}$, $[\sigma] = \frac{S}{m}$

$\mathrm{rot}\,\bar{E} = 0 \Rightarrow \bar{E} = -\mathrm{grad}\,\varphi$

$\mathrm{div}\,\bar{J} = 0$



$U_{12} = \int_\ell \bar{E}\,d\bar{\ell}$

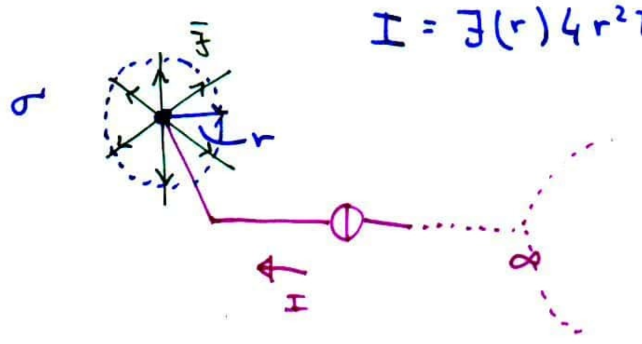
$\frac{\partial}{\partial t} = 0$ miatt

$R \triangleq \frac{U_{12}}{I}$

$$\oint_A \bar{J}\,d\bar{A} = 0 = \int_{A \setminus A_0} \bar{J}\,d\bar{A} + \underbrace{\int_{A_0} \bar{J}\,d\bar{A}}_{-I}$$

$-I$ („–", mert $\hat{n}$ kifelé mutat)

Pontforrás:



$I = J(r)\,4r^2\pi$

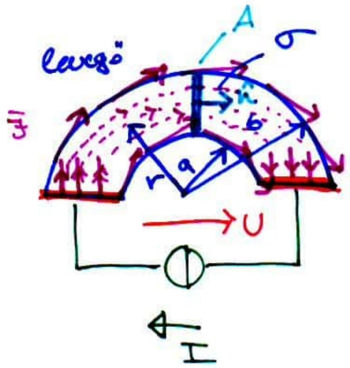
$$J(r) = \frac{I}{4\pi r^2}$$

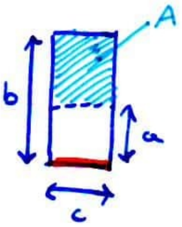
$$E(r) = \frac{I}{4\pi\sigma} \cdot \frac{1}{r^2}$$

$$\varphi(r) = \frac{I}{4\pi\sigma} \cdot \frac{1}{r} \left(+ \underbrace{\varphi(\infty)}_{0} \right)$$

Példatár: 3.3 – 3.7

$\boxed{1}$ Ellenállásszámítás: félgyűrű $R = ?$



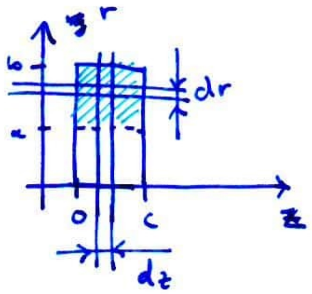


$$\bar{J} = \sigma \bar{E}$$

$$U = \int_{\ell} \bar{E}\, d\bar{\ell} = r \cdot \pi \cdot E(r) = r\pi \frac{J(r)}{\sigma}$$

$$\boxed{J(r) = \frac{U\sigma}{r\pi}}$$: Áramsűrűség helyfüggő

$$I = \int_A \bar{J}\, d\bar{A}$$

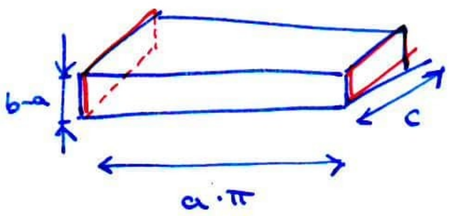


$$I = \int_A \bar{J}\, d\bar{A} = \int_{z=0}^{c}\int_{r=a}^{b} J(r) \cdot dr \cdot dz =$$

$$= c \cdot \frac{U\sigma}{\pi} \int_a^b \frac{1}{r}\, dr = c \frac{U\sigma}{\pi} \ln \frac{b}{a}$$

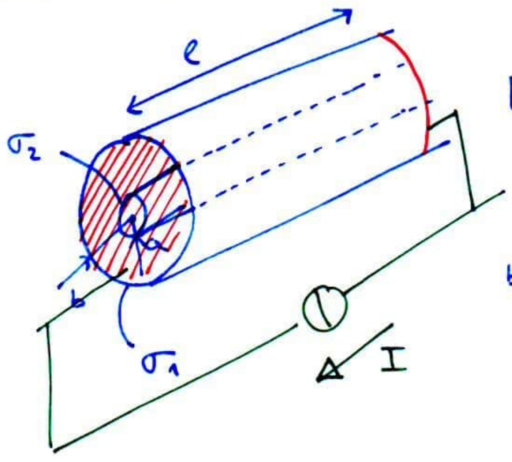
$$\Rightarrow \underline{\underline{R}} \triangleq \frac{U}{I} = \underline{\underline{\frac{\pi}{c \cdot \sigma \cdot \ln \frac{b}{a}}}}$$

Közelítés: $b - a \ll a$

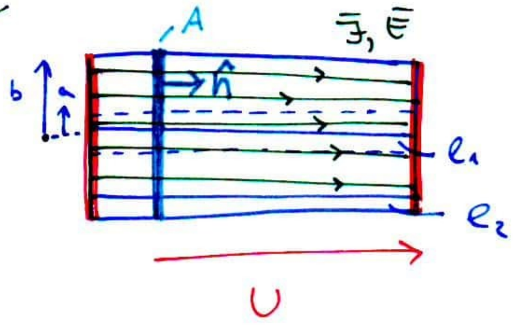


$$R = \frac{a\pi}{\sigma(b-a) \cdot c}$$

2 Rétegzett vezeték



$R' = \frac{R}{l}$: egységhosszára jutó ellenállás ?



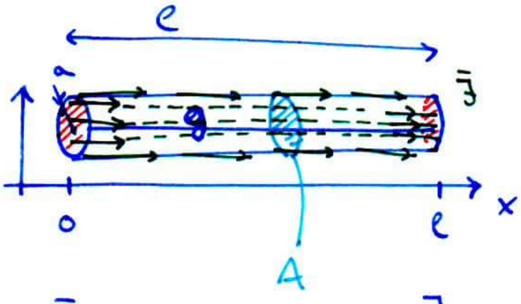
$$U = \int_{l_1} \bar{E}\, d\bar{l} = \int_{l_2} \bar{E}\, d\bar{l} \Rightarrow \bar{E} = \text{konst.} =$$

$$\left.\begin{array}{ll} \text{ér:} & \bar{J}_1 = \sigma_1 \bar{E} \\ \text{köpeny:} & \bar{J}_2 = \sigma_2 \cdot \bar{E} \end{array}\right\} \longrightarrow I = a^2\pi \bar{J}_1 + (b^2 - a^2)\pi \bar{J}_2$$

$$R = \frac{U}{I} = \frac{l \cdot \bar{E}_{(1)}}{a^2\pi J_1 + (b^2 - a^2)\pi \bar{J}_2} = \frac{l \cdot E}{a^2\pi \sigma_1 \cdot E + (b^2 - a^2) \cdot \pi \sigma_2 E} = \frac{l}{a^2\pi\sigma_1 + (b^2 - a^2)\pi\sigma_2}$$

$$R' = \frac{R}{l}$$

$\boxed{3}$ Változó vezetőképességű kábel



$R = ?$

$\sigma(x) := \sigma_0 \cdot e^{-\alpha x}$

$\bar{J} = \text{const.} \Rightarrow \bar{E} \neq \text{const.}$

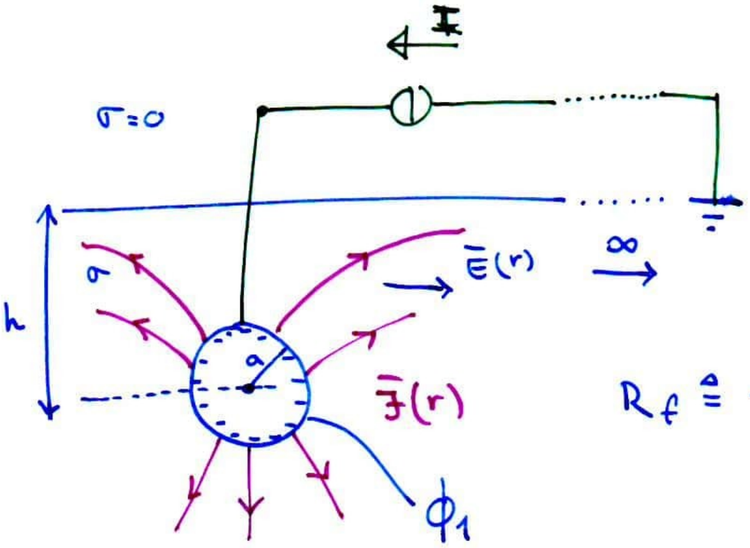
$$\bar{E} = \frac{\bar{J}}{\sigma(x)} \Rightarrow E(x) = \frac{J}{\sigma(x)}$$

$$U = \int_g \bar{E}\,d\bar{g} = \int_0^{\ell} E(x)\,dx = \frac{J}{\sigma_0}\int_0^{\ell}\frac{dx}{e^{-\alpha x}} = \frac{J}{\sigma_0}\cdot\frac{e^{\alpha\cdot\ell}-1}{\alpha}$$

$I = J \cdot a^2\pi \longleftarrow$ „A”-ból

$$\underline{\underline{R}} = \frac{U}{I} = \underline{\underline{\frac{e^{\alpha\ell}-1}{\sigma_0\alpha a^2\pi}}}$$

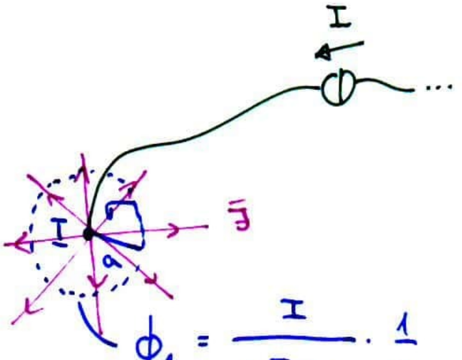
$\boxed{4}$ Földelési ellenállás



$\phi(\infty) = 0$

$$R_f \triangleq \frac{\phi_1 - \overbrace{\phi(\infty)}^{0}}{I}$$

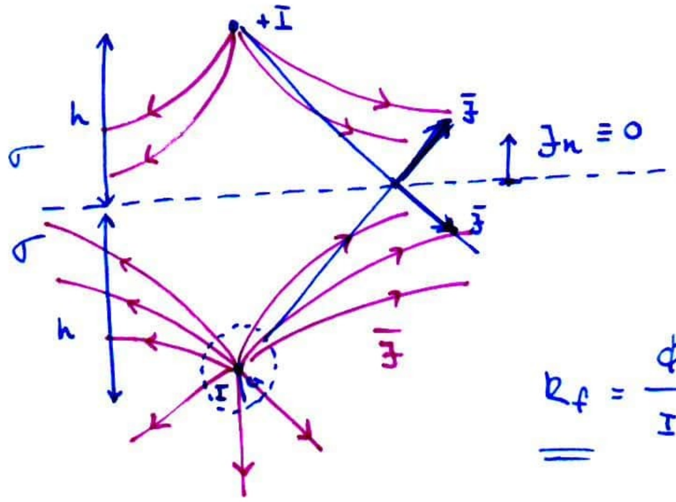
a) $h \ggg a$



$$\phi_1 = \frac{I}{4\pi\sigma} \cdot \frac{1}{a} \Rightarrow R_f = \frac{1}{4\pi\sigma a}$$

b) $h \gg a$ (földfelszín hatása nem elhanyagolható)

Helyettesítő pontforrás



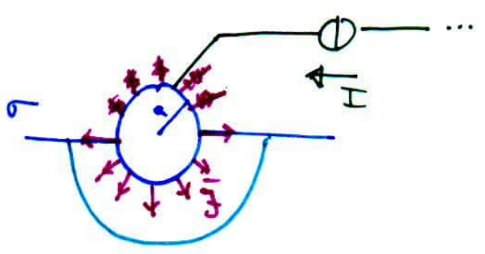
$$\phi_1 \cong \frac{I}{4\pi\sigma a} + \frac{I}{4\pi\sigma 2h} = \frac{I}{4\pi\sigma} \frac{2h+a}{2ha}$$

$$R_f = \frac{\phi_1}{I} = \frac{1}{4\pi\sigma} \frac{2h+a}{2ha} \xrightarrow{h \to \infty} \frac{1}{4\pi\sigma} \cdot \frac{1}{a}$$

c) $h \approx a$

∄ zárt alakú megoldás

d) $h = 0$

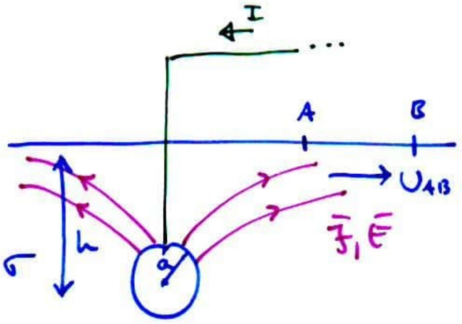


$$I = J(r) \cdot \frac{4r^2\pi}{2}$$

„◡" miatt

$$R_f = 2 \cdot \frac{1}{4\pi\sigma \cdot a}$$

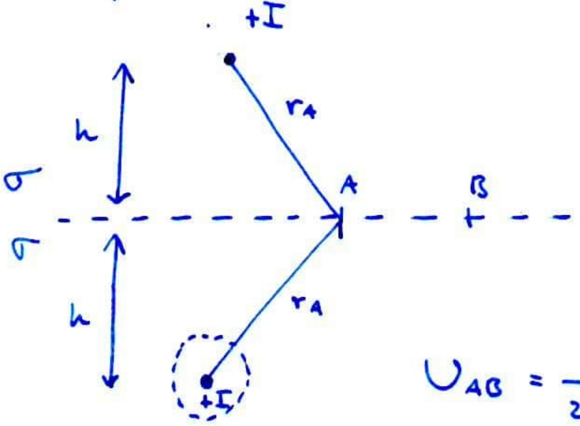
5 Lépésfeszültség



$U_{AB} = ?$

$$U_{AB} = \int_{l=A}^{B} \bar{E}\, d\bar{l}$$

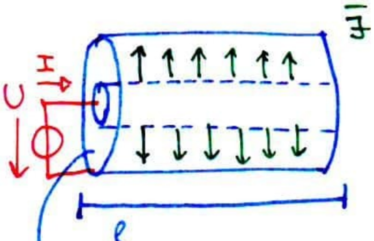
$$\bar{E} = \frac{\bar{J}}{\sigma}$$



$$\left.\begin{aligned} \phi_A &= 2 \cdot \frac{I}{4\pi\sigma} \cdot \frac{1}{r_A} \\ \phi_B &= 2 \cdot \frac{I}{4\pi\sigma} \cdot \frac{1}{r_B} \end{aligned}\right\} U_{AB} = \phi_A - \phi_B$$

$$U_{AB} = \frac{I}{2\pi\sigma}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

6 Elektrosztatikai analógia



Szivárgási ellenállás = ?

$R_{sz} = ?$ $l, C', \varepsilon_r, \sigma$ adott

$$R_{sz} = \frac{U}{I} \qquad R_{sz} = \frac{1}{G' \cdot l}$$

$$\boxed{\frac{C'}{G'} \equiv \frac{\varepsilon}{\sigma} = \frac{\varepsilon_0 \cdot \varepsilon_r}{\sigma}}$$

$$G' = C' \cdot \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_r}$$

$$R_{sz} = \frac{1}{G' l}$$

Terek E.
15. 10. 21.

Stacionárius áramok mágneses tere

$\frac{\partial}{\partial t} \equiv 0$, $\bar{J}$ kelti a teret.

$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$

$\mathrm{rot}\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

$\mathrm{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$\mathrm{div}\,\bar{B} = 0$

$\mathrm{div}\,\bar{D} = \varrho$

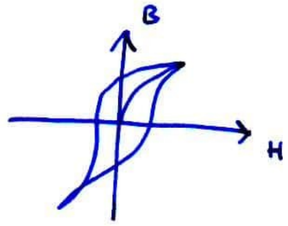
$\rightarrow$

$\mathrm{rot}\,\bar{H} = \bar{J}$

$\mathrm{div}\,\bar{B} = 0$

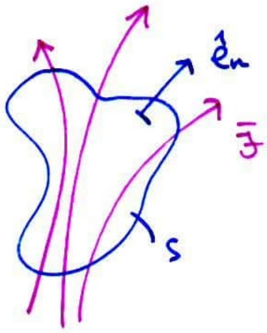
$\bar{B} = \mu_0 \bar{H} + \mu_0 \bar{M}$

Lágymágnes: $\bar{B} = \mu_0 \mu_r \bar{H} = \mu \bar{H}$



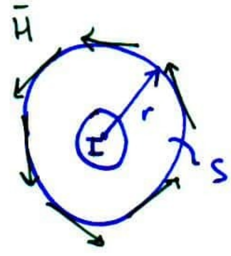
Hiszterézis

$$\int_S \mathrm{rot}\,\bar{H}\, d\bar{S} = \int_S \bar{J}\, d\bar{S} \xrightarrow{\text{Stokes}} \oint_\ell \bar{H}\, d\bar{\ell} = \int_S \bar{J}\, d\bar{S} = I$$



1)

- Egyenes vezető által keltett mágneses tér:



$$\oint_\ell \bar{H}\, d\bar{\ell} = \int_S \bar{J}\, d\bar{A} = I$$

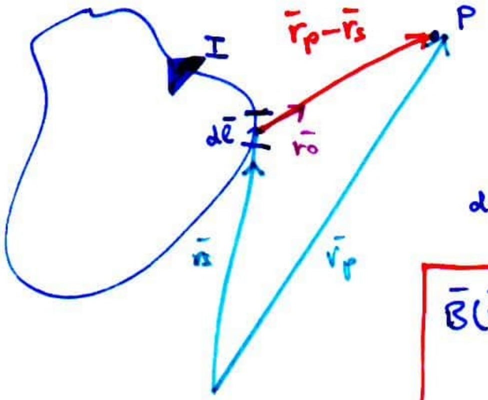
$\bar{H}$ tangenciális. ← Mert $\mathrm{div}\,\bar{B} = 0$

(Ha $\bar{H}$ nem tangenciális lenne $\Rightarrow$ $\mathrm{div}\,\bar{B} \neq 0$ $\Rightarrow$ ↯)

$$2\pi H_\varphi(r)\, r = I$$

$$H_\varphi(r) = \frac{I}{2\pi r}$$

2) • Vonalszerű áramok mágneses tere :

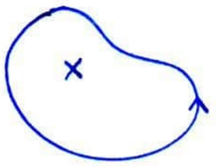


$$\frac{\bar{r}_p - \bar{r}_s}{|\bar{r}_p - \bar{r}_s|} = \bar{r}_0 = \text{egység}$$

$$d\bar{B}(\bar{r}_p) = \frac{\mu I}{4\pi} \cdot \frac{d\bar{e} \times \bar{r}_0}{|\bar{r}_p - \bar{r}_s|^2}$$

$$\boxed{\bar{B}(\bar{r}_p) = \mu \frac{I}{4\pi} \oint_e \frac{d\bar{e}' \times (\bar{r}_p - \bar{r}')}{\underbrace{|\bar{r}_p - \bar{r}'|^3}_{\text{egységvektor lesz}}}}$$: Biót - Savart

Következmény:

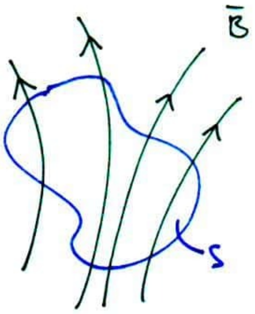


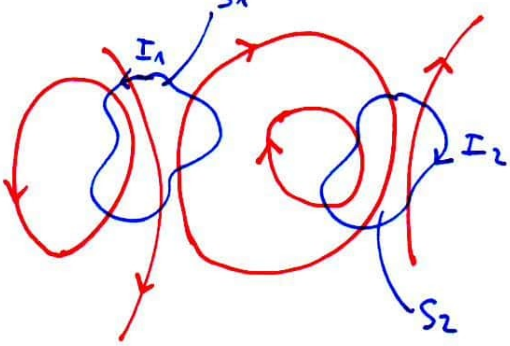
Síkban fekvő vezető mágneses tér mindig merőleges a vezető által meghatározott síkra.

3) • Mágneses fluxus, mágneses indukció

Vonalszerű áramok fluxusa

$$\phi = \int_S \bar{B}\, d\bar{S}$$





$$\begin{cases} \phi_1 = L_{11} I_1 + L_{12} I_2 \\ \phi_2 = L_{21} I_1 + L_{22} I_1 \end{cases}$$

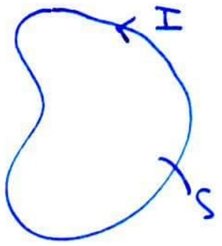
L_{11} : önindukciós együttható

L_{12} : kölcsönös indukciós együttható

$L_{12} = L_{21}$

$$\phi_i = \sum_{j=1}^{N} L_{ij} I_j \quad i = 1, \dots, N$$

Bizonyítás $\left(W_E = \frac{1}{2}\sum_i \sum_j L_{ij} I_j I_i \right)$



$$W_M = \frac{1}{2}\mu_0 H^2 \quad \text{(levegőben)}$$

$$W_M = \int_V w_M \, dV$$

$\mathrm{rot}\,\bar{H} = \bar{J}$

$\mathrm{div}\,\bar{B} = 0 \xrightarrow{\text{def.}} \bar{B} \triangleq \mathrm{rot}\,\bar{A} \Rightarrow \mathrm{div}\,\bar{B} = 0$ automatikusan teljesül

azonosság: $\mathrm{div}(\bar{H}\times\bar{A}) = \bar{A}\cdot\mathrm{rot}\,\bar{H} - \bar{H}\,\mathrm{rot}\,\bar{A}$

$$w_M = \frac{1}{2}\mu_0 H^2 = \frac{1}{2}\bar{H}\cdot\bar{B} = \frac{1}{2}\bar{H}\cdot\mathrm{rot}\,\bar{A} = \frac{1}{2}\bar{A}\cdot\mathrm{rot}\,\bar{H} - \frac{1}{2}\mathrm{div}(\bar{H}\times\bar{A})$$

$$W_M = \int_V w_M \, dV = \int_V \frac{1}{2}\bar{A}\cdot\underbrace{\mathrm{rot}\,\bar{H}}_{\bar{J}}\, dV - \underbrace{\int_V \frac{1}{2}\mathrm{div}(\bar{H}\times\bar{A})\, dV}_{\oint_{S_\infty} \frac{1}{2}\bar{H}\times\bar{A}\, d\bar{S}}$$

S_∞: $r \to \infty$

$H \sim \frac{1}{r^2}, \quad A \sim \frac{1}{r}$

$$W_m = \underbrace{\int_V \frac{1}{2}\bar{A}\bar{J}\, dV}_{= \oint_l \frac{1}{2}\bar{A}\bar{I}\, d\bar{l}} - \underbrace{\oint_{S_\infty} \frac{1}{2}\bar{H}\times\bar{A}\, d\bar{S}}_{=0} \quad (=)$$

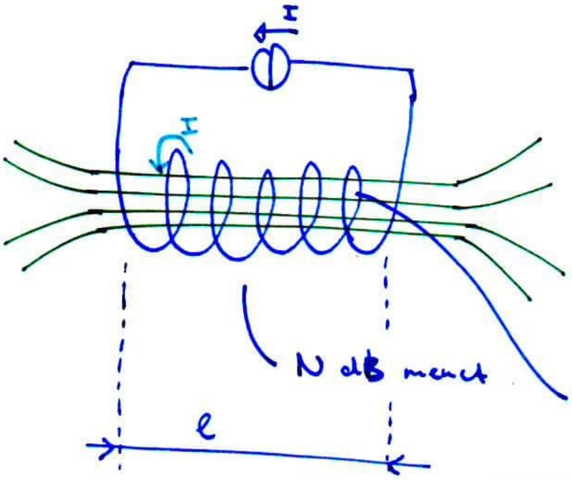
$= 0 \to$ mert $\bar{H}, \bar{A} \to 0$

vonalszerű alakzatokra

$$(=)\ \oint_l \frac{1}{2}\bar{A}\bar{I}\, d\bar{l} \underset{\text{Stokes}}{=} \int_S \frac{1}{2}\bar{B} I\, d\bar{S} = I\cdot\frac{1}{2}\underbrace{\int_S \bar{B}\, d\bar{S}}_{\Phi} = \boxed{\frac{1}{2}\Phi\cdot I = W_m}$$

N db keret esetében: $\boxed{W_M = \sum_{i=1}^{N} \frac{1}{2}\Phi_i I_i} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} L_{ij} I_j I_i$

Speciális eset: tekercs



$$l \cdot H = N \cdot I$$

$$H = \frac{N \cdot I}{l}$$

$$\Phi = S_0 \cdot B = S_0 \cdot \mu \cdot \frac{N \cdot I}{l}$$

$$\boxed{\Psi = N \cdot \Phi = \frac{S_0 \mu N^2 I}{l} = L \cdot I \Rightarrow L = \frac{S_0 \mu N^2}{l}}$$

tekercsfluxus = (menetszám)·(egy db menetfluxus)

4) Mágneses energia

$$w_M = \frac{1}{2} \bar{H} \cdot \bar{B} = \frac{1}{2} \mu \cdot \hat{H}^2 = \frac{1}{2} \frac{B^2}{\mu}$$

lineáris, izotróp közegben

Teljes tér energiája:

$$\boxed{W_M = \int_V \frac{1}{2} \bar{H} \bar{B} \, dV = \int_V \frac{1}{2} \mu \cdot H^2 dV}$$

lin., izotróp

: közelhatás alapján

$$W_M = \frac{1}{2} \sum_{i=1}^{N} \Phi_i I_i$$

: vékony I_i-vel átjárt vezető keretek esetében

$$\frac{1}{2} \sum_{i=1}^{N} \Phi_i I_i = \frac{1}{2} \sum_{i=1}^{N} \underbrace{\sum_{j=1}^{N} (L_{ij} I_j)}_{\Phi_i} I_i$$

: távolhatás esetén

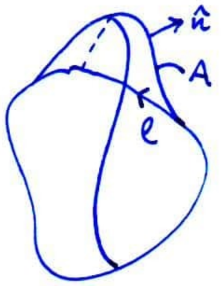
Terek E.
15.10.22.

Indukálási jelenségek

gyakorlatban: generátor, transzformátor

Faraday-törvény:

$$\mathrm{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t} \Rightarrow \oint_{l} \bar{E}\,d\bar{l} = -\int_{A} \frac{\partial \bar{B}}{\partial t}\,d\bar{A}$$



Nyugalmi indukció

„álló" görbe:
$$\underbrace{\oint_{l} \bar{E}\,d\bar{l}}_{u_i(t)} = -\frac{d}{dt}\underbrace{\int_{A} \bar{B}\,d\bar{A}}_{\Phi} = -\frac{d\Phi}{dt}$$

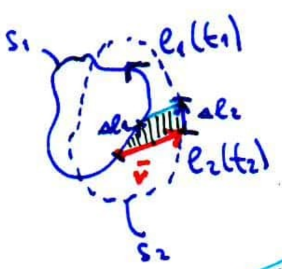
$u_i(t)$: indukált feszültség

Φ fluxus

$$\boxed{u_i(t) = -\frac{d\Phi}{dt}}$$

Mozgási indukció

„mozgó" görbe: $\bar{B} = \bar{B}(\bar{r})$, $\bar{B}$ időben állandó



$\Delta t = t_2 - t_1$

$\Delta\Phi = ?$

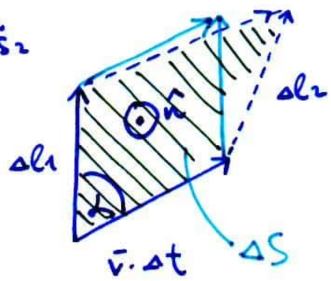
$\Delta S = \Delta l_1 (v \cdot \Delta t \cdot \sin\alpha)$

vektorszorzás

$\Delta\bar{S} = (\bar{v}\cdot\Delta t) \times \Delta\bar{l}_1$

vektoros azonosság

$\Delta\bar{S}\cdot\bar{B} = \bar{B}(\bar{v}\times\Delta\bar{l}_1)\Delta t = \Delta\bar{l}_1(\bar{B}\times\bar{v})\Delta t$



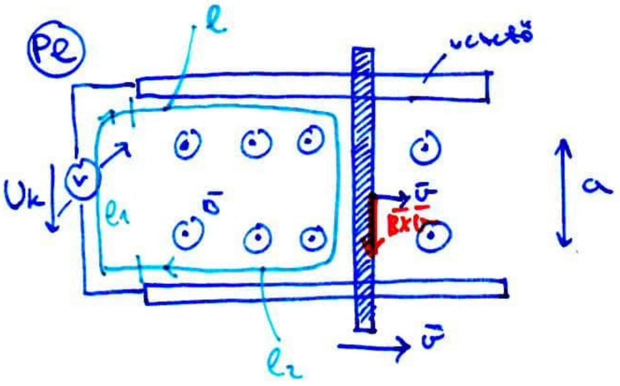
$\Delta\Phi = \left(\oint_{l} d\bar{l}(\bar{B}\times\bar{v})\right)\Delta t$

$$\boxed{u_i(t) = -\frac{d\Phi}{dt} = -\oint_{l} (\bar{B}\times\bar{v})\,d\bar{l}}$$

Eredő: $\bar{B} = \bar{B}(\bar{r}, t)$ + ℓ mozog

$$u_i(t) = \underbrace{-\frac{\partial}{\partial t}\int_S \bar{B}\,d\bar{s}}_{\text{nyugalmi ind.}} \underbrace{- \oint_\ell (\bar{B}\times\bar{v})\,d\bar{\ell}}_{\text{mozgási ind.}}$$

Pl.



$\bar{B}$ homogén

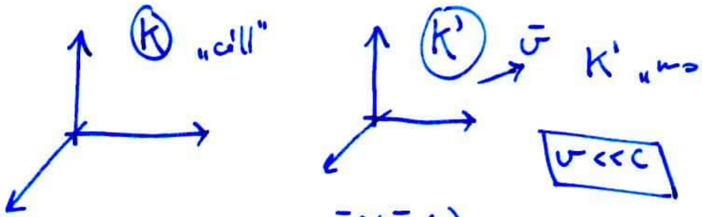
$\ell = \ell_1 \cup \ell_2$

$$u_i = -\oint_\ell (\bar{B}\times\bar{v})\,d\bar{\ell} = -a\cdot B\cdot v$$

$$\oint_\ell \bar{E}\,d\bar{\ell} = \int_{\ell_1} \bar{E}\,d\ell + \underbrace{\int_{\ell_2} \bar{E}\,d\bar{\ell}}_{0} = -U_k$$

$$\underline{\underline{U_{ind} = -U_k}}$$

$\bar{E}$ transzformálása mozgó vonatkoztatási rendszerben



K' „mozog" K-hoz képest $\bar{v}$-vel.

$\boxed{v \ll c}$

$\bar{E}(\bar{r}, t)$, $\bar{B}(\bar{r}, t)$

$\bar{E}'(\bar{r}, t)$, $\bar{B}'(\bar{r}, t)$

$\boxed{\bar{B} \equiv \bar{B}'}$ $(v \ll c)$

rot $\bar{E}$

$$\underbrace{-\int_S \frac{\partial \bar{B}}{\partial t}\,d\bar{s} - \oint_\ell (\bar{B}\times\bar{v})\,d\bar{\ell}}_{u_i \text{ K-ban}} \equiv \underbrace{\oint_\ell \bar{E}'\,d\ell}_{\text{K'-ben}}$$

ℓ görbe K'-ban áll.

$$\Leftarrow \quad \oint_e \bar{E}\,d\bar{e} = \oint_e (\bar{B}\times\bar{v})\,d\bar{e} + \oint_e \bar{E}'\,d\bar{e}$$

$\Rightarrow \bar{E} = \bar{B}\times v + \bar{E}'$ -nek kell lennie

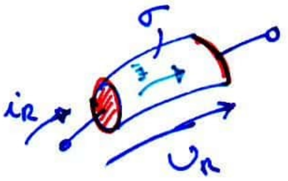
$$\Rightarrow \boxed{\bar{E}' \equiv \bar{E} + \bar{v}\times\bar{B}}$$

Koncentrált ~~[illegible]~~ paraméterű hálózatok törvényei

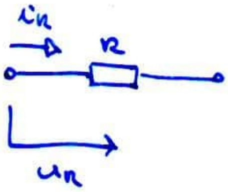
Kétpólusok karakterisztikái + Kirchhoff-törvények

I. Kétpólusok

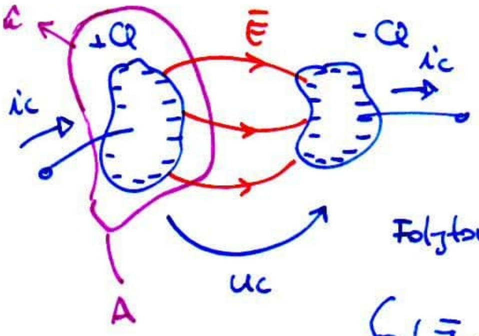
1) Ellenállás



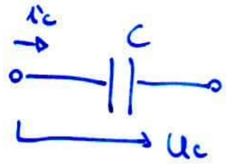
$$R = \frac{u_R}{i_R} \quad :$$



2) Kapacitás



$$\frac{Q}{u_C} = C \quad :$$



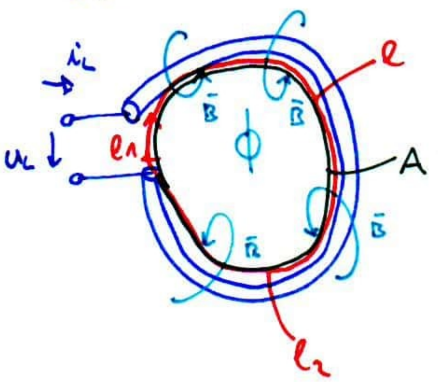
Folytonossági tv. „A"-ra: $\mathrm{div}\left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right) = 0$

$$\oint_A \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right) d\bar{A} = 0$$

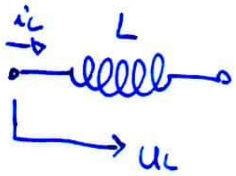
$$-i_C + \oint_A \frac{\partial \bar{D}}{\partial t}\, d\bar{A} = 0 \quad \Rightarrow \quad -i_C + \frac{dQ}{dt} = 0$$

$$\boxed{\Rightarrow i_C = C \cdot \frac{du_C}{dt}}$$

3) Induktivitás



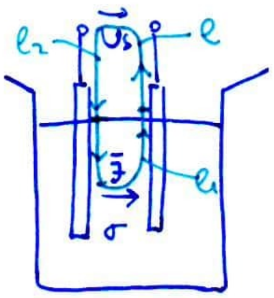
$$\oint_l \bar{E}\,d\bar{l} = -\frac{d\Phi}{dt}$$



$$\int_{l_1} \bar{E}\,d\bar{l} + \underbrace{\int_{l_2} \bar{E}\,d\bar{l}}_{0} = -\frac{d\Phi}{dt}$$

$$u_L = -\left(-\frac{d\Phi}{dt}\right) \Rightarrow \boxed{u_L = L\cdot\frac{di_L}{dt}}$$

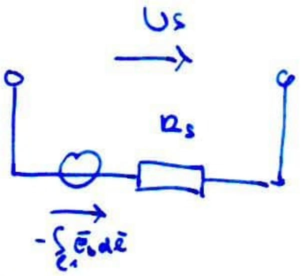
4.) Feszültségforrás



$$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b \quad , \quad \bar{J}_b := 0$$

$$\bar{E} = \frac{1}{\sigma}\bar{J} - \bar{E}_b$$

$$\oint_l \bar{E}\,d\bar{l} = 0$$



U_s = adott

$$\int_{l_1} \bar{E}\,d\bar{l} + \int_{l_2} \bar{E}\,d\bar{l} = 0 \neq$$

ideális fesz.forr. : $\sigma \approx \infty$

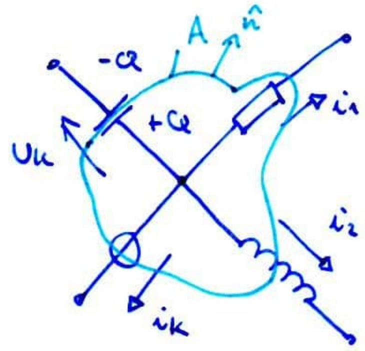
$\Rightarrow R_s = 0$

$$\int_{l_1} \left(\frac{1}{\sigma}\bar{J} - \bar{E}_b\right) dl - U_s = 0$$

$$U_s = -\int_{l_1} \bar{E}_b\,d\bar{l} + \int_{l_1} \frac{1}{\sigma}\bar{J}\,dl$$

II. Kirchhoff-törvények

1) Csomóponti-törvény (töltésmegmaradás elve)



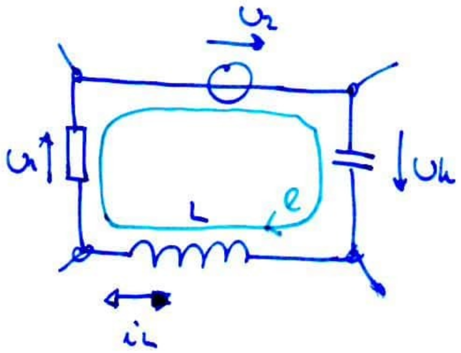
$$\oint_A \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right) d\bar{A} = 0$$

$$\sum_{k=1}^{K} i_k + \underbrace{C \cdot \frac{du_k}{dt}}_{i_C} = 0$$

tetszőleges vágatra:

$$\sum_{\text{összes}} i \equiv 0$$

2.) Huroktörvény



$$\underbrace{\oint_\ell \bar{E}\, d\bar{\ell}}_{\circledast} = -\underbrace{\frac{d\Phi}{dt}}_{\frac{d}{dt}(L \cdot i_L)}$$

$$\sum_{k=1}^{k} u_k + L \cdot \frac{di_L}{dt} = 0$$

$\circledast$: $\sum_{k=1}^{k} u_k$

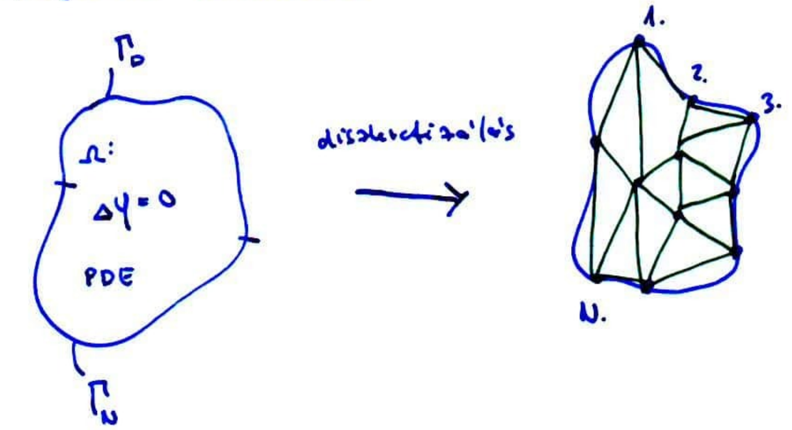
tetszőleges hurokra:

$$\sum_{\text{összes}} u_k \equiv 0$$

Terek E.
15.10.28.

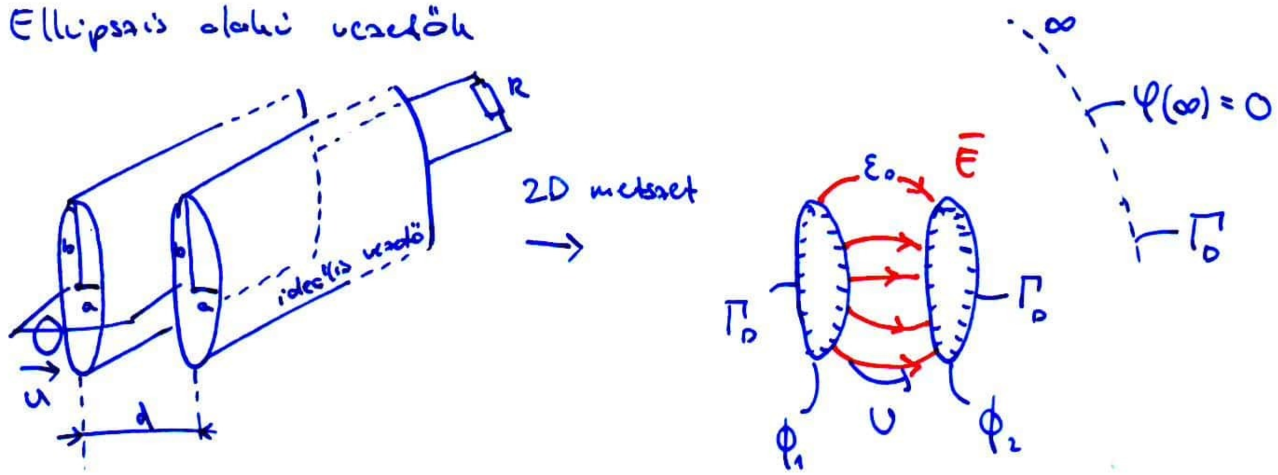
Számítógépes demonstráció

Végeselem-módszerrel ↑ $\Delta\varphi \equiv \mathrm{div\,grad}\,\varphi$



$\varphi(x,y) = ?$ → $\varphi_1, \varphi_2, \ldots, \varphi_N = ?$: Lineáris algebrai egyenletrendszer

① Ellipszis alakú vezetők

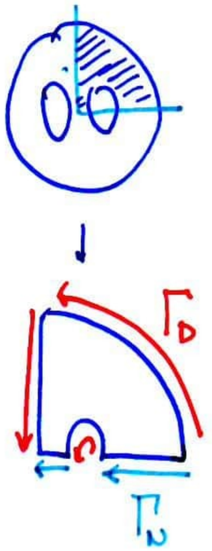


Hosszegységre eső kapacitás?

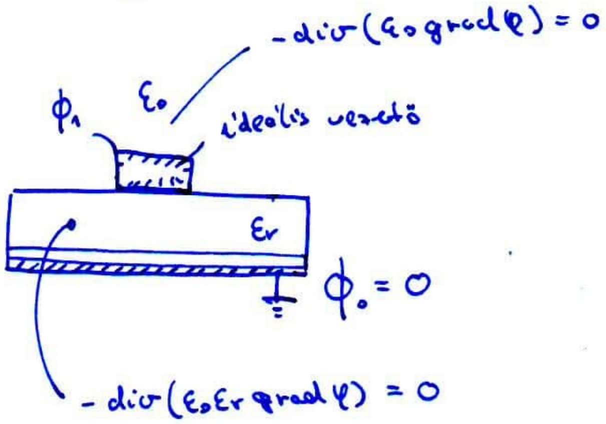
$$W_E = \frac{1}{2} C U^2$$

$$\int_V w\,dV = W_E \;, \quad w = \frac{1}{2}\varepsilon_0 |\bar{E}|^2$$

$$\bar{E} = -\mathrm{grad}\,\varphi$$



② Mikroszalag-vonal



Távvezetékek

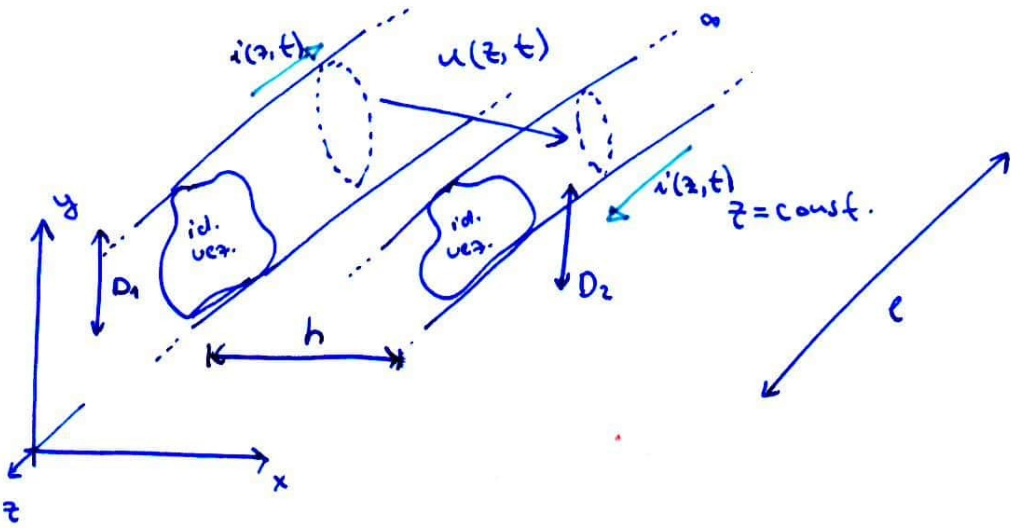
Elosztott paraméterű hálózat ⊕ 1D hullámterjedés

hálózat → fesz., áram

Elosztott → $u = u(z,t)$

↑ hely, időfüggés

$i = i(z,t)$



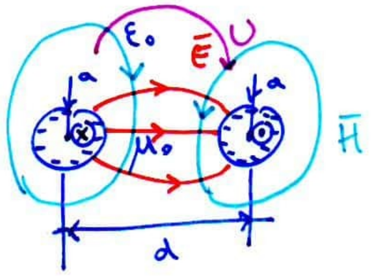
! Hullámhossz: $\lambda = \dfrac{c}{f}$, $\lambda \gg D_1, D_2, h \rightarrow x, y$: sztatikus/stacionárius tér

$\ell \gtrapprox \lambda \Rightarrow z$: hullám pl.: $f = 50\,Hz \Rightarrow \lambda = \dfrac{3\cdot 10^8 \frac{m}{s}}{50 \frac{1}{s}} = 6000\,km$

$f = 300\,MHz \Rightarrow \lambda \approx \dfrac{3\cdot 10^8}{3\cdot 10^8} = 1\,m$: összemérhető a kábel hosszával

Gyakorlatban:

- LECHER-vezeték:

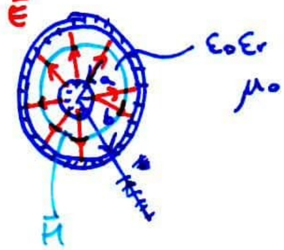


(közeg lehetne $\varepsilon_0 \cdot \varepsilon_r$)

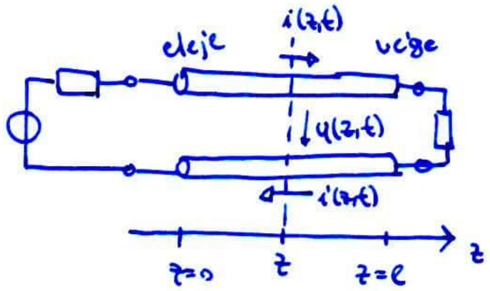
⊗ : I befelé

⊙ : I kifelé

- KOAXIÁLIS kábel:

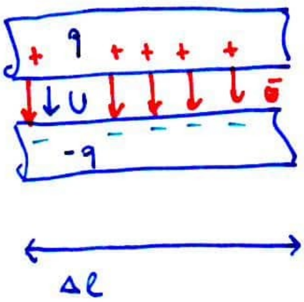


Sematikus ábra:



Vezetékparaméterek

1.) Hosszegységre eső kapacitás : C'

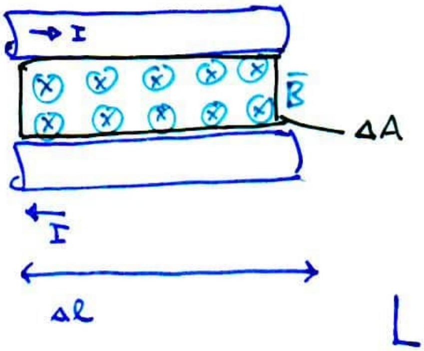


$$\Delta Q = q \cdot \Delta l$$

$$C_{\Delta l} = \frac{\Delta Q}{U} \Rightarrow \boxed{C' = \frac{C_{\Delta l}}{\Delta l} = \frac{q}{U}}$$

$$\boxed{\text{LECHER: } C' = \frac{\pi \varepsilon_0 \varepsilon_r}{\ln \frac{d}{a}}} , \quad \boxed{\text{KOAXIÁLIS: } C' = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln \frac{b}{a}}}$$

2.) Hosszegységre eső induktivitás : L'



fluxus:

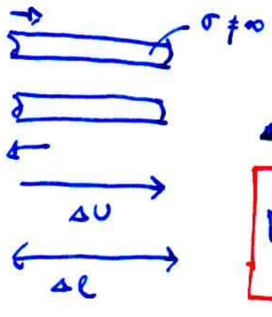
$$\Phi_{\Delta A} = \int_{\Delta A} \bar{B}\, d\bar{A}$$

$$L_{\Delta A} = \frac{\Phi_{\Delta A}}{I} \Rightarrow \boxed{L' = \frac{L_{\Delta A}}{\Delta l} = \frac{\Phi_{\Delta A}}{I \cdot \Delta l}}$$

$$\boxed{\begin{aligned} &\text{LECHER: } L' = \frac{\mu_0}{\pi} \ln \frac{d}{a} \\ &\text{KOAX: } L' = \frac{\mu_0}{2\pi} \ln \frac{b}{a} \end{aligned}}$$

$$\boxed{\text{Bizonyítható: } L' \cdot C' \equiv \mu_0 (\varepsilon_0 \cdot \varepsilon_r)}$$

3.) Hosszegységre eső ellenállás:

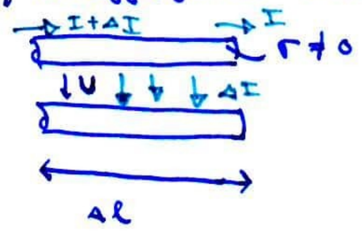


$$R_{\Delta l} = \frac{\Delta U}{I}$$

$$\boxed{R' = \frac{R_{\Delta l}}{\Delta l}}$$

4.) Hosszegységre eső vezetés

ΔI : szivárgási áram



$$G_{\Delta l} = \frac{\Delta I}{U} \Rightarrow \boxed{G' = \frac{G_{\Delta l}}{\Delta l}}$$

Terek Gy.
15. 10. 29.

Stacionárius áramlási tér

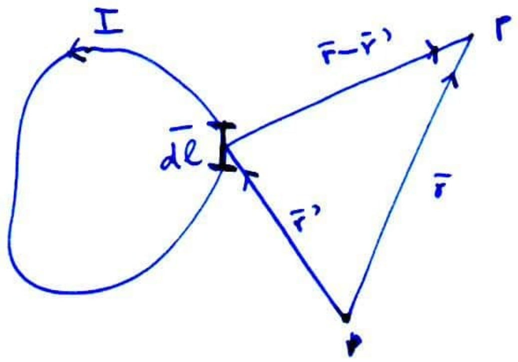
$\mathrm{rot}\,\bar{H} = \bar{J}$

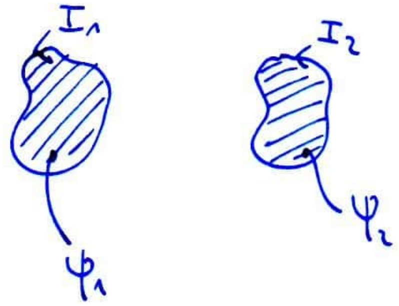
$\mathrm{div}\,\bar{B} = 0$

$\bar{B} = \mu \bar{H}$ (lineáris, izotróp anyagok)

Biot-Savart: $\bar{H}(\bar{r}) = \dfrac{I}{4\pi} \oint_{l} \dfrac{d\bar{l}' \times (\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$ ($\oint$, mert $\mathrm{div}\,\bar{B}=0$)

$$d\bar{H}(\bar{r}) = \frac{I}{4\pi} \frac{d\bar{l}' \times (\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$$





$\Psi_1 = L_{11} I_1 + L_{12} I_2$

$\Psi_2 = L_{21} I_1 + L_{22} I_2$

$L_{12} = L_{21}$

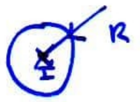
Indukálás:

$$\mathrm{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\oint \bar{E}\,d\bar{l} = -\frac{\partial}{\partial t} \int_A \bar{B}\,d\bar{A}$$

$$u = -\frac{d\Psi}{dt}$$

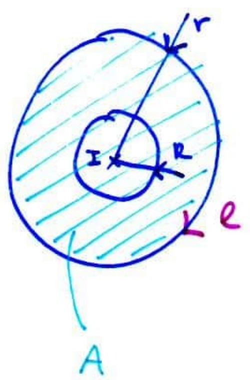
1 Végtelen hosszú vezető tere



$\bar{H} = ? \Big|_{r>R,\ r<R}$

Szimmetria van. => $\bar{H}$ állandó

I.



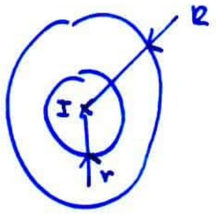
$$\int_A \mathrm{rot}\,\bar{H}\,d\bar{A} = \int_A \bar{J}\,d\bar{A}$$

$$\oint_l \bar{H}\,d\bar{l} = \int_A \bar{J}\,d\bar{A}$$

$$H_\varphi(r)\,2r\pi = I$$

$$\boxed{H_\varphi(r) = \frac{I}{2\pi r}} \quad , \quad \bar{H} = H_\varphi \cdot \bar{e}_\varphi$$

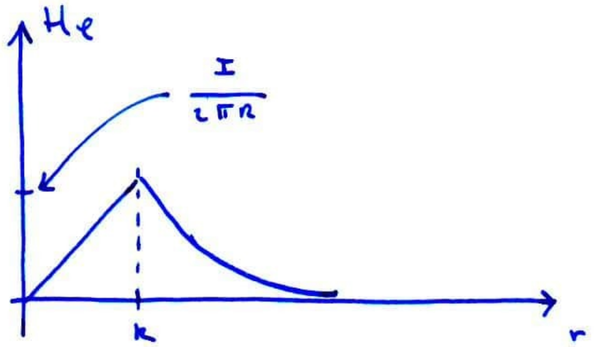
II.



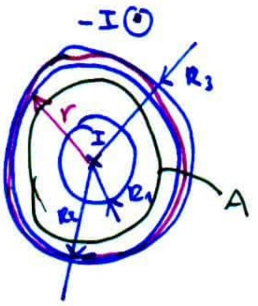
$$\oint_l \bar{H}\,d\bar{l} = \int_A \bar{J}\,d\bar{A}$$

$$H_\varphi(r)\cdot 2\pi r = \frac{I}{R^2\pi}\cdot r^2\pi$$

$$\Rightarrow \boxed{H_\varphi(r) = \frac{I\cdot r}{R^2 2\pi}}$$



2 Koaxiális kábel, axiális töltéseloszlás

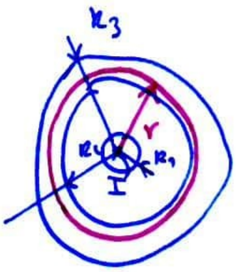


$$H_\varphi = \begin{cases} \dfrac{I \cdot r}{2\pi R_1^2}, & 0 < r < R \\ \dfrac{I}{2\pi r}, & R_1 < r < R_2 \end{cases}$$

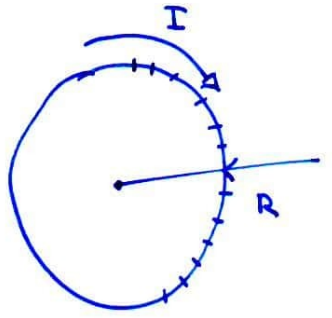
$$\oint_\ell \bar{H} d\bar{\ell} = \int_A \bar{J} d\bar{A} \quad \Rightarrow \quad H_\varphi(r) \cdot 2\pi r = I - \frac{I}{\pi (R_3^2 - R_2^2)} \cdot (r^2 - R_2^2) \cdot \pi$$

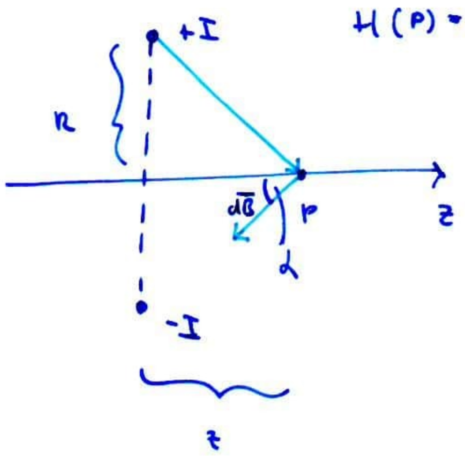
$$H_\varphi(r)\, 2\pi r = I \cdot \frac{R_3^2 - R_2^2 - r^2 + R_2^2}{R_3^2 - R_2^2}$$

$$\boxed{H_\varphi(r) = I \frac{R_3^2 - r^2}{2\pi r (R_3^2 - R_2^2)}} \qquad R_2 < r < R_3$$



3 Vékony gyűrű mágneses tere





$H(P) = ?$

$$\bar{B} = \frac{I\mu}{4\pi} \oint_l \frac{d\bar{l}' \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}$$

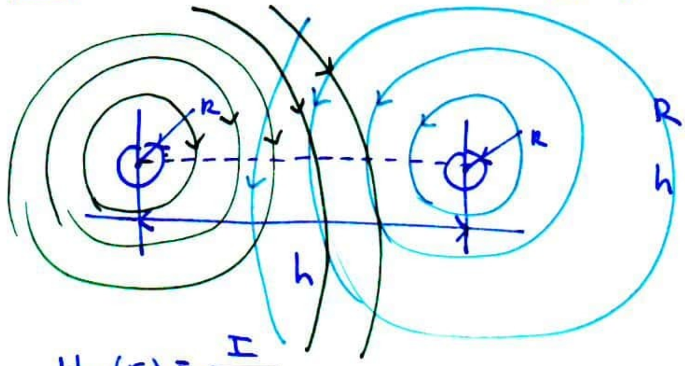
$$\bar{B}(z=0) = B_z \cdot \hat{e}_z \Rightarrow B_z = -\mu \cdot \frac{I}{4\pi} \frac{2\pi R}{R^2} = -\mu \cdot \frac{I}{2R}$$

$$d\bar{B} = \mu \frac{I}{4\pi} \frac{d\bar{l} \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3} \; ; \; \mu \frac{I}{4\pi} \frac{dl}{R^2 + z^2} = |d\bar{B}|$$

$$dB_z = \mu \frac{I}{4\pi} \frac{dl}{R^2 + z^2} \cdot \underbrace{\frac{R}{\sqrt{R^2 + z^2}}}_{\cos\alpha}$$

$$\boxed{B_z} = \oint_l B_z \, dl = \mu \cdot \frac{I}{4\pi} \cdot \frac{R}{(R^2 + z^2)^{\frac{3}{2}}} \cdot 2\pi R = \boxed{\mu \frac{I}{2} \frac{R^2}{(R^2 + z^2)^{\frac{3}{2}}}}$$

4 Lecher-vezeték hosszegységre eső induktivitása



$R = 8\,mm$

$h = 1{,}2\,m$

$$H_\varphi(r) = \frac{I}{2\pi R}$$

$$\Psi = 2 \cdot \int_R^{h-R} \frac{I\mu}{2\pi r}\,dr = \mu \frac{I}{\pi} \ln \frac{h-R}{R}$$

$$\Psi = L \cdot I \,, \quad L' = \frac{L}{\ell}$$

$$\frac{\Psi}{\ell} = L' \cdot I \quad \Rightarrow \quad \boxed{L'_k = \frac{\mu \cdot \ln \frac{h-R}{R}}{\pi}}$$

L'_k : külső induktivitás

Belül:

I, R $\qquad H_\varphi = \dfrac{I \cdot r}{2\pi R^2}$

$$W = \frac{1}{2} L \cdot I^2$$

$$W_b = \frac{1}{2} L_b \cdot I^2$$

$$W_b = \ell \int_0^R \frac{1}{2}\mu \left(\frac{I \cdot r}{2\pi R^2}\right)^2 \cdot 2\pi r \cdot dr = \frac{\ell \cdot I^2 \mu}{4\pi R^4} \int_0^R r^3\,dr = \frac{\ell \cdot I^2 \mu}{4\pi R^4} \cdot \frac{R^4}{4} =$$

$$= \ell \cdot \mu \cdot \frac{I^2}{16\pi}$$

$W_b = \frac{1}{2} L_b \cdot I^2 \Rightarrow L_b = \ell \cdot \mu \cdot \frac{1}{8\pi} \Rightarrow L'_b = \frac{\mu}{8\pi}$; ← egy vezeték esetben

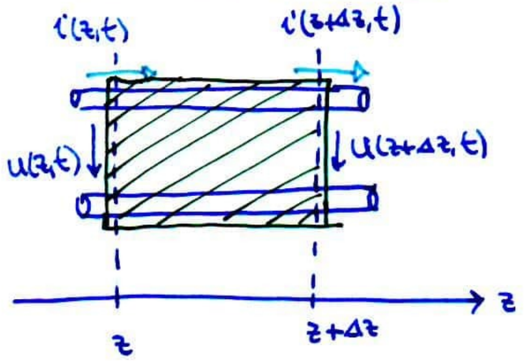
Terek E.
15. 11. 04.

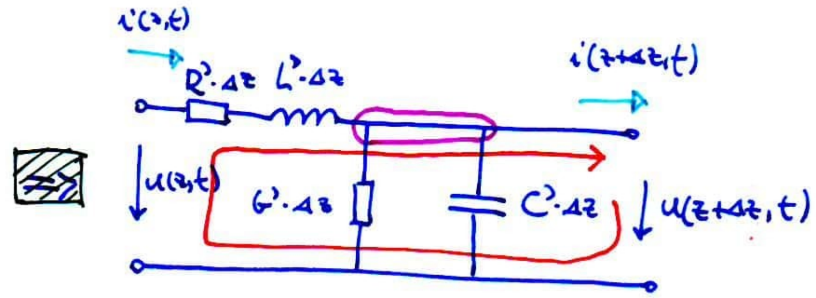
Távvezetékek +

Vezetékparaméterek: L', R', C', G'

$u(z,t)$, $i(z,t)$ = ?

Távíróegyenletek





koncentrált paraméterű helyettesítőkép

Kirchhoff-tv.:

$$\text{I.}\quad -u(z,t) + i(z,t)\cdot R'\Delta z + \frac{\partial i(z,t)}{\partial t} L'\cdot\Delta z + u(z+\Delta z,t) = 0$$

$$\text{II.}\quad -i(z,t) + i(z+\Delta z,t) + u(z+\Delta z,t)\cdot G'\Delta z + \frac{\partial u(z+\Delta z,t)}{\partial t} = 0$$

$$\Delta z \to 0 \quad\to\quad \frac{u(z+\Delta z,t) - u(z,t)}{\Delta z} = \frac{\partial u(z,t)}{\partial z}$$

$$\boxed{\begin{aligned} \text{I:}\quad -\frac{\partial u(z,t)}{\partial z} &= R' i(z,t) + L'\cdot\frac{\partial i(z,t)}{\partial t} \\ \text{II.}\quad -\frac{\partial i(z,t)}{\partial z} &= G' u(z,t) + C'\cdot\frac{\partial u(z,t)}{\partial t} \end{aligned}}$$

: 2 változós elsőrendű parciális differenciálegyenlet-rendszer

töltési áram

szivárgási áram

35

megoldása:

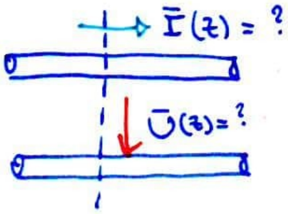
- általában nehéz
- szinuszos időbeli változás (ω)

$\left\{ \text{ált.: Fourier-tr.} \right\}$

$$u(z,t) \triangleq \hat{U}(z)\cdot\cos(\omega t + \varphi_u(z)) \equiv Re\{\underbrace{\hat{U}(z)e^{j\varphi_u(z)}}\, e^{j\omega t}\}$$

$$i(z,t) = \hat{I}(z)\cdot\cos(\omega t + \varphi_i(z)) \equiv Re\{\underbrace{\hat{I}(z)e^{j\varphi_i(z)}}_{\bar{I}(z)}\, e^{j\omega t}\}$$

komplex amplitúdó : $\bar{U}(z)$ (fazor)



- $u(z,t) \longrightarrow \bar{U}(z)$
- $i(z,t) \longrightarrow \bar{I}(z)$
- $\dfrac{\partial u(z,t)}{\partial t} \longrightarrow j\omega\bar{U}(z)$
- $\dfrac{\partial i(z,t)}{\partial t} \longrightarrow j\omega\bar{I}(z)$
- $\dfrac{\partial u(z,t)}{\partial z} \longrightarrow \dfrac{d\bar{U}(z)}{dz}$
- $\dfrac{\partial i(z,t)}{\partial z} \longrightarrow \dfrac{d\bar{I}(z)}{dz}$

=> Távíróegyenletek =>

$$\boxed{\begin{aligned} \text{I.}\quad -\frac{d\bar{U}(z)}{dz} &= (R'+j\omega L')\bar{I}(z) \\ \text{II.}\quad -\frac{d\bar{I}(z)}{dz} &= (G'+j\omega C')\bar{U}(z) \end{aligned}}$$

$$\to \text{I.}:\quad -\frac{d^2\bar{U}(z)}{dz^2} = (R'+j\omega L')\underbrace{(-G'-j\omega C')\bar{U}(z)}_{-\frac{dI(z)}{dz}} \Rightarrow$$

$$\Rightarrow \quad \frac{d^2\bar{U}(z)}{dz^2} - \underbrace{(R'+j\omega L')(G'+j\omega C')}_{:=\gamma^2}\cdot\bar{U}(z) = 0$$

$$\boxed{\begin{aligned} \text{I.}:\quad \frac{d^2\bar{U}(z)}{dz^2} - \gamma^2\bar{U}(z) &= 0 \\ \text{II.}:\quad \frac{d^2\bar{I}(z)}{dz^2} - \gamma^2\bar{I}(z) &= 0 \end{aligned}}$$

Helmholtz-egyenletek

közönséges másodrendű diff. egy.

Re

$$\left.\begin{aligned}&\frac{d^2\bar{U}(z)}{dz^2}-\gamma^2\bar{U}(z)=0\\[1em]&\frac{d^2\bar{I}(z)}{dz^2}-\gamma^2\bar{I}(z)=0\end{aligned}\right\}\quad \lambda^2-\gamma^2=0 \;\to\; \lambda=\pm\gamma$$

Általános megoldás:

$$\left[\begin{aligned}&\bar{U}(z)=\underline{U_0^+e^{-\gamma z}}+U_0^-e^{\gamma z}\\&\bar{I}(z)=I_0^+e^{-\gamma z}+I_0^-e^{\gamma z}\end{aligned}\right.$$

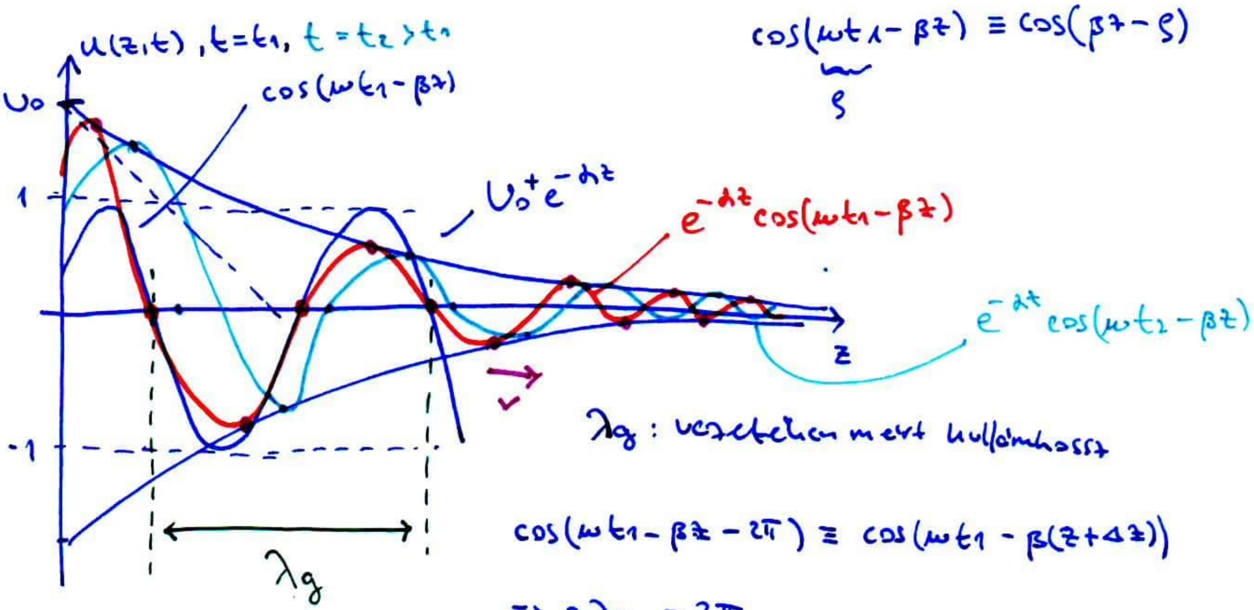
, ahol $\left.\begin{matrix}U_0^+, U_0^-\\ I_0^+, I_0^-\end{matrix}\right\}$ tetszőleges konstansok

Megoldás értelmezése

$$u(z,t)=\underbrace{Re\{U_0^+e^{-\gamma z}e^{j\omega t}\}}_{u^+(z,t)}+\underbrace{Re\{U_0^-e^{\gamma z}e^{j\omega t}\}}_{u^-(z,t)}$$

U_0^+ valós, jelölés: $\gamma=\alpha+j\beta$

$$\boxed{u^+(z,t)=U_0^+\cdot e^{-\alpha z}\cdot\cos\left(\omega t-\beta z\right)}\quad : \alpha,\beta>0$$



$$\cos(\underbrace{\omega t_1}_{\varphi}-\beta z)\equiv\cos(\beta z-\varphi)$$

λ_g: vezetéken mért hullámhossz

$$\cos(\omega t_1-\beta z-2\pi)\equiv\cos(\omega t_1-\beta(z+\Delta z))$$

$$\Rightarrow \beta\lambda_g=2\pi$$

$$\Rightarrow \boxed{\lambda_g=\frac{2\pi}{\beta}}$$

„+z" irányban haladó exponenciálisan csillapodó hullám hullámhossz

$\Leftarrow$

$$\omega \Rightarrow f = \frac{\omega}{2\pi} \rightarrow T = \frac{2\pi}{\omega}$$

$$v = \frac{\lambda_g}{T} = \frac{2\pi}{\beta} \cdot f = \frac{\omega}{\beta}$$

$$\boxed{v = \frac{\omega}{\beta}}$$: Hullám fázissebessége $\sim c$

$\alpha = Re\{\gamma\}$: csillapítási tényező $[\alpha] = \frac{1}{m}$

$\beta = Im\{\gamma\}$: fázistényező $[\beta] = \frac{1}{m}$

Teljes megoldás :

$$u(z,t) = u^+(z,t) + u^-(z,t)$$

→ „$-z$" irányban terjedő csill. hull.

→ „$+z$" irányban terjedő csill. hull.

Feszültség és áram kapcsolata :

$$\bar{U}(z) = U_0^+ e^{-\gamma z} \quad , \quad -\frac{d\bar{U}(z)}{dz} = (R' + j\omega L') \cdot \bar{I}(z)$$

$$\bar{I}(z) = -\frac{1}{R' + j\omega L'} - \gamma U_0^+ e^{-\gamma z} + \gamma U_0^- e^{\gamma z} \quad (=)$$

$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$: terjedési tényező

$$(=) \; \underbrace{\sqrt{\frac{G' + j\omega C'}{R' + j\omega L'}} \cdot U_0^+ e^{-\gamma z}}_{I_0^+} - \underbrace{\sqrt{\frac{G' + j\omega C'}{R' + j\omega L'}} \cdot U_0^- e^{\gamma z}}_{I_0^-}$$

$\Rightarrow$

ez

$$\underbrace{\sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}}}_{} = \frac{U_0^+}{I_0^+} = -\frac{U_0^-}{I_0^-}$$

Z_0 : hullámimpedancia

Def: $Z_0 \triangleq \frac{U_0^+}{I_0^+} = -\frac{U_0^-}{I_0^-}$

[A $+z$ irányban terjedő fesz. és áram hullám komplex amplitúdójának a hányadosa.

Ideális tápvezeték (veszteségmentes)

- $R' = 0$
- $G' = 0$
- Terjedési tényező : $\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')} = j\omega\sqrt{L'C'}$ → $\alpha = 0$, $\boxed{\beta = \omega\sqrt{L'C'}}$
- Hullámimpedancia : $\boxed{Z_0 = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}} = \sqrt{\frac{L'}{C'}}}$: tisztán valós konstans
- Fázissebesség: $v = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}$: frekvencia független

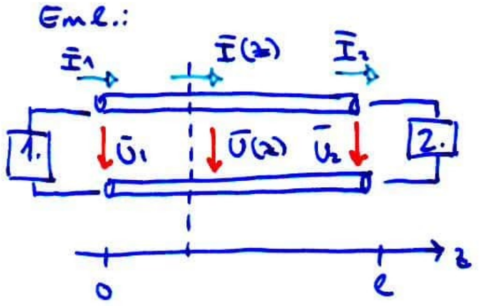
tetszőleges közegben : $L'C' = \mu_0(\varepsilon_0\varepsilon_r)$

$$v = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\varepsilon_r}} \cdot \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{c}{\sqrt{\varepsilon_r}}$$

Légszigetelésű tápvez.: $\varepsilon_r = 1 \Rightarrow \boxed{v = c}$

Terch E.
15. 11. 05.

Eml.:



$$\bar U(z) = U_0^+ e^{-\gamma z} + U_0^- e^{\gamma z}$$

$$\bar I(z) = \frac{U_0^+}{Z_0} e^{-\gamma z} - \frac{U_0^-}{Z_0} e^{\gamma z}$$

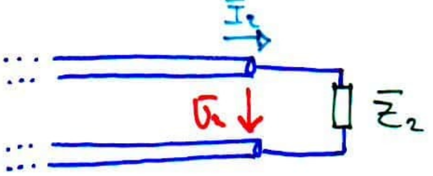
Lezárt távvezeték

$$\left.\begin{aligned} \bar U_1 &= \bar U(z=0), \quad \bar U_2 = \bar U(z=l) \\ \bar I_1 &= \bar I(z=0), \quad \bar I_2 = I(z=l) \end{aligned}\right\}$$

adott: kétpólus karakterisztikák

$$\left.\begin{aligned} \bar U_1 &= \bar U_1(\bar I_1) \\ \bar U_2 &= \bar U_2(\bar I_2) \end{aligned}\right\} \Rightarrow U_0^+, U_0^- \text{ meghatározható}$$

Reflexiós tényező a vezeték végén



Z_0, γ adott.

$\bar U_2 = \bar Z_2 \cdot \bar I_2$

$$\bar U_2 = U_0^+ e^{-\gamma l} + U_0^- e^{\gamma l}$$

$$\bar I_2 = \frac{U_0^+}{Z_0} e^{-\gamma l} - \frac{U_0^-}{Z_0} e^{\gamma l}$$

$\vdots$

$$\frac{1}{\frac{1}{\bar Z_2}} = \frac{U_0^+ e^{-\gamma l} + U_0^- e^{\gamma l}}{\frac{U_0^+}{Z_0} e^{-\gamma l} - \frac{U_0^-}{Z_0} e^{\gamma l}}$$

$\vdots$

$$\underbrace{\frac{U_0^- e^{\gamma l}}{U_0^+ e^{-\gamma l}}} = \frac{\bar Z_2 - Z_0}{\bar Z_2 + Z_0}$$

$$\boxed{\bar r_2 \triangleq \frac{\bar Z_2 - Z_0}{\bar Z_2 + Z_0}}$$ Reflexiós tényező a vezeték végén

$U_0^- e^{\gamma l}$: „-z" irányba haladó fesz. hullám komplex amplitúdója a vezeték **végén**

Passzív impedancia:

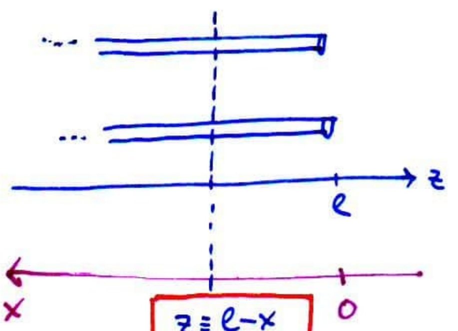
$$\bar{Z}_2 = R_2 + jX_2, \quad R_2 \geq 0, \quad Z_0 > 0 \text{ és valós}$$

$$\bar{\Gamma}_2 = \frac{R_2 + jX_2 - Z_0}{R_2 + jX_2 + Z_0} = \frac{(R_2 - Z_0) + jX_2}{(R_2 + Z_0) + jX_2}$$

$\rightarrow \underline{|\bar{\Gamma}_2| \leq 1}$ és $|\bar{\Gamma}_2| = 1 \Longleftrightarrow R_2 = 0$: teljes visszaverődés

és $|\bar{\Gamma}_2| = 0 \Longleftrightarrow R_2 = Z_0$: illesztett lezárás

Új koordinátarendszer:



$$\boxed{\begin{gathered} z \equiv l - x \\ \downarrow \\ x \equiv l - z \end{gathered}}$$

$$\bar{U}(x) = U_0^+ e^{-\gamma(l-x)} + U_0^- e^{\gamma(l-x)} =$$

$$= \underbrace{U_0^+ e^{-\gamma l}}_{\text{jobbra}} e^{\gamma x} + \underbrace{U_0^- e^{\gamma l}}_{\text{balra haladó hullám}} e^{-\gamma x}$$

$$\downarrow \qquad\qquad \downarrow$$

U_2^+ $\qquad$ U_2^- : vezeték végén értelmezett komplex amplitúdók

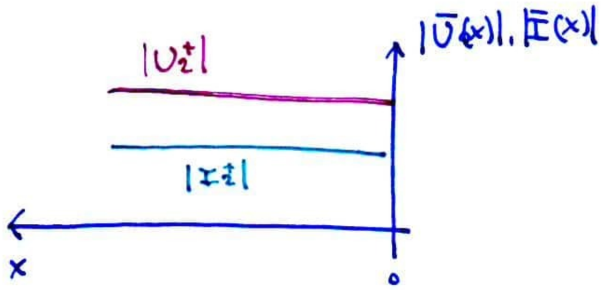
$$\bar{I}(x) = \frac{U_2^+}{Z_0} e^{\gamma x} - \frac{U_2^-}{Z_0} e^{-\gamma x}$$

Feszültség és áramelosztás speciális lezárások mellett ideális távvezetékben, x koord. rendszerben

a) <u>Illesztett</u>: $Z_2 = Z_0 \Rightarrow r_2 = 0$

$$r_2 \triangleq \frac{U_2^-}{U_2^+} \Rightarrow \boxed{U_2^- = 0}$$

$$\Rightarrow \bar{U}(x) = U_2^+ e^{j\beta x} \quad ; \quad \bar{I}(x) = \frac{U_2^+}{Z_0} e^{j\beta x}$$



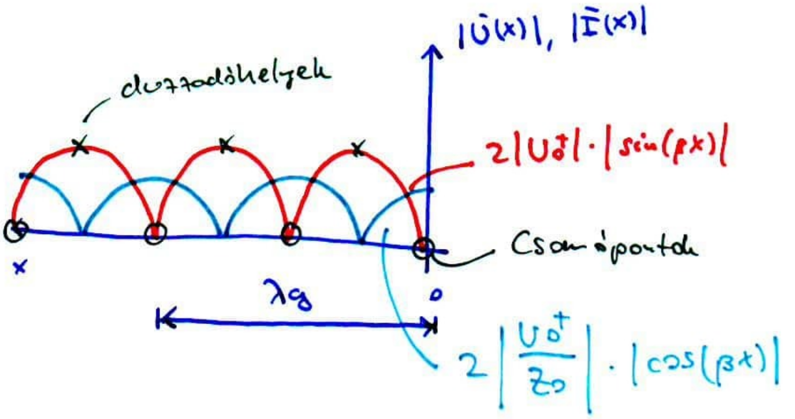
: Haladó hullám

b) <u>Rövidzár</u> ($Z_2 = 0$) $\Rightarrow r_2 = -1$

$$\boxed{U_2^- = -U_2^+}$$

$$\Rightarrow \bar{U}(x) = U_2^+ \left(e^{j\beta x} - e^{-j\beta x}\right) = 2j\, U_2^+ \sin(\beta x)$$

$$\Rightarrow \bar{I}(x) = \frac{U_2^+}{Z_0}\left(e^{j\beta x} + e^{-j\beta x}\right) = 2 \cdot \frac{U_2^+}{Z_0} \cdot \cos(\beta x)$$



: Állóhullámok

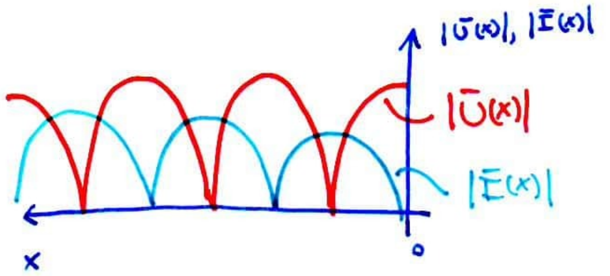
- $\bar{U}$ és $\bar{I}$ között $\frac{\lambda g}{4}$ eltolás
- Időben $\frac{T}{4}$ időnyi fáziskülönbség adódik (90°)

c) <u>Szakadás</u>: $Z_2 \to \infty \Rightarrow r_2 = 1$

$$\boxed{U_2^- = U_2^+}$$

$$\Rightarrow \bar{U}(x) = U_2^+ \left(e^{j\beta x} + e^{-j\beta x}\right) = 2U_2^+ \cos(\beta x)$$

$$\Rightarrow \bar{I}(x) = \frac{U_2^+}{Z_0}\left(e^{j\beta x} - e^{-j\beta x}\right) = 2j\frac{U_2^+}{Z_0}\sin(\beta x)$$



d) <u>Reaktancia</u>: $Z_2 = jX_2$

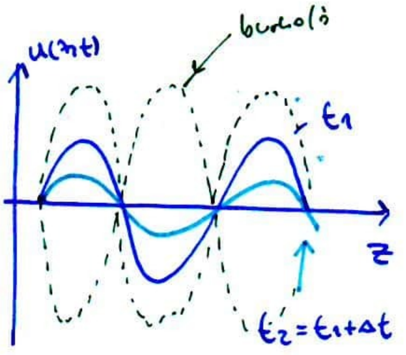
$$r_2 = \frac{jX_2 - Z_0}{jX_2 + Z_0} = -\underbrace{\frac{Z_0 - jX_2}{Z_0 + jX_2}}_{\text{komplex konjugáltak}} = e^{j\sigma} \rightarrow \boxed{U_2^- = e^{j\sigma} U_2^+}$$

$$\Rightarrow \bar{U}(x) = U_2^+\left(e^{j\beta x} + e^{j\sigma} \cdot e^{-j\beta x}\right) = U_2^+ \cdot e^{j\frac{\sigma}{2}}\left(e^{j(\beta x - \frac{\sigma}{2})} + e^{-j(\beta x - \frac{\sigma}{2})}\right) =$$

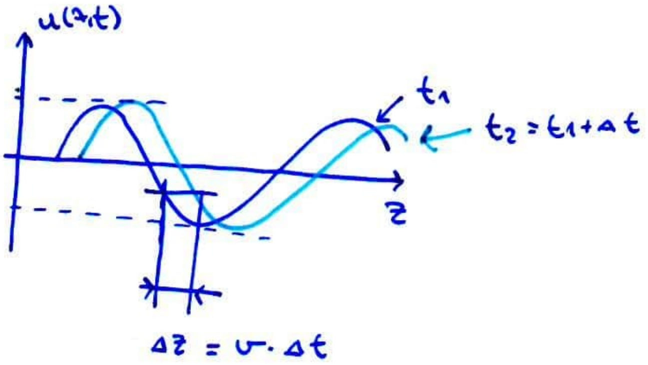
$$= 2U_2^+ e^{j\frac{\sigma}{2}} \cdot \cos\left(\beta x - \frac{\sigma}{2}\right)$$: állóhullám (nem szállít hatásos teljesítményt)

veszteségmentes

Állóhullám / haladóhullám felbontás



- álló -



- haladó -

• Ideális távvezeték: $(\gamma = j\beta)$ lezárás: $\bar{Z}_2 \Rightarrow \underline{\bar{r}_2 = \rho e^{j\delta}}$

$0 \le \rho \le 1, \quad -\pi \le \delta \le \pi$

$$\bar{U}(x) = U_2^+ e^{j\beta x} + \underbrace{U_2^-}_{\underline{r_2} \cdot U_2^+} e^{-j\beta x} \Leftrightarrow$$

$$\Leftrightarrow U_2^+\left[e^{j\beta x} + \underline{\rho e^{j\delta}} e^{-j\beta x}\right] = U_2^+\left[\underbrace{e^{j\beta x} - \rho e^{j\beta x}}_{(1-\rho)e^{j\beta x}} + \underbrace{\rho e^{j\beta x} + \rho e^{j\delta} \cdot e^{-j\beta x}}_{\rho e^{j\frac{\delta}{2}}\left(e^{j(\beta x - \frac{\delta}{2})} + e^{-j(\beta x - \frac{\delta}{2})}\right)}\right] \Leftrightarrow$$

$$\Leftrightarrow \underbrace{U_2^+ (1-\rho) e^{j\beta x}}_{\text{haladó h.}} + \underbrace{U_2^+ \rho e^{j\frac{\delta}{2}} \cdot 2\cos\left(\beta x - \frac{\delta}{2}\right)}_{\text{álló h.}}$$

Ha illesztés: $\rho = 0 \Rightarrow$ állóhullám $= 0$

Ha teljes lezárás: $\rho = 1 \Rightarrow$ haladóhullám $= 0$

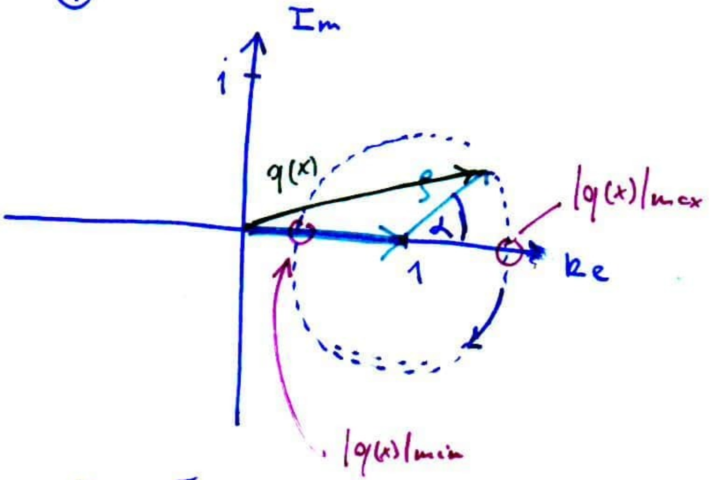
Állóhullám arány

9

- mennyire van AH és HH.

$$|\bar{U}(x)| = |U_2^+ e^{j\beta x} + \rho e^{j\delta} U_2^+ e^{-j\beta x}| =$$

$$= \underbrace{|U_2^+ e^{j\beta x}|}_{|U_2^+|} \cdot \underbrace{|1 + \rho e^{j(-2\beta x + \delta)}|}_{q(x)}$$



$$\alpha = -2\beta x + \delta$$

$$|q(x)|_{max} = 1 + \delta \quad : \quad \alpha = 0 \Rightarrow -2\beta x + \delta = 0 + k \cdot 2\pi \, , \quad k \in \mathbb{Z}$$

$$x_{max} = \frac{1}{2\beta}(\delta - 2k\pi)$$

$$|q(x)|_{min} = 1 - \delta \quad : \quad \alpha = \pi \Rightarrow -2\beta x + \delta = \pi + k 2\pi \, , \quad k \in \mathbb{Z}$$

$$x_{min} = \frac{1}{2\beta}(\delta - \pi - 2k\pi) \quad \longrightarrow \quad \frac{\pi}{2\beta} = \frac{\lambda_g}{4} \text{ különbség}$$

Def: Állóhullámarány:

$$\boxed{\sigma \triangleq \frac{|U(x)|_{max}}{|U(x)|_{min}} = \frac{1+\rho}{1-\rho} = \frac{1+|r_2|}{1-|r_2|}}$$

$$\boxed{1 \le \sigma \le \infty}$$

tiszta állóhullám

tiszta haladóhullám

Terek E.
15.11.11.

Tápvezetékek +

ism.:

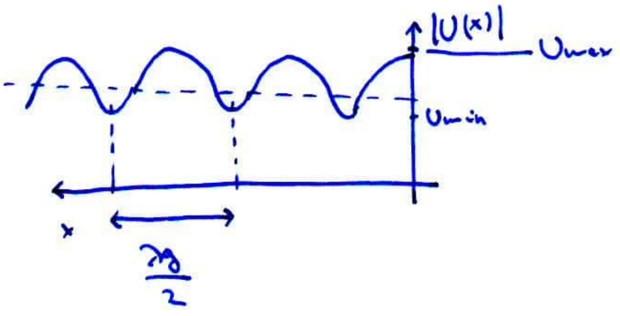
$$|U(x)| = |U_2^+| \cdot \underbrace{|1+\rho e^{-j(2\beta x-\sigma)}|}_{q(x)}$$

$$|q(x)| = |\underbrace{(1+\rho\cdot\cos(-2\beta x+\sigma))}_{a(x)} + j\underbrace{\rho\sin(-2\beta x+\sigma)}_{b(x)}| = \sqrt{a^2(x)+b^2(x)} =$$

$$= \sqrt{1+\rho^2\cos^2(-2\beta x+\sigma) + 2\rho\cos(-2\beta x+\sigma) + \rho^2\sin^2(-2\beta x+\sigma)} =$$

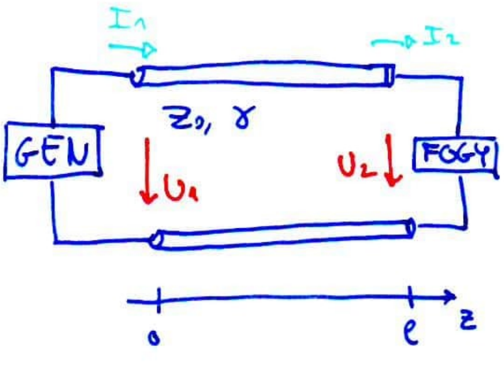
$$\rho^2$$

$$= \sqrt{1+\rho^2+2\rho\cos(-2\beta x+\sigma)}$$



A távvezeték mint kétkapu

- belső visszaverődések nem érdekesek
- generátor – terhelés kapcsolat



$$U(z) = U_0^+ e^{-\gamma z} + U_0^- e^{\gamma z}$$

$$I(z) = \frac{U_0^+}{Z_0} e^{-\gamma z} - \frac{U_0^-}{Z_0} e^{\gamma z}$$

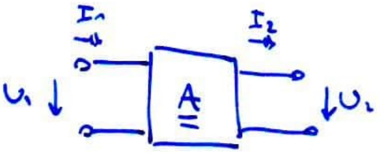
$$U_1 = U(0) \qquad U_2 = U(\ell)$$

$$I_1 = I(0) \qquad I_2 = I(\ell)$$

$\underline{\underline{A}}$ láncmátrix

$$\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 \\ \frac{1}{Z_0} & -\frac{1}{Z_0} \end{pmatrix}}_{\underline{\underline{M_1}}} \begin{pmatrix} U_0^+ \\ U_0^- \end{pmatrix} = \boxed{\underline{\underline{M_1}} \cdot \underline{\underline{M_2}}^{-1}} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$$

$$\begin{pmatrix} U_2 \\ I_2 \end{pmatrix} = \underbrace{\begin{pmatrix} e^{-\gamma \ell} & e^{\ell \gamma} \\ \frac{e^{-\gamma \ell}}{Z_0} & -\frac{e^{\gamma \ell}}{Z_0} \end{pmatrix}}_{\underline{\underline{M_2}}} \begin{pmatrix} U_0^+ \\ U_0^- \end{pmatrix} \Rightarrow \begin{pmatrix} U_0^+ \\ U_0^- \end{pmatrix} = \underline{\underline{M_2}}^{-1} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$$



$$\boxed{\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = \underline{\underline{A}} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}}$$

$$\underline{\underline{A}} = \underline{\underline{M_1}} \cdot \underline{\underline{M_2}}^{-1} = \begin{pmatrix} ch(\gamma \ell) & Z_0 \cdot sh(\gamma \ell) \\ \frac{1}{Z_0} \cdot sh(\gamma \ell) & ch(\gamma \ell) \end{pmatrix} :$$

$A_{11} \equiv A_{22} \Rightarrow$ szimmetrikus

$\det \underline{\underline{A}} = ch^2(\gamma \ell) - sh^2(\gamma \ell) \equiv 1$

$\Rightarrow$ reciprok kétkapu

$$ch(\gamma \ell) = \frac{e^{\gamma \ell} + e^{-\gamma \ell}}{2}$$

$$sh(\gamma \ell) = \frac{e^{\gamma \ell} - e^{-\gamma \ell}}{2}$$

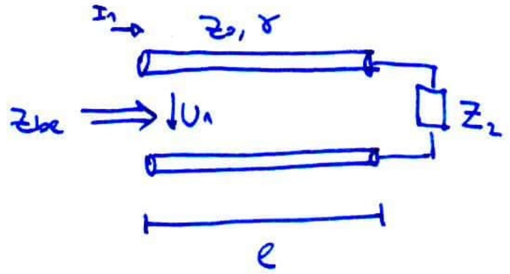
$\Rightarrow$ Reciprok és szimmetrikus kétkapu

Ideális távvezeték esetén:

$$\boxed{\begin{aligned} \gamma &= j\beta \\ ch(j\beta l) &\equiv \cos(\beta l) \\ sh(j\beta l) &\equiv j\sin(\beta l) \end{aligned}} \Rightarrow \underline{\underline{A}} = \begin{pmatrix} \cos(\beta l) & jZ_0 \sin(\beta l) \\ -\frac{j}{Z_0}\sin(\beta l) & \cos(\beta l) \end{pmatrix}$$

: $\left.\begin{aligned} A_{11} &\equiv A_{22} \\ \det \underline{\underline{A}} &= 1 \end{aligned}\right\} \Rightarrow$ Két független lánc-paraméter.

Bemeneti impedancia:



$$Z_{be} \triangleq \frac{U_1}{I_1} = ?$$

$$\boxed{\frac{U_1}{I_1}} = \frac{U_2\, ch(\gamma l) + I_2 Z_0\, sh(\gamma l)}{U_2 \cdot \frac{1}{Z_0}\, sh(\gamma l) + I_2\, ch(\gamma l)} = \frac{Z_2 \cdot ch(\gamma l) + Z_0 \cdot sh(\gamma l)}{\frac{Z_2}{Z_0}\, sh(\gamma l) + ch(\gamma l)} = \boxed{Z_0 \frac{Z_2 + Z_0\, th(\gamma l)}{Z_0 + Z_2\, th(\gamma l)}}$$

Ideális távvezeték:

$$\gamma = j\beta \longrightarrow th(\gamma l) \equiv j\, tg(\gamma l)$$

$$\boxed{Z_{be} = Z_0 \frac{Z_2 + jZ_0\, tg(\gamma l)}{Z_0 + jZ_2\, tg(\gamma l)}}$$

(Pl)

① Ideális: $Z_2 = Z_0$ (illesztett lezárás)

$\Rightarrow Z_{be} = Z_0$

② Ideális: $Z_2 = jX$ (tisztán reaktáns: $X>0 \Rightarrow$ tekercs)

$$Z_{be} = Z_0 \frac{jX + jZ_0 \cdot tg(\beta l)}{Z_0 - X\, tg(\beta l)}$$: Képzetes!

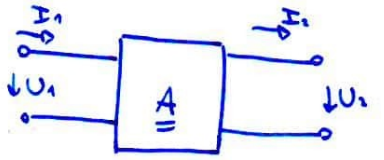
a) $l = \lambda_g \Rightarrow \beta l = 2\pi \longrightarrow tg\, 2\pi = 0$

$\Rightarrow Z_{be} = jX$

b) $l = \frac{\lambda_g}{4} \Rightarrow \beta l = \frac{\pi}{2} \longrightarrow tg\,\frac{\pi}{2} \longrightarrow \infty$

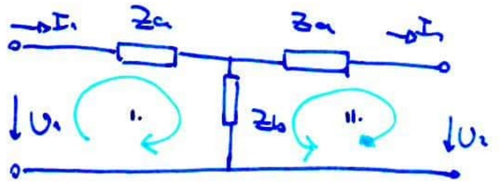
$$\Rightarrow Z_{be} = Z_0 \frac{Z_2\cos(\beta l) + jZ_0\sin(\beta l)}{Z_0\underbrace{\cos(\beta l)}_{0} + jZ_2\underbrace{\sin(\beta l)}_{1}} = Z_0 \frac{jZ_0}{jjX} = -j\frac{Z_0^2}{X}$$

<u>Távvezeték helyettesítőképei</u>



egyszerű, koncentrált paraméterű helyettesítőkép

① T-tag

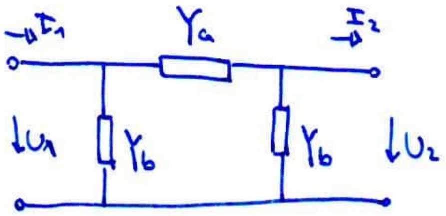


$$\left.\begin{aligned} U_1 &= A_{11}U_2 + A_{12}I_2 \\ I_1 &= A_{21}U_2 + A_{22}I_2 \end{aligned}\right\}$$

$$\left.\begin{aligned} \text{I.}: &\quad -U_1 + I_1 Z_a + (I_1 - I_2)Z_b = 0 \\ \text{II.}: &\quad -U_2 + Z_a(-I_2) + Z_b(I_1 - I_2) = 0 \end{aligned}\right\} \quad Z_a = Z_0\frac{ch(\gamma l) - 1}{sh(\gamma l)} \qquad Z_b = Z_0\frac{1}{sh(\gamma l)}$$

$\Rightarrow$ Egy adott ω-n működik!

② Π-tag:

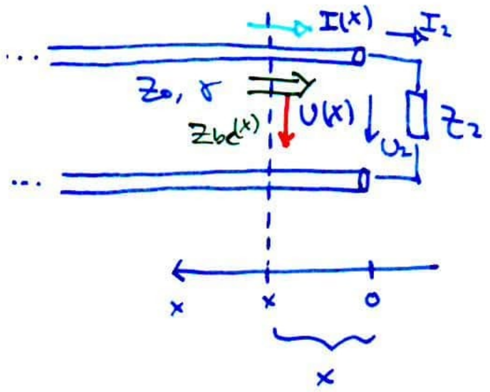


$Y_a = \dots$

$Y_b = \dots$

Egy adott ω-n működik.

Bemeneti impedancia és reflexiós tényező, mint a hely függvénye



$$\begin{pmatrix} U(x) \\ I(x) \end{pmatrix} = \begin{pmatrix} ch(\gamma x) & Z_0 \cdot sh(\gamma x) \\ \frac{1}{Z_0} sh(\gamma x) & ch(\gamma x) \end{pmatrix} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$$

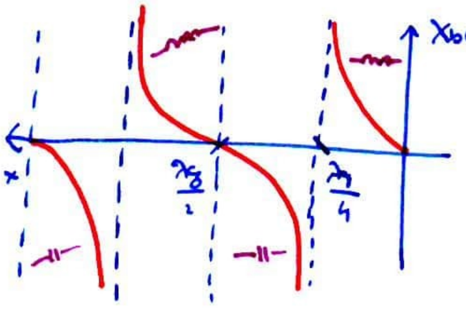
$$Z_{be}(x) \triangleq \frac{U(x)}{I(x)} = Z_0 \frac{Z_2 + Z_0 \, th(\gamma x)}{Z_0 + Z_2 \, th(\gamma \underset{x}{x})}$$

Pl. id. TV. rz.-ral lezárva: $Z_2 = 0$

↳ $\gamma = j\beta$

$$Z_{be}(x) = Z_0 \frac{0 + jZ_0 tg(\beta x)}{Z_0 + 0} = \underbrace{jZ_0 tg(\beta x)}_{X_{be}(x)}$$: tisztán képzetes

⇐ rz. nem disszipál (teljes sem)



(R2) $Z_2 = R_2$ rezisztív

$$Z_{be}(x) = Z_0 \frac{R_2 + jZ_0 \operatorname{tg}(\beta x)}{Z_0 + jR_2 \operatorname{tg}(\beta x)} = R_{be}(x) + jX_{be}(x)$$

$$\boxed{\min\left(R_2, \frac{Z_0^2}{R_2}\right) \leq R_{be}(x) \leq \max\left(R_2, \frac{Z_0^2}{R_2}\right)}$$

$$r_2 \triangleq \frac{U_2^-}{U_2^+}$$

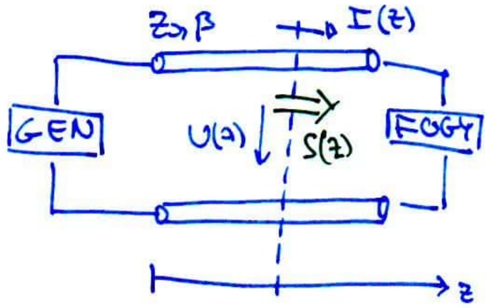
$$r(x) \triangleq \frac{U_2^- e^{-\gamma x}}{U_2^+ e^{\gamma x}}$$: Helmholtz-ból $\left(U(x) = U_2^+ e^{\gamma x} + U_2^- e^{-\gamma x}\right)$.

$$\rightarrow = \frac{Z_{be}(x) - Z_0}{Z_{be}(x) + Z_0}$$

Ideális: $\gamma = j\beta \longrightarrow r_2(x) = \frac{U_2^-}{U_2^+} e^{-2j\beta x} \Rightarrow$

$\Rightarrow |r_2(x)| = |r_2| =$ konstans.

Teljesítményáramlás ideális tápvezetékben



$$S(z) = \frac{1}{2} U(z) I^*(z) =$$

$$U(z) = U_0^+ e^{-j\beta z} + U_0^- e^{j\beta z}$$

$$I(z) = \frac{U_0^+}{Z_0} e^{-j\beta z} - \frac{U_0^-}{Z_0} e^{j\beta z}$$

$$= \frac{1}{2}\left[\underbrace{\frac{|U_0^+|^2}{Z_0} - \frac{|U_0^-|^2}{Z_0}}_{\text{valós}} \underbrace{- \frac{U_0^+ U_0^{-*}}{Z_0} e^{-2j\beta z} + \frac{U_0^- U_0^{+*}}{Z_0} e^{2j\beta z}}_{\text{tisztán képzetes}} \right]$$

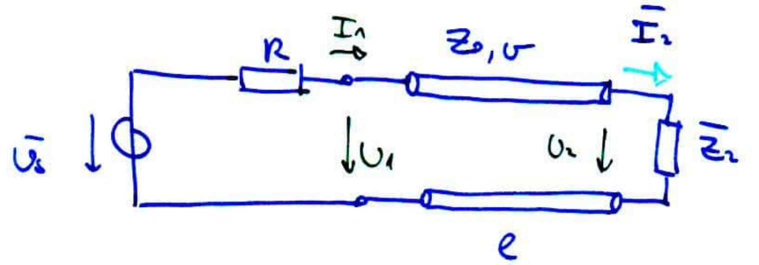
$$\Rightarrow P(z) = \mathrm{Re}\{S(z)\} = \frac{|U_0^+|^2 - |U_0^-|^2}{2Z_0} =$$

$$= |U_0^+|^2 \cdot \frac{1}{2Z_0}\left(1 - \underbrace{|\Gamma(x)|^2}_{\text{konstans}}\right) \Rightarrow \text{helyfüggetlen}$$

Terek Gy.
15. 11. 12.

Távvezetékek 6.2 – 6.9

1 Láncharakterisztika



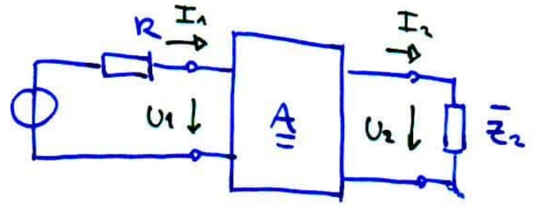
$\ell = 1\,km$ $\quad R = 10\,\Omega$

$Z_0 = 50\,\Omega$ $\quad \bar{Z}_2 = (50 + j50)\,\Omega$

$v = 2{,}2 \cdot 10^8 \frac{m}{s}$ $\quad \bar{U}_s = 100\,V$

$\omega = 2\pi \cdot 10^5 \frac{1}{s}$ $\quad \bar{I}_2 = ?$

$\Rightarrow$



$\rightarrow \begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = \underline{\underline{A}} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$ I., II.

$U_1 = U_s - R I_1$ III.

$U_2 = Z_2 \cdot I_2$ IV.

4 függ. egy.

id. TV:

$$\underline{\underline{A}} = \begin{pmatrix} \cos\beta\ell & j\sin\beta\ell \\ \frac{j}{Z_0}\sin\beta\ell & \cos\beta\ell \end{pmatrix}$$

$$\boxed{\beta = \frac{\omega}{v}} = \frac{2\pi \cdot 10^5}{2{,}2 \cdot 10^8} \frac{1}{m} = \dots \quad \beta \checkmark$$

U_1: III. $\rightarrow$ I.

U_2: IV. $\rightarrow$ I., II.

I., II. : I_1, I_2 ✓

$\Rightarrow$

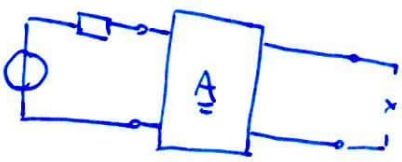
$$\underline{\underline{A}} = \begin{pmatrix} -0{,}9595 & j14{,}09\,\Omega \\ j5{,}635\,mS & -0{,}9595 \end{pmatrix} \nearrow$$

$\Rightarrow$

$$I_2 = 1{,}473 e^{j2{,}666}\,A$$

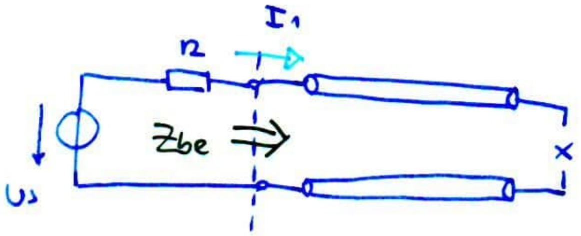
$$i_2(t) = 1{,}473 \cdot \cos(\omega t + 2{,}666)\,A$$

2 Bemeneti impedancia

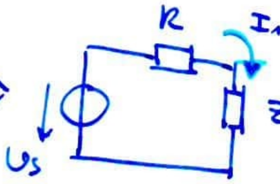


Előző feladat : $Z_2 \to \infty$

$I_1 = ?$



$\Rightarrow$



$$I_1 = \frac{U_s}{R + Z_{be}}$$

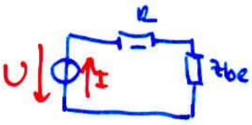
$$Z_{be} = Z_0 \frac{Z_2 + Z_0 j \operatorname{tg}\beta\ell}{Z_0 + Z_2 j \operatorname{tg}\beta\ell} \xrightarrow[Z_2 \to \infty]{} Z_0 \frac{1}{j \operatorname{tg}\beta\ell} = j170{,}3\ \Omega \quad \Rightarrow$$

tisztán képzetes: id. TV.-en ~~nem~~ visszaverődik teljesítmény.

$$\Rightarrow \quad I_1 = \frac{100}{10 + j170{,}3} = 0{,}568\, e^{-j1{,}512}\ \text{A}$$

3 Forrás hatásos és meddő teljesítménye az előző példában

$\bar{S} = \frac{1}{2} \bar{U}_s \cdot \bar{I}_1^*$ $\Rightarrow$ CSAK, HA A FESZ. ÉS AZ ÁRAM EGY IRÁNYBA FOLYIK!!!



$\Leftrightarrow \quad \bar{S} = \frac{1}{2} \bar{U}_s (-\bar{I}_1^*)$

$\Rightarrow \quad \bar{S} = -\frac{1}{2} \bar{U}_s \cdot \bar{I}_1^* = (-1{,}72 - j29{,}3)\ \text{VA}$

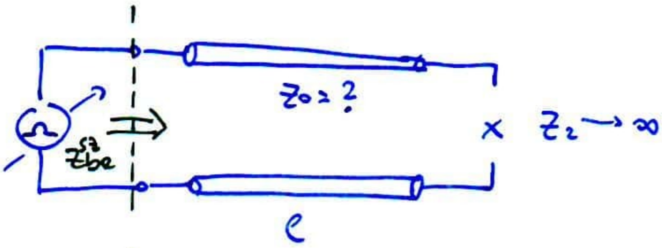
$P = \mathrm{Re}\{\bar{S}\} = -1{,}72\ \text{W} < 0 \Rightarrow$ TERMEL

$Q = \mathrm{Im}\{\bar{S}\} = -29{,}3\ \text{var}$

$P_R = \frac{1}{2} R |\bar{I}_1|^2 = \dots = 1{,}72\ \text{W}$

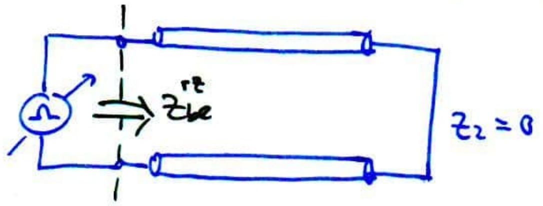
$Q_{TV} = \frac{1}{2} \mathrm{Im}\{Z_{be}\} \cdot |\bar{I}_1|^2 = \dots = 29{,}3\ \text{var}$

$\boxed{4}$ Hullámimpedancia mérése Ω-mérővel



$$Z_{be}^{sz} = Z_0 \frac{1}{j\,tg\beta l}$$

$$Z_{be}^{rz} = Z_0 \frac{Z_2 + jZ_0\,tg\beta l}{Z_0 + jZ_2\,tg\beta l} = jZ_0\,tg\beta l$$



$$\boxed{Z_0^2 = Z_{be}^{sz} \cdot Z_{be}^{rz}}$$

$$\omega = ? \leftarrow \frac{tg\beta l}{tg\beta l} = 1$$

$$\beta = \frac{\omega}{v} = \frac{2\pi f}{v}$$

$$\left.\begin{array}{l} v \sim 10^8 \frac{m}{s} \\ l \sim 10\,m \\ f \sim 100\,Hz \end{array}\right\} \quad \beta l \sim 10^{-5}$$

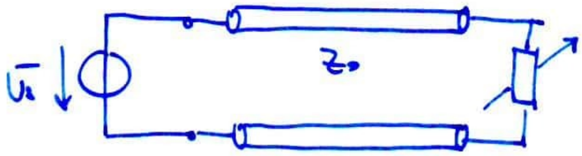
$$\downarrow$$

$$tg\beta l \sim 10^{-5} \Rightarrow$$

$$\Rightarrow \frac{\sim 10^{-5}}{\sim 10^{-5}} \stackrel{?}{=} 1$$

Ha $\boxed{\lambda \sim l}$ $\Rightarrow$ lehet jó mérést végezni

5 Állóhullámarány



$R_2 = n \cdot Z_0$

$0 \le n \le \infty$

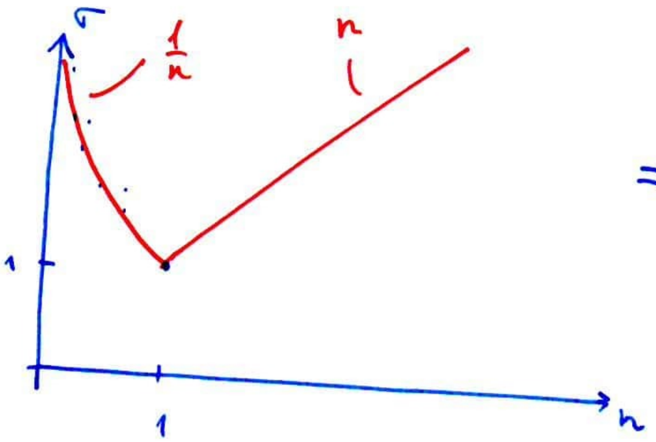
$\sigma(n) = ?$

$$\sigma \triangleq \frac{U_{max}}{U_{min}} = \frac{1+|r_2|}{1-|r_2|}$$

$$r_2 : \frac{R_2 - Z_0}{R_2 + Z_0} = \frac{n-1}{n+1} \longrightarrow |r_2| \begin{cases} \dfrac{n-1}{n+1}, & n \ge 1 \\ \dfrac{1-n}{n+1}, & n < 1 \end{cases} \Rightarrow$$

$$\Rightarrow \quad \sigma = \frac{1+|r_2|}{1-|r_2|} \begin{cases} \dfrac{1+\frac{n-1}{n+1}}{1-\frac{n-1}{n+1}} = n \\ \dfrac{1+\frac{1-n}{n+1}}{1-\frac{1-n}{n+1}} = \dfrac{1}{n} \end{cases} \Rightarrow$$

$\Rightarrow$



$\Rightarrow \boxed{\sigma \ge 1}$

$\boxed{6}$ Előzőből: $n = 0{,}2$, l adott

$|U(z)| = \frac{1}{2}|U_0^+|$ $\longleftarrow$ $z = ?$ (hol teljesül)

$$U(z) = \underbrace{U_0^+ e^{-j\beta z}}_{+z} + \underbrace{U_0^- e^{j\beta z}}_{-z} \quad \text{irány}$$

$$|U(z)| = \left| U_0^+ e^{-j\beta z} + U_0^- e^{j\beta z} \right| = \frac{1}{2}|U_0^+|$$

$$r_2 \triangleq \frac{U_0^- e^{j\beta l}}{U_0^+ e^{-j\beta l}} = \frac{R_2 - Z_0}{R_2 + Z_0} = \frac{0{,}2 - 1}{0{,}2 + 1} = -\frac{2}{3} \quad \Longrightarrow$$

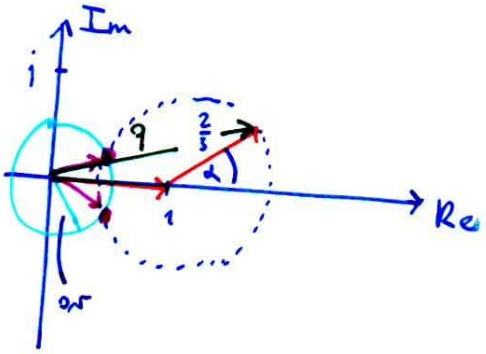
$$\Longrightarrow \quad U_0^- = U_0^+ e^{-j2\beta l} \cdot \left(-\frac{2}{3}\right)$$

$$\Longrightarrow \quad |U_0^+| \cdot \underbrace{\left| e^{-j2\beta z} - \frac{2}{3} e^{j\beta z} \cdot e^{-j2\beta l} \right|}_{p(z)} = \frac{1}{2}|U_0^+|$$

$$p(z) = e^{-j\beta z}\left(1 - \frac{2}{3} e^{+j2(\beta z - \beta l)}\right)$$

$\alpha = 2\beta(z - l) + \pi$

$$p(z) = \underbrace{\left| 1 \pm \frac{2}{3} e^{2j\beta(z-l) + j\pi} \right|}_{q(z)} \quad \longrightarrow$$



$\Longrightarrow$ $\cos^{-1}(\ldots) \Rightarrow 2\beta(z-l) + \pi = \ldots + 2k\pi$

● : ahol teljesül a feltétel

◯ : 0,5 sugarú kör ⇒ ahol metszi

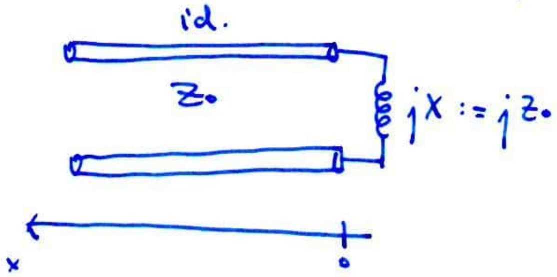
◌ - t, ott teljesül az egyenlet.

$$q = 1 + \frac{2}{3}\cos(2\beta(z-l) + \pi) + j\,\frac{2}{3}\sin(2\beta(z-l) + \pi)$$

$$|q| = \sqrt{\left(1 + \frac{2}{3}\cos(\ldots)\right)^2 + \left(\frac{2}{3}\sin(\ldots)\right)^2} = \sqrt{1 + \frac{4}{3}\cos(\ldots) + \frac{4}{9}} \quad \Longrightarrow$$

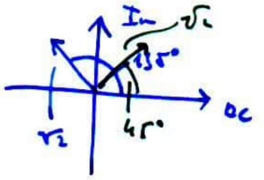
$\boxed{7}$ Tekerccsel lezárt távvezeték

$|U(x)| = ?$



$$U(x) = U_2^+ e^{j\beta x} + U_2^- e^{-j\beta x} \Leftrightarrow$$

$$\frac{U_2^-}{U_2^+} = r_2 = \frac{jZ_0 - Z_0}{jZ_0 + Z_0} = \frac{j-1}{j+1} :$$

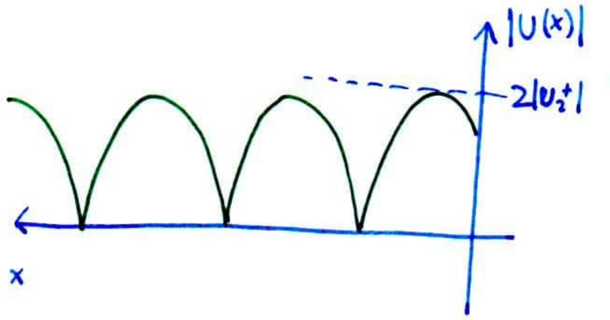


$$\Rightarrow r_2 = e^{j\frac{\pi}{2}} = j \rightarrow$$

$|r_2| = 1$

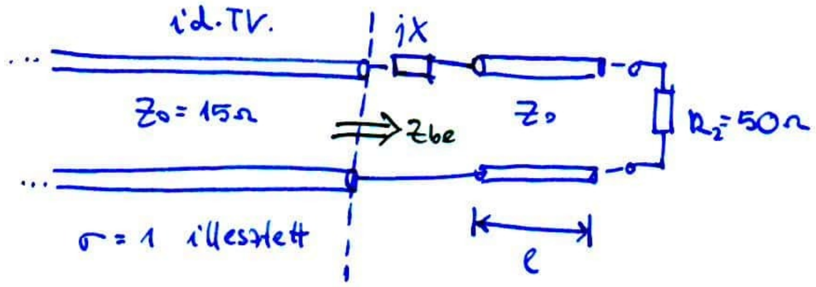
$$\Leftrightarrow U_2^+ \left(e^{j\beta x} + e^{j\frac{\pi}{2}} e^{-j\beta x}\right) = U_2^+ e^{j\frac{\pi}{4}} \left(e^{j(\beta x - \frac{\pi}{4})} + e^{-j(\beta x - \frac{\pi}{4})}\right) \Leftrightarrow$$

$$\Leftrightarrow |U(x)| = |U_0^+| \cdot 2\cos\left(\beta x - \frac{\pi}{4}\right)$$



• : $\frac{\pi}{4}$-gyel eltolt cos(...)

8 Illesztés. adott ω-n $\Rightarrow$ $\beta = \frac{\omega}{v}$ adott



$jX = ?$, $\ell = ?$: $Z_{be} = Z_0$ legyen!

$$Z_{be} = jX + Z_0 \frac{R_2 + jZ_0 \operatorname{tg}\beta\ell}{Z_0 + jR_2 \operatorname{tg}\beta\ell} := Z_0$$

Re: $$\operatorname{Re}\left\{ Z_0 \frac{R_2 + jZ_0 \operatorname{tg}\beta\ell}{Z_0 + jR_2 \operatorname{tg}\beta\ell} \right\} := Z_0 \quad \text{(I)}$$

Im: $$X + \operatorname{Im}\left\{ Z_0 \frac{\cdots}{\cdots} \right\} := 0 \quad \text{(II)}$$

(I) $\rightarrow$ $\operatorname{tg}\beta\ell = \ldots \rightarrow \beta\ell = \ldots \rightarrow \ell = \ldots$ ✓

$\ell \rightarrow$ (II) $\rightarrow$ $X = \ldots$ ✓

Terek E.
15. 11. 18.

Elektromágneses hullámok

Maxwell:

meghatározó szerepű

I. $\operatorname{rot}\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

II. $\operatorname{rot}\bar{E} = -\frac{\partial \bar{B}}{\partial t}$

III. $\operatorname{div}\bar{B} = 0$

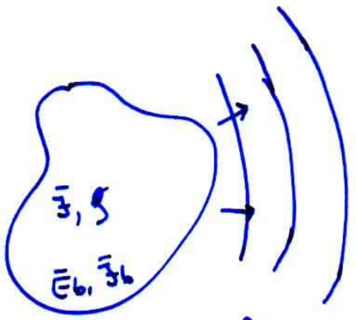
IV. $\operatorname{div} D = \rho$

$\bar{D} = \varepsilon\bar{E}$

$\bar{B} = \mu\bar{H}$

$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$

$\frac{\partial}{\partial t}$ nem hanyagolható el.



forrásmentes tér

homogén közeg, anizotróp : ε, μ, σ

Alapegyenletek

I. $\operatorname{rot}\operatorname{rot}\bar{H} = \sigma\cdot\operatorname{rot}\bar{E} + \varepsilon\frac{\partial}{\partial t}\operatorname{rot}\bar{E} \overset{\text{II.}}{=} -\mu\cdot\sigma\frac{\partial}{\partial t}\bar{H} - \varepsilon\mu\frac{\partial^2}{\partial t^2}\bar{H}$

II. $\operatorname{rot}\operatorname{rot}\bar{E} = -\mu\frac{\partial}{\partial t}\operatorname{rot}\bar{H} \underset{\text{I.}}{=} -\mu\sigma\frac{\partial}{\partial t}\bar{E} - \varepsilon\mu\frac{\partial^2}{\partial t^2}\bar{E}$

$\boxed{\operatorname{rot}\operatorname{rot}\bar{x} = \operatorname{grad}\operatorname{div}\bar{x} - \Delta\bar{x}}$: azonosság $\Rightarrow$

$$\Rightarrow \boxed{\begin{aligned} \Delta\bar{E} - \mu\sigma\frac{\partial}{\partial t}\bar{E} - \mu\varepsilon\frac{\partial^2}{\partial t^2}\bar{E} = 0 \\ \Delta\bar{H} - \mu\sigma\frac{\partial}{\partial t}\bar{H} - \mu\varepsilon\frac{\partial^2}{\partial t^2}\bar{H} = 0 \end{aligned}}$$

: Hullámegyenletek

homogén (nincs gerjesztés)

parciális diff. egyenletek

(másodrendű)

Megj.:

skalárok

$\Delta\bar{E} \hat{=} \text{grad div}\,\bar{E} - \text{rot rot}\,\bar{E}$

$\bar{E}(x,y,z,t) \Rightarrow \Delta\bar{E} = \hat{e}_x\Delta E_x + \hat{e}_y\Delta E_y + \hat{e}_z\Delta E_z$

$\parallel$

$\hat{e}_x E_x(x,y,z,t) + \hat{e}_y E_y(x,y,z,t) + \hat{e}_z E_z(x,y,z,t)$

$$\Delta E_x(x,y,z,t) = \frac{\partial^2}{\partial x^2}E_x + \frac{\partial^2}{\partial y^2}E_x + \frac{\partial^2}{\partial z^2}E_x$$

„Legegyszerűbb" megoldás : a síkhullám

$\bar{E}(x,y,z,t) \hat{=} \hat{e}_x E_x(z,t)$: z irányba terjed

Hullámegyenlet:

$$\Delta\bar{E} = \hat{e}_x\Delta E_x = \hat{e}_x\frac{\partial^2}{\partial z^2}E_x(z,t)$$

$$\frac{\partial}{\partial t}\bar{E} = \hat{e}_x\frac{\partial}{\partial t}E_x(z,t)$$

$$\frac{\partial^2}{\partial t^2}\bar{E} = \hat{e}_x\frac{\partial^2}{\partial t^2}E_x(z,t)$$

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} - \mu\sigma\frac{\partial E_x}{\partial t} - \mu\varepsilon\frac{\partial^2 E_x}{\partial t^2} = 0}$$

[$\sigma = 0$: ideális szigetelő]

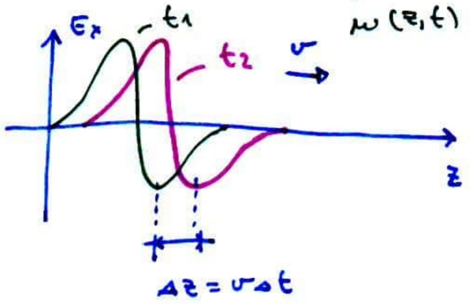
($\sigma = \infty$: ideális vezető)

σ „kicsi": veszteséges szigetelő

σ „nagy" : veszteséges vezető

Terjedő hullám-e?

$$E_x(z,t) \overset{?}{=} E_x(\underbrace{z-vt}_{w(z,t)})$$



$$\frac{\partial}{\partial z}E_x(w(z,t)) = E_x' \underbrace{\frac{\partial w}{\partial z}}_{1}$$

$$\Rightarrow$$

$$\frac{\partial}{\partial t}E_x(w(z,t)) = E_x' \underbrace{\frac{\partial w}{\partial t}}_{v}$$

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon\frac{\partial^2 E_x}{\partial t^2} = \left(\frac{\partial}{\partial z} - \sqrt{\mu\varepsilon}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t} + \sqrt{\mu\varepsilon}\frac{\partial}{\partial t}\right)E_x = 0$$

$$\frac{\partial E_x}{\partial z} \pm \sqrt{\mu\varepsilon}\frac{\partial E_x}{\partial t} = 0$$

$$\Rightarrow \quad E_x' \pm v\sqrt{\mu\varepsilon}\,E_x' = 0$$

$$\left(1 \pm v\sqrt{\mu\varepsilon}\right) = 0 \quad \begin{cases} v = \dfrac{1}{\sqrt{\mu\varepsilon}} \\[2ex] v = -\dfrac{1}{\sqrt{\mu\varepsilon}} \end{cases} \quad \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c = 3\cdot 10^8\,\frac{m}{s}$$

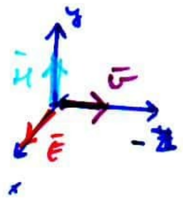
$$\bar{E} = \hat{e}_x E_x(z-vt) \longrightarrow \bar{H} = ?$$

$$\mathrm{rot}\,\bar{E} = -\mu\frac{\partial \bar{H}}{\partial t} \longrightarrow \frac{\partial \bar{H}}{\partial t} = -\frac{1}{\mu}\mathrm{rot}\,\bar{E}$$

$$-\frac{1}{\mu}\begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z-vt) & 0 & 0 \end{vmatrix} = -\frac{1}{\mu}\underbrace{\frac{\partial}{\partial z}E_x(z-vt)}_{\frac{1}{-v}\frac{\partial}{\partial t}E_x(z-vt)}\hat{e}_y = +\frac{\sqrt{\mu\varepsilon}}{\mu}\frac{\partial}{\partial t}E_x(z-vt)\,\hat{e}_y$$

$\bar{H}(z,t)$

$$\bar{H}(z,t) = \hat{e}_y \underbrace{\sqrt{\frac{\varepsilon}{\mu}}\,E_x(z-vt)}_{H_y(z-vt)}$$



Def: Síkhullám: a terjedés irányra $\perp$ síkban $\bar{E}$ és $\bar{H}$ helyfüggetlen.

- $\bar{E} \perp \bar{H}$
- $\bar{E} \perp \bar{v}$ és $\bar{H} \perp \bar{v}$

Időben szinuszos változás

$$\bar{E}(x,y,z,t) \triangleq \mathrm{Re}\left\{ \underbrace{\tilde{\bar{E}}(x,y,z)\, e^{j\omega t}}_{\text{komplex időfüggvény}} \right\}$$

$$\tilde{\bar{E}} = \tilde{E}_x \hat{e}_x + \tilde{E}_y \hat{e}_y + \hat{E}_z \hat{e}_z \quad : \text{komplex vektor}$$

komplex fazorok

Helmholtz-egyenlet

$$\Delta \bar{E} - \mu\sigma \frac{\partial \bar{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0 \xrightarrow{\frac{\partial}{\partial t} \to j\omega} \boxed{\Delta \tilde{\bar{E}} - j\omega\sigma\mu \tilde{\bar{E}} + \omega^2\mu\varepsilon \tilde{\bar{E}} = 0}$$

$$\Delta \tilde{\bar{E}} + \underbrace{(\omega^2\mu\varepsilon - j\omega\mu\sigma)}_{-\gamma^2} \bar{E} = 0 \quad \Big| \quad \gamma : \text{terjedési együttható}$$

Síkhullámok:

$$\frac{d^2 \tilde{E}_x}{dz^2} - \gamma^2 \tilde{E}_x = 0$$

$$\left(\text{TV:} \quad \frac{d^2 U}{dz^2} - \gamma^2 U = 0 \right)$$

$\Rightarrow$ általános megoldás:

$$\tilde{E}_x(z) = E_{x0}^+ e^{-\gamma z} + E_{x0}^- e^{\gamma z}$$

$$\tilde{H}_y(z) = H_{y0}^+ e^{-\gamma z} + H_{y0}^- e^{\gamma z}$$

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}$$

- $\to \gamma = j\omega\sqrt{\mu\varepsilon} = j\beta$ (ideális szigetelő)
- $\to \gamma = \alpha + j\beta$ (veszteséges szigetelő)
- $\to \gamma = \alpha + j\beta$ $(\alpha = \beta)$ (VEZETŐ)

α : csillapítási tényező

β : fázistényező

II. $\operatorname{rot}\tilde{\bar{E}} = -\mu j\omega \tilde{\bar{H}}$

$$\tilde{\bar{H}} = -\frac{1}{j\omega\mu}\operatorname{rot}\tilde{\bar{E}} = -\frac{1}{j\omega\mu}\left[\begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_{x0}^{+}e^{-\gamma z} & 0 & 0 \end{vmatrix} + \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_{x0}^{-}e^{\gamma z} & 0 & 0 \end{vmatrix}\right] =$$

$$= \frac{1}{j\omega\mu}\left(\gamma E_{x0}^{+} e^{-\gamma z} - \gamma E_{x0}^{-} e^{\gamma z}\right)\hat{e}_y$$

$\frac{j\omega\mu}{\gamma} \hat{=} Z_0$: hullámimpedancia $\Rightarrow$ $Z_0 \hat{=} \frac{E_{x0}^{+}}{H_{y0}^{+}} \equiv -\frac{E_{x0}^{-}}{H_{y0}^{-}}$ $\Rightarrow$

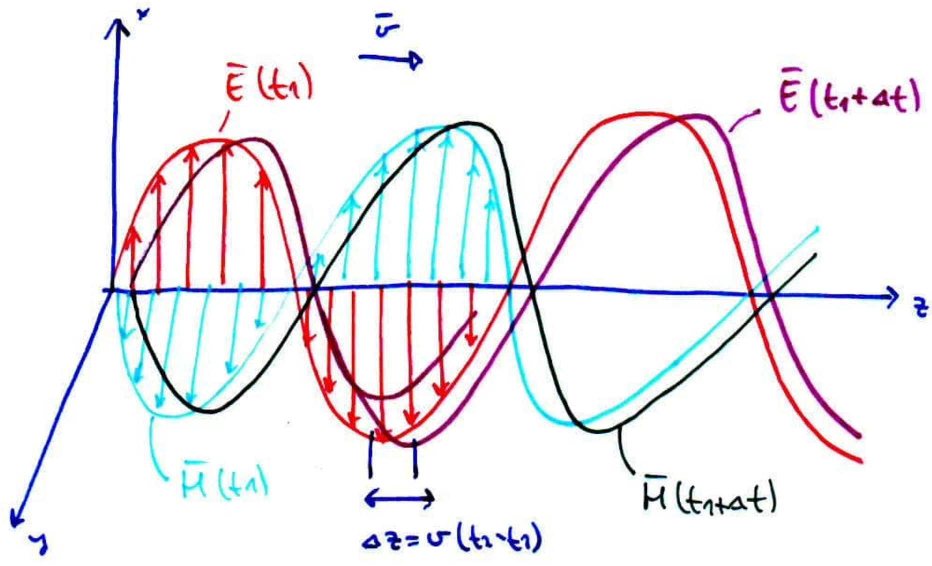
$$H_y(z) = \underbrace{\boxed{\frac{E_{x0}^{+}}{Z_0}}}_{H_{y0}^{+}} e^{-\gamma z} + \underbrace{\boxed{-\frac{E_{x0}^{-}}{Z_0}}}_{H_{y0}^{-}} e^{\gamma z}$$

$\Rightarrow$ $Z_0 = \frac{j\omega\mu}{\sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon}}$

szig. $\rightarrow$ $Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$: tisztán valós

levegő : $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$: vákuum hullámimpedancia

vezet. szig. $\rightarrow$ $Z_0 \in \mathbb{C}$



$\tilde{E}_x(z) = E_{x0}^{+} e^{-j\beta z}$

$\tilde{H}_y(z) = \frac{E_{x0}^{+}}{Z_0} e^{-j\beta z}$

$\left.\begin{array}{l} \lambda = T\cdot v = \frac{v}{f} \\ \lambda = \frac{2\pi}{\beta} \end{array}\right\}$ hullámhossz

Terek E.
15. 11. 19.

Elektromágneses hullámok ⊕

- Szinuszos időfüggés ⟶ komplex amplitúdó
- Síkhullám $\nrightarrow z$ $\bar{E} = \hat{e}_x E_x$, $\bar{H} = \hat{e}_y H_y$

$$\tilde{E}_x(z) = E_{x0}^+ e^{-\gamma z} + E_{x0}^- e^{\gamma z}$$

$$\tilde{H}_y(z) = \frac{E_{x0}^+}{Z_0} e^{-\gamma z} - \frac{E_{x0}^-}{Z_0} e^{\gamma z}$$

Teljesítményáramlás síkhullámban

Poynting-vektor: $\bar{S} = \bar{E} \times \bar{H}$: valós, időfüggő

$$E_x(z,t) = \hat{E}(z) \cos(\omega t + \varphi_E(z))$$

$$H_y(z,t) = \hat{H}_y(z) \cdot \cos(\omega t + \varphi_H(z))$$

⇒ $E \times H$: z irányú lesz.

$$S_z(t) = E_x(z,t) \cdot H_y(z,t) = \hat{E}_x(z)\hat{H}_y(z)\cos(\omega t + \varphi_E(z)) \cdot \cos(\omega t + \varphi_H(z)) =$$

$$= \frac{1}{2}\hat{E}_x(z)\hat{H}_y(z)\Big[\underbrace{\cos(2\omega t + \varphi_E + \varphi_H)}_{\text{középértéke } 0.} + \underbrace{\cos(\varphi_E - \varphi_H)}_{\text{időfüggetlen}}\Big]$$

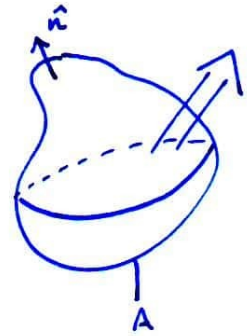
$$\boxed{S_z^{\text{átl.}}(z) = \frac{1}{2}\hat{E}_x\hat{H}_y \cdot \cos(\varphi_E - \varphi_H)}$$

$$S_z^{\text{átl.}}(z) = \mathrm{Re}\Big\{\underbrace{\frac{1}{2}\tilde{E}_x\tilde{H}_y^*}_{}\Big\}$$

komplex Poynting-vektor „z" irányú rendezője

Ált.: komplex P.-vektor:

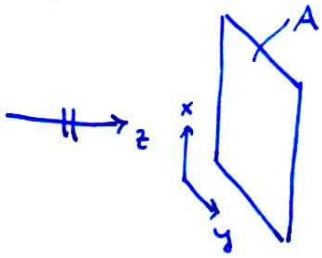
$$\boxed{\bar{S} = \frac{1}{2}\tilde{\bar{E}} \times \tilde{\bar{H}}^*}$$



$$P_s = \oint_A \mathrm{Re}\{\bar{S}\}\, d\bar{A}$$

sugárzó hatásos teljesítmény

Síkhullám:



$$P_s = A \cdot \mathrm{Re}\{\tilde{S}_z\}$$

komplex P.-vektor

Síkhullám vezetőben

$\sigma \gg \omega \cdot \varepsilon \quad \left(\mathrm{rot}\,\bar{H} = \bar{J} + \cancel{j\omega \bar{D}} = (\sigma + \cancel{j\omega\varepsilon})\bar{E}\right)$

elhanyagolható

Hullámparaméterek:

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon} = \sqrt{j\omega\mu(\sigma + \cancel{j\omega\varepsilon})}$$

$$\Rightarrow \gamma = \sqrt{j\omega\mu\sigma} = \alpha + j\beta \; ; \; \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

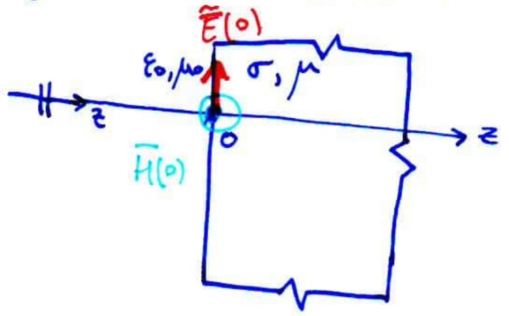
$$Z_0 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \Rightarrow Z_0 = \sqrt{\frac{j\omega\mu}{\sigma}} = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$$

pl.: réz: $\sigma = 57\,\frac{MS}{m}$

$f = 500\,kHz \rightarrow |Z_0| = 0{,}26\,m\Omega$

$f = 5\,kHz \rightarrow |Z_0| = 0{,}026\,m\Omega$

• Végtelen vezető féltér:



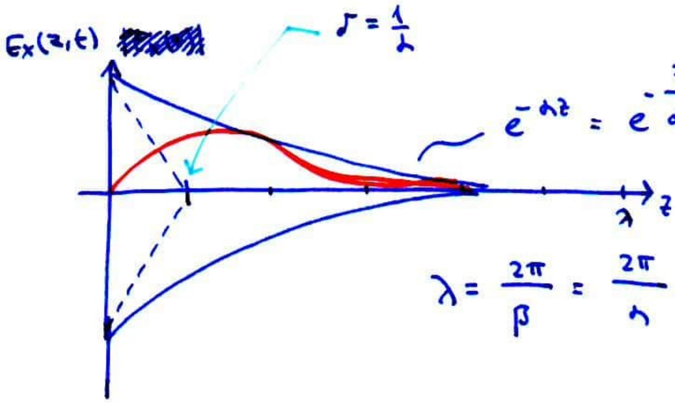
$$\bar{E}_x = \underbrace{E_{x0}^+ e^{-\gamma z}}_{+z \text{ irány}} + \cancel{E_{x0}^- e^{\gamma z}}$$

nem alakul ki

$$\bar{H}_y(z) = \underbrace{\frac{E_{x0}^+}{Z_0}}_{H_{y0}^+} e^{-\gamma z}$$

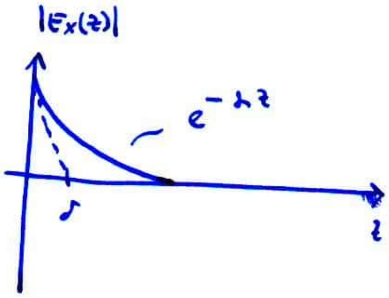
$$E_x(z) = E_x(0) \cdot e^{-\alpha z} \cdot e^{-j\beta z}$$

$$|E_x(z)| = |E_x(0)| \cdot e^{-\alpha z} \cdot 1$$



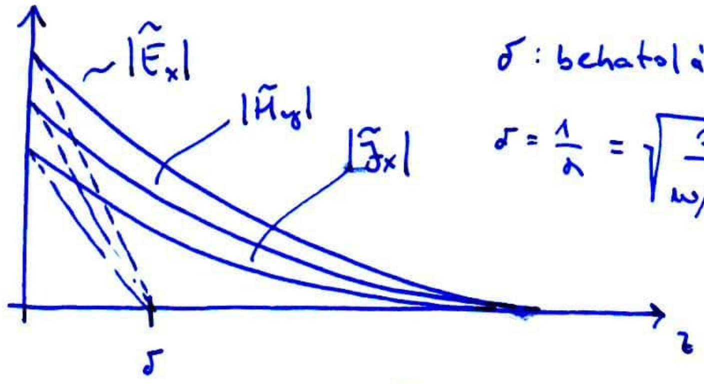
$e^{-\alpha z} = e^{-\frac{z}{\delta}}, \quad \delta = \frac{1}{\alpha}$

$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\alpha} = 2\pi\delta$



$$|\tilde{H}_y(z)| = \frac{|\tilde{E}_x(z)|}{|Z_0|}$$

$$\bar{J} = \sigma \bar{E} \longrightarrow \tilde{J}_x(z) = \sigma \tilde{E}_x(z)$$



δ : behatolási mélység (skin-hatás)

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

pl.: réz: $\sigma = 57 \frac{MS}{m}$

f	δ (mm)
50 Hz	9,43
5000 Hz	0,943
500 kHz	0,0943

• $z=0$ sík „A" keresztmetszeten átáramló hatásos teljesítmény:
(féltérben disszipálódó teljesítmény)

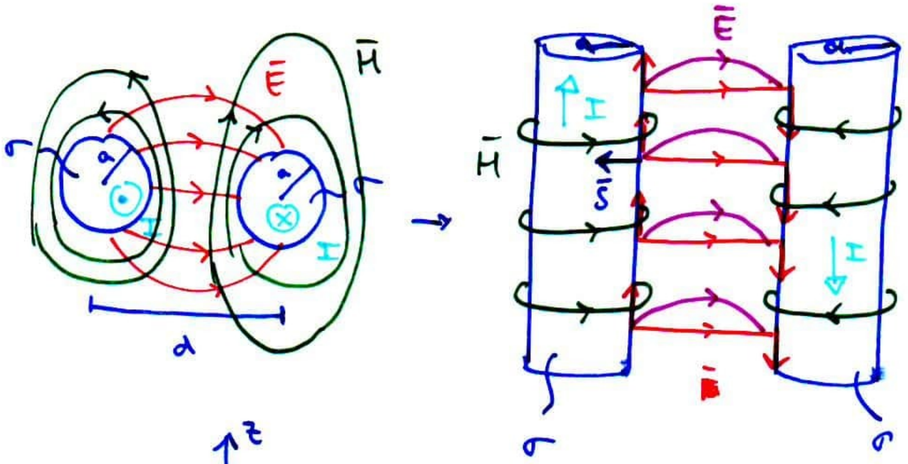
$$\bar S = \frac{1}{2}\underbrace{\bar E}_{Z_0\cdot\bar H}\times\bar H^* = \frac{1}{2}(\hat e_x E_x)\times(\hat e_y H_y)^* = \boxed{\frac{1}{2}\hat e_z Z_0|H_y|^2}$$

$$P_A = \mathrm{Re}\{S_z\}\cdot A = \frac{1}{2}\mathrm{Re}\{Z_0\}|H_y|^2\cdot A = \boxed{\frac{1}{2}\sqrt{\frac{\omega\mu}{2\sigma}}\,|H_y|^2\cdot A}$$

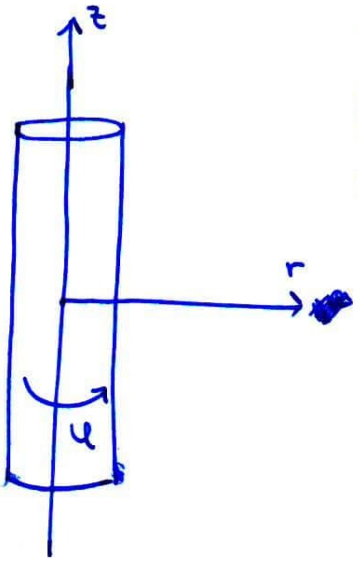
$$\hookrightarrow Z_0 = \sqrt{\frac{j\omega\mu}{\sigma}}$$

Hengeres vezető váltóáramú ellenállása

Lecher-vezeték:



$d >> a$

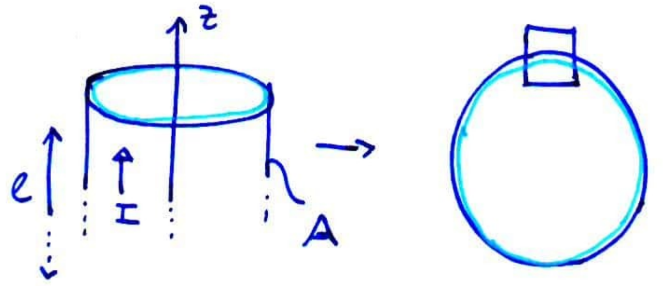


$$\bar H = \hat e_\varphi H_\varphi$$

$$\bar E = \hat e_z E_z\,(+\hat e_r + \dots + \hat e_\varphi)$$

$$\bar S = \frac{1}{2}\bar E\times\bar H^*$$

$$S_r = -\frac{1}{2}E_z H_\varphi^*$$



: felszínhez nagyon közel halad az EM hullám.

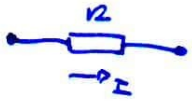
a) $\delta \ll a$

Amper-tv.

$$P_A = \frac{1}{2}\sqrt{\frac{\omega\mu}{2\sigma}} \cdot \left(\frac{|I|}{2\pi a}\right)^2 \cdot l\,2a\pi = \frac{1}{2}|I|^2 \frac{1}{\delta\sigma}\frac{l}{2\pi a}$$

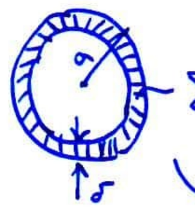
váltóáramú ellenállás „l" hosszú, „a" sugarú, σ fajlagos vez. kép. hengeres vezető

$\downarrow \sim R$



$P = \frac{1}{2}R|I|^2$

Egyenáramon: $R_0 = \frac{l}{\sigma a^2 \pi}$



$\sum 2a\pi$

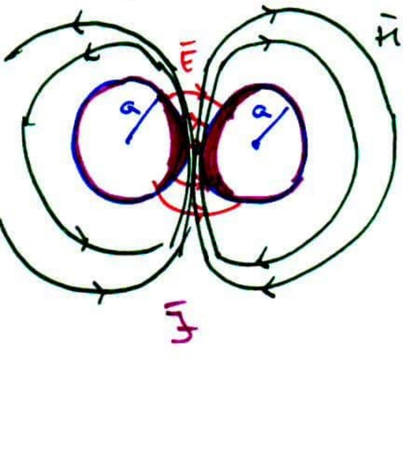
Egyenáramon akkora az ellenállása, mint váltóáramon a tömör hengernek.

b) $\delta \gg a$

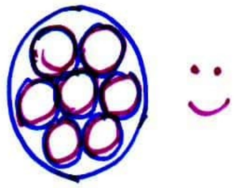
$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \Rightarrow R \cong R_0 \text{ (egyenáramú)}$$

c) $\delta \simeq a$: bonyolult eset

d) $d \gg a$



közelségi hatás



Terek E.
15.11.25.

Síkhullám – távvezeték analógia

TV:

$$\frac{d^2U}{dz^2} - \gamma^2 U = 0$$

$$U(z) = U_0^+ e^{-\gamma z} + U_0^- e^{\gamma z}$$

$$I(z) = \frac{U_0^+}{Z_0} e^{-\gamma z} - \frac{U_0^-}{Z_0} e^{\gamma z}$$

$$\gamma = \sqrt{(j\omega L' + R')(j\omega C' + G')}$$

$$Z_0 = \sqrt{\frac{j\omega L' + R'}{j\omega C' + G'}}$$

ideális TV.: $\gamma = j\omega\sqrt{L'C'}$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

SH. ($\rightarrow z$)

$$\frac{d^2E_x}{dz^2} - \gamma^2 E_x = 0$$

$$E_x(z) = E_{x0}^+ e^{-\gamma z} + E_{x0}^- e^{\gamma z}$$

$$H_y(z) = \frac{E_{x0}^+}{Z_0} e^{-\gamma z} - \frac{E_{x0}^-}{Z_0} e^{\gamma z}$$

$$\gamma = \sqrt{j\omega\mu(j\omega\varepsilon + \sigma)}$$

$$Z_0 = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon + \sigma}}$$

ideális szigetelő: $\sigma = 0$

$$\gamma = j\omega\sqrt{\mu\varepsilon}$$

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$$

TV	L'	C'	R'	G'	U	I	γ	Z_0	... származtatott mennyiségek
SH	μ	ε	$-$ *	σ	E_x	H_y	γ	Z_0	... —— " ——
	TV./közeg				terjedelmzők		hullámpor.		

$-$ * oka: TV.-re a távíróegyenletek:

$$-\frac{dU}{dz} = j\omega L' \cdot I + R' \cdot I$$

$$-\frac{dI}{dz} = j\omega C' \cdot U + G' \cdot U$$

SH.-ra:

$$\text{rot}\,\bar{E} = -j\omega\mu\bar{H} + ?$$

$$\text{rot}\,\bar{H} = j\omega\varepsilon\bar{E} + \sigma\bar{E}$$

: Maxwell-egy. asszimetriája miatt ($\nexists$ mágneses áram) R'-nek nincs párja.

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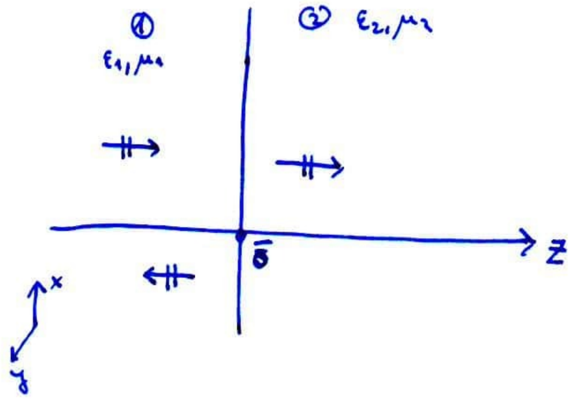
Síkhullám viselkedése közeghatáron

(S.H. merőlegesen esik a sík határfelületre)

$\boxed{\text{I}}$. Reflexió végtelen féltér határáról



1.: $E_x(z) = \boxed{E_{x01}^+} e^{-\gamma_1 z} + \boxed{E_{x01}^-} e^{\gamma_1 z}$

$H_y(z) = \frac{E_{x01}^+}{Z_{01}} e^{-\gamma_1 z} - \frac{E_{x01}^-}{Z_{01}} e^{\gamma_1 z}$

$\square$: 3 változónk van

2.: $E_x(z) = \boxed{E_{x02}^+} e^{-\gamma_2 z}$

$H_y(z) = \frac{E_{x02}^+}{Z_{02}} e^{-\gamma_2 z}$

Ha pl.: E_{x01}^+ adott => 2 ismeretlen : $E_{x02}^+, E_{x01}^- = ?$

=> két ismeretlenre 2 folytonossági egyenlet E_x-re és H_y-ra

$$\left.\begin{aligned} &\text{I.} \quad E_{x01}^+ + E_{x01}^- = E_{x02}^+ \\ &\text{II.} \quad \frac{E_{x01}^+}{Z_{01}} - \frac{E_{x01}^-}{Z_{01}} = \frac{E_{x02}^+}{Z_{02}} \end{aligned}\right\}$$

Reflexiós tényező ($z=0$) :

$$r(0) \triangleq \frac{E_{x01}^-}{E_{x01}^+} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

Feladatokban tipikusan: $\mu_1 = \mu_2 = \mu_0$

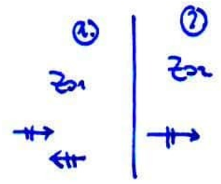
$\varepsilon = \varepsilon_r \varepsilon_0$

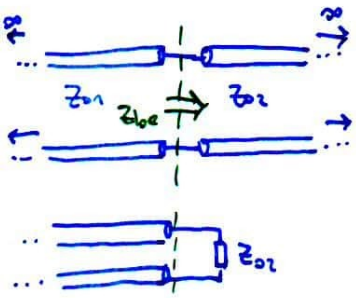
id. szig.: $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} = \frac{1}{\sqrt{\varepsilon_r}} \cdot 377\,\Omega$

$$\gamma = j\omega \sqrt{\mu_0 \varepsilon_0} \sqrt{\varepsilon_r} = \underbrace{j\omega \sqrt{\varepsilon_r} \cdot \frac{1}{c}}_{\beta \text{ fázistényező}}$$

$\Rightarrow v = \frac{c}{\sqrt{\varepsilon_r}}$, $\sqrt{\varepsilon_r} = n$: törésmutató

TV. modell :

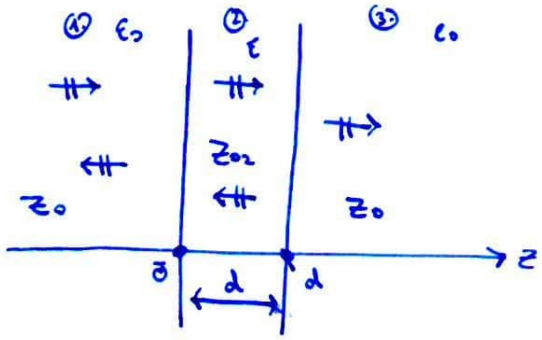




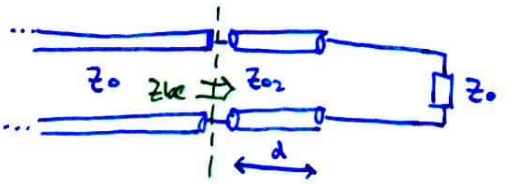
$$\left.\begin{aligned} U(z) &= U_{02}^{+} e^{-\gamma z} \\ I(z) &= \frac{U_{02}^{+}}{Z_0} e^{-\gamma z} \end{aligned}\right\} \quad Z_{be} = \frac{U(z)}{I(z)} \equiv Z_{02}$$

$$r(0) = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

$\boxed{\text{II.}}$ Reflexió végtelen lemezfelületről :



$$r(0) = ?$$



$$r(0) = \frac{Z_{be} - Z_0}{Z_{be} + Z_0}$$

$$Z_{be} = Z_{02} \frac{Z_0 + j Z_{02} \operatorname{tg} \beta_2 d}{Z_{02} + j Z_0 \operatorname{tg} \beta_2 d}$$

$$Z_{02} = \frac{Z_0}{\sqrt{\varepsilon_r}} \qquad \beta_2 = \frac{\omega}{v} = \frac{\omega}{c \cdot \sqrt{\varepsilon_r}}$$

Korlátok:

- csak merőleges beesés sík határfelületek
- E_x-szel jellemezhető
- lineárisan polarizált síkhullám

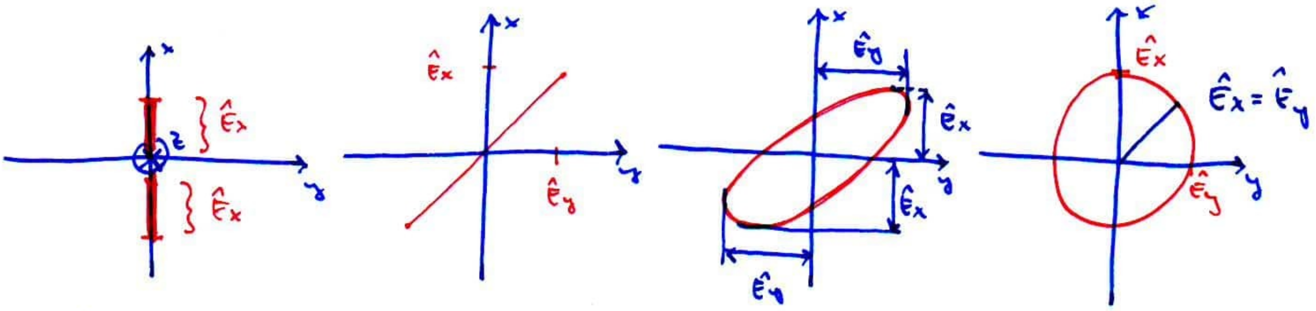
SH.-ok polarizációja

$$E_1(z,t) = \hat{e}_x E_x(z,t) = \hat{e}_x \cdot \hat{E}_x \cdot \cos(\overbrace{\omega t}^{-\beta z} + \varphi_x) \quad : SH$$

$$E_2(z,t) = \hat{e}_y E_y(z,t) = \hat{e}_y \cdot \hat{E} \cdot \cos(\underbrace{\omega t}_{-\beta z} + \varphi_y) \quad : SH$$

$E_1 + E_2$ is SH. lesz.

$$\bar{E} = \bar{E}_1 + \bar{E}_2$$



$\hat{E}_y = 0$
Lineáris pol.
x irányú

$\varphi_x = \varphi_y$
Lineáris pol.
általános irányú

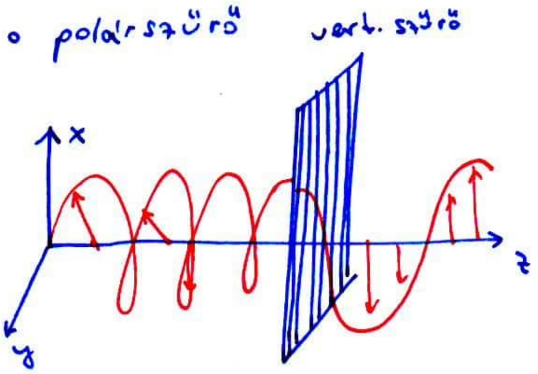
$\varphi_x \neq \varphi_y$
$\hat{E}_x \neq \hat{E}_y$
Elliptikus polarizáció

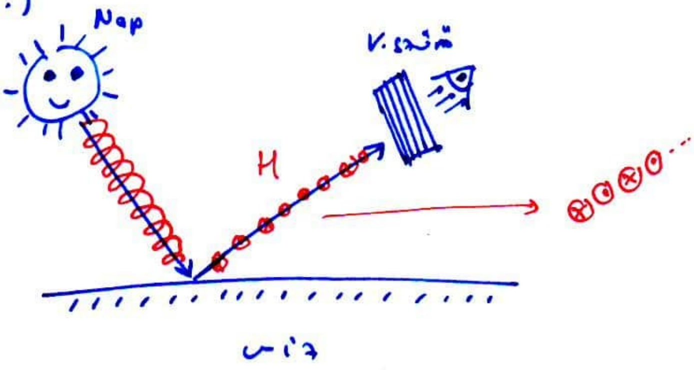
$\hat{E}_x = \hat{E}_y$
$\varphi_x - \varphi_y = \frac{\pi}{2}$
Cirkuláris polarizáció

Vízszintes = horizontális
Függőleges = vertikális

Alkalmazások:

- különböző csatornák (horiz./vertik.)
- polárszűrő

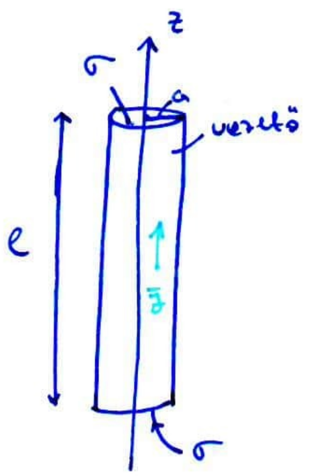




Hullámok keltése – a Hertz-dipólus

eddig: forrásmentes közegben a hullámok terjedése

most: források hatása: $\bar{J}, \rho \longrightarrow \underbrace{\bar{A}, \varphi}_{\text{potenciálok}} \longrightarrow \bar{E}, \bar{B}$



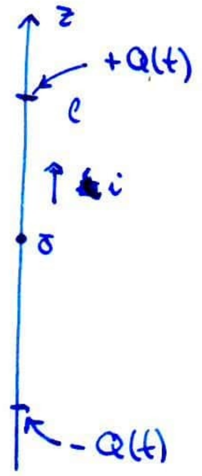
$\bar{J} = \hat{e}_z \cdot \hat{J} \cdot \cos(\omega t)$

$\mathrm{div}\, \bar{J} + \dfrac{\partial \rho}{\partial t} = 0$

$\sigma = \dfrac{\hat{J}}{\omega} \cdot \sin(\omega t)$

$a \ll \ell$

$\Rightarrow$

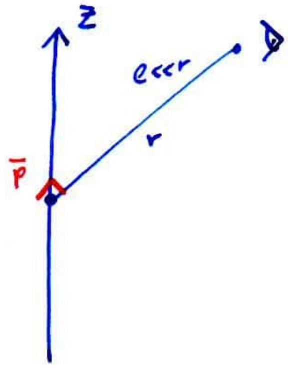


$\Rightarrow$

$i(t) = a^2 \pi \cdot \hat{J} \cdot \cos(\omega t)$

helyfüggetlen!

$\Rightarrow$ megfigyelő „r" távolsága: $r \gg \ell$

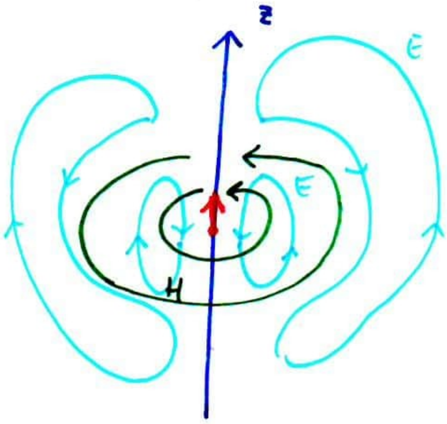


$\left.\begin{array}{l} \ell \to 0 \\ \hat{I} \to \infty \end{array}\right\} \ell \cdot \hat{I} = \text{const.}$

$\bar{p} = \hat{e}_z \cdot \ell \cdot \hat{I}$: nyomaték (Hertz-dipólus) $\quad (\bar{p} = \hat{e}_z \cdot \ell \cdot \bar{I}$ komplex$)$

Hertz-dipólussal jellemezhető:

- egyenes antenna
- $a \ll \ell$: vonalszerű
- $r \gg \ell$: pontszerű
- $\lambda \gg \ell$: állandó (helyfüggetlen) áramú (időtől függ)

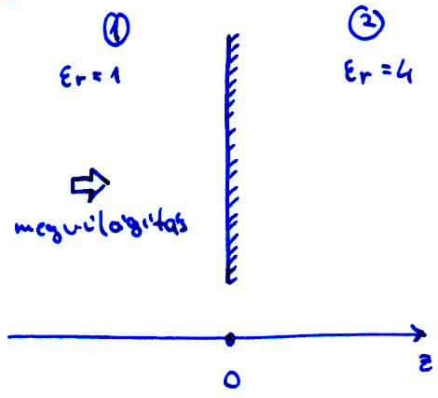


Terek. Gy.
15. 11. 26.

EM hullámok

Gym.

$\boxed{1}$ $\bar{E}(z) = ?$ $H(z) = ?$



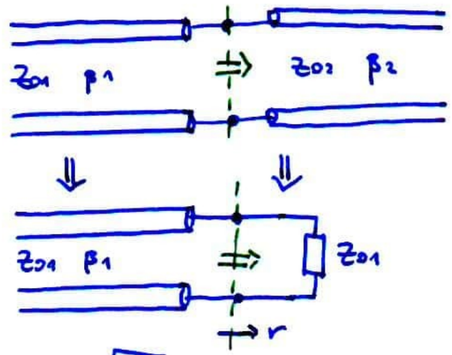
$f = 10\,\text{MHz}$

$E = 120\,\text{V/m}$ amplitúdójú

$\bar{E} = E_x(z)\,\hat{e}_x$

$\Rightarrow |E_x(0)| = 120\,\frac{V}{m}$

Távvezetékes helyettesítés : csak merőleges beesés esetén.



$$Z_{01} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\,\Omega \cong 377\,\Omega$$

$$Z_{02} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \frac{Z_{01}}{\sqrt{\varepsilon_r}} = \frac{Z_{01}}{\sqrt{4}} = \frac{1}{2} Z_{01} = 60\pi\,\Omega$$

$$r_{12} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = -\frac{1}{3}$$

$$\beta_1 = \frac{2\pi \cdot f}{c} = 0{,}209\,\frac{1}{m}$$: 1m-en hány radiánt változik a fázisa

$$\beta_2 = \frac{2\pi f}{v} = \frac{2\pi f}{\frac{c}{\sqrt{\varepsilon_r}}} = 0{,}419\,\frac{1}{m}$$

① $(z<0)$

$$E_x(z) = E_{01}^+ e^{-j\beta_1 z} + E_{01}^- e^{j\beta_1 z}$$

$$E_x(0) = E_{01}^+ + E_{01}^- \Leftrightarrow 120 \frac{V}{m}$$

$$\Leftrightarrow E_1^+ + r_{12} E_1^+ = E_1^+ (1 + r_{12})$$

$$\Rightarrow \underline{E_1^+ = 180 \frac{V}{m}} \quad \text{e's} \quad \underline{E_1^- = -60 \frac{V}{m}}$$

$$\Rightarrow \underline{\underline{E_x(z) = \left(180\, e^{-j0{,}209z} - 60\, e^{j0{,}209z}\right) \frac{V}{m}}}$$

$$\bar{H} = H_y \hat{e}_y$$

$$H_y(z) = H_1^+ e^{-j\beta_1 z} + H_1^- e^{j\beta_1 z}$$

$$H_1^+ = \frac{E_1^+}{Z_{01}} \qquad H_1^- = -\frac{E_1^-}{Z_{01}}$$

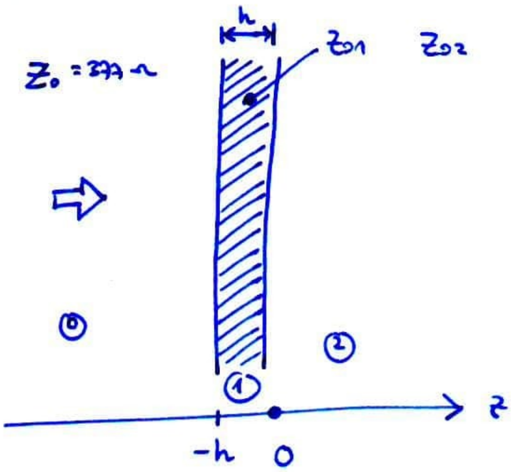
$$\underline{\underline{H_y(z) = \left(0{,}478\, e^{-j0{,}209z} + 0{,}159\, e^{j0{,}209z}\right) \frac{A}{m}}}$$

② $(z>0)$

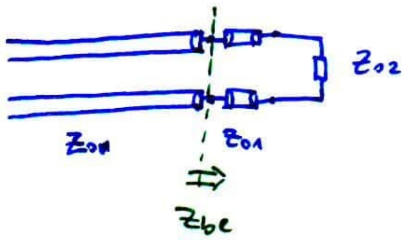
$$E_x(z) = E_2^+ e^{-j\beta_2 z} = 120 \frac{V}{m}\, e^{-j0{,}419z}$$

$$H_y(z) = \frac{E_2^+}{Z_{02}} e^{-j\beta_2 z} = 0{,}637 \frac{A}{m} \cdot e^{-j0{,}419z}$$

$\boxed{2}$ Reflexiómentes (illesztett) réteg



$h = ?$ $Z_{01} = ?$, hogy $z = -h$-nál ne legyen reflexió.



$Z_{be} = Z_0$

$\Downarrow$

$$Z_{01}\frac{Z_{02} - Z_{01} j\,\mathrm{tg}\,\beta h}{Z_{01} - Z_{02} j\,\mathrm{tg}\,\beta h} = Z_0 \Rightarrow$$

$$\Rightarrow Z_{01}Z_{02} - jZ_{01}^2\,\mathrm{tg}\,\beta h = Z_0 \cdot Z_{01} - jZ_0Z_{02}\,\mathrm{tg}\,\beta h \quad \Big/ \div\,\mathrm{tg}\,\beta h$$

$$\frac{Z_{01}Z_{02}}{\mathrm{tg}\,\beta h} - jZ_{01}^2 = \frac{Z_0Z_{01}}{\mathrm{tg}\,\beta h} - jZ_0Z_{02}$$

Ha $\mathrm{tg}\,\beta h \to \infty \Leftrightarrow$ valós részek egyenlőek

$\Rightarrow \beta h = \frac{\pi}{2} + k\pi.$ $\quad \lambda_1 = \frac{2\pi}{\beta}$

$$\underline{\underline{h = \frac{\lambda_1}{4} + k \cdot \frac{\lambda_1}{2}}}$$ (λ negyedes illesztés)(szűk frekvenciatartományban működik)

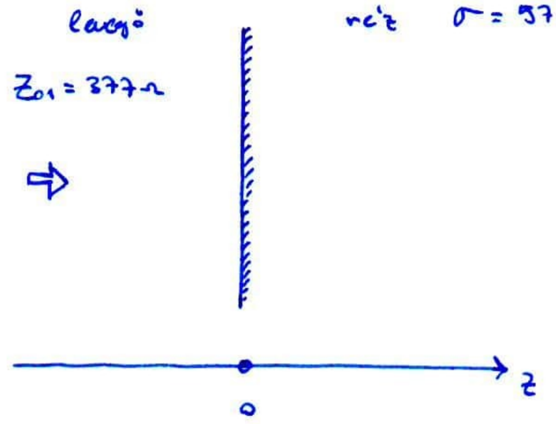
$Z_{01}^2 = Z_0 Z_{02} \to \underline{\underline{Z_{01} = \sqrt{Z_0 Z_{02}}}}$: másik kettő mértani közepe

$\Rightarrow \varepsilon_{r1}$ meghatározható.

3 Veszteséges közegek

levegő $\quad$ réz $\quad \sigma = 57\,\frac{MS}{m}$

$Z_{01} = 377\,\Omega$



$f = 1\,MHz$

$r_{12} = ? \quad E_x(z=0) = ?$

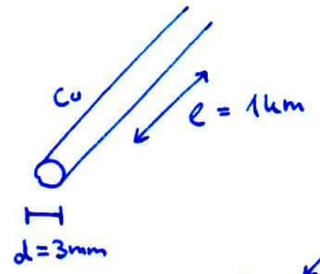
$$Z_{02} = \sqrt{\frac{j\omega\mu_0}{\sigma}} = 0{,}372\, e^{j\frac{\pi}{4}}\ m\Omega$$

$$\underline{\underline{r_{12}}} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = -1 + 1{,}974\cdot 10^{-6}\, e^{j\frac{\pi}{4}} \approx \underline{\underline{-1}}$$

$$\underline{\underline{E(0)}} = E_1^+ + E_1^- = E_1^+(1 + r_{12}) = 1{,}974\cdot 10^{-6}\, e^{j\frac{\pi}{4}} \cdot E_1^+ \approx \underline{\underline{0}}$$

PEC: perfect electric conductor

4 Vezetékek ellenállása

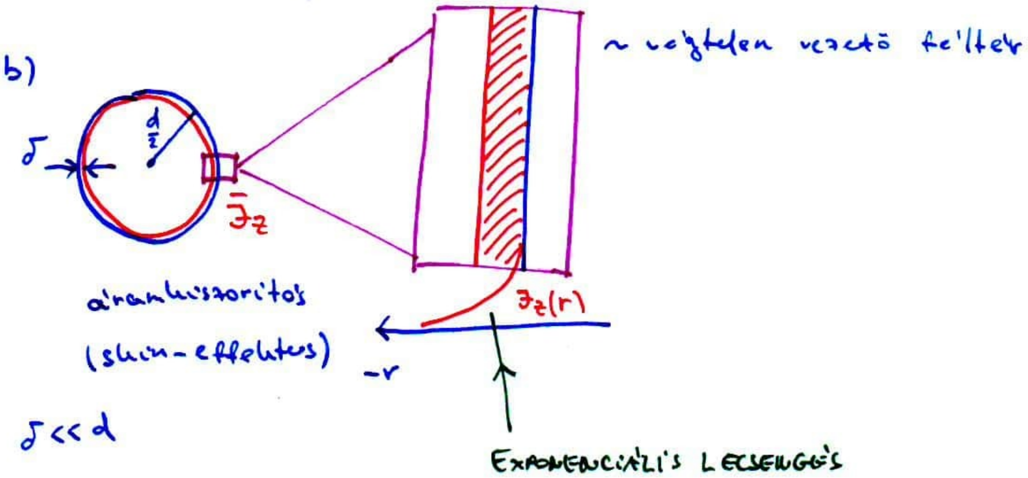


a) $R_0 = ?\ (f = 0)$

b) $R = ?\ (f = 3\,GHz)$

a) $$R_0 = \frac{1}{\sigma_{Cu}} \cdot \frac{\overset{l}{1000}}{\frac{d^2}{4}\cdot \pi} = 2{,}48\,\Omega$$

b)

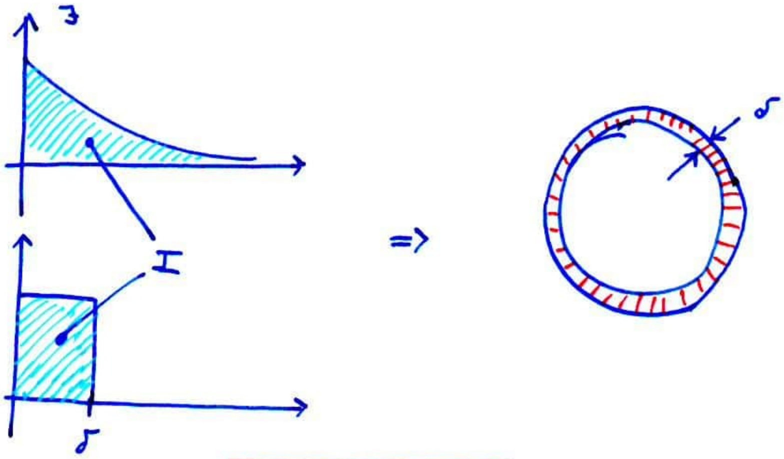


áramkiszorítás (skin-effektus)

$\delta \ll d$

δ

$\ll$



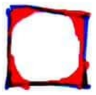
$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \quad , \quad \boxed{\delta_{Cu}^{mm} = \frac{66{,}7}{\sqrt{f^{Hz}}}} = 0{,}38\,mm \ll d$$

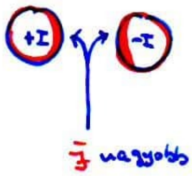
$$R = \frac{1}{\sigma_{Cu}} \frac{l}{d\pi\delta} = 5{,}52\,\Omega > R_0$$

↑ közelítés

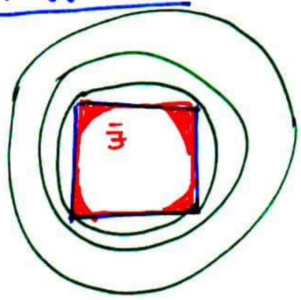
$\boxed{5}$

$\delta = ?$
- $\delta \gg d, r \Rightarrow R_0$
- $\delta \ll d, r \Rightarrow R$
- $\delta \sim d, r \Rightarrow$ Numerikus módszer

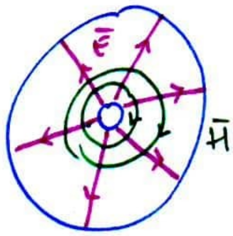
Négyszög keresztmetszetű vezetőnél:  δ

Lecher-vezeték: 

Magyarázat



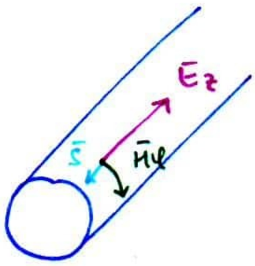
Koax:



külső palást $\to \infty$ visszavezeti az áramot (szimmetria miatt)

TEM : transzverz. elektr. mágneses

$$\bar S = \bar E \times \bar H = \bar S_z \hat e_z$$

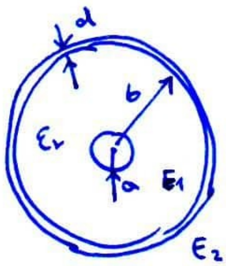


$\longrightarrow$: töltéseket hajtja

$\bar S = -S_r \hat e_r$ ($\bar S$ „r" irányú)

$\bar S$ és $\bar H$ merőleges a vezetőre

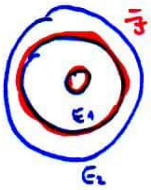
$\boxed{6}$ Koaxiális kábel árnyékoló hatása



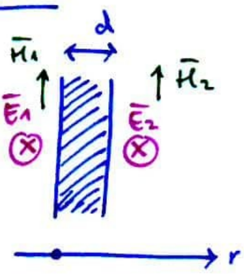
$a = 2{,}07\,mm$ $\quad d = 0{,}6\,mm$ $\quad tg\,\delta = 80 \cdot 10^{-4}$

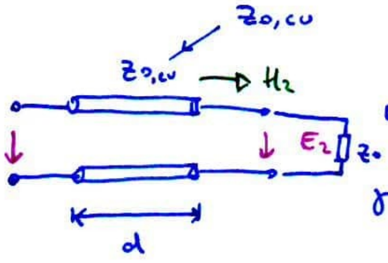
$b = 4a = 8{,}28\,mm$ $\quad \varepsilon_r = 3{,}5$ $\quad \sigma_{cu} = 57\,\frac{MS}{m}$

$f_{min} = ?$, hogy $\frac{E_2}{E_1} < 10^{-3}$ $\quad Z_0 = 377\,\Omega$



Közelítés:





$$E_1 = E_2 \cdot ch(\gamma d) + H_2 \cdot Z_0 \cdot sh(\gamma d)$$

$$\gamma = \sqrt{j\omega\mu_0\sigma}, \quad Z_{0,cu} = \sqrt{\frac{j\omega\mu_0}{\sigma}}$$

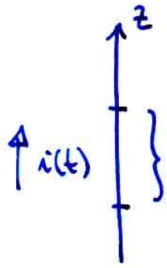
$$H_2 = \frac{E_2}{Z_0} \Rightarrow E_2 = \frac{E_1}{\underbrace{ch(\gamma d) + \frac{Z_{0,cu}}{Z_0} \cdot sh(\gamma d)}_{=10^{-3}}} \Rightarrow$$

iteratív megoldás

$\Rightarrow$ erre a geometriára: $f = 7{,}15 \cdot 10^5\,Hz$

Terek E.
15. 12. 02.

EM hullámok



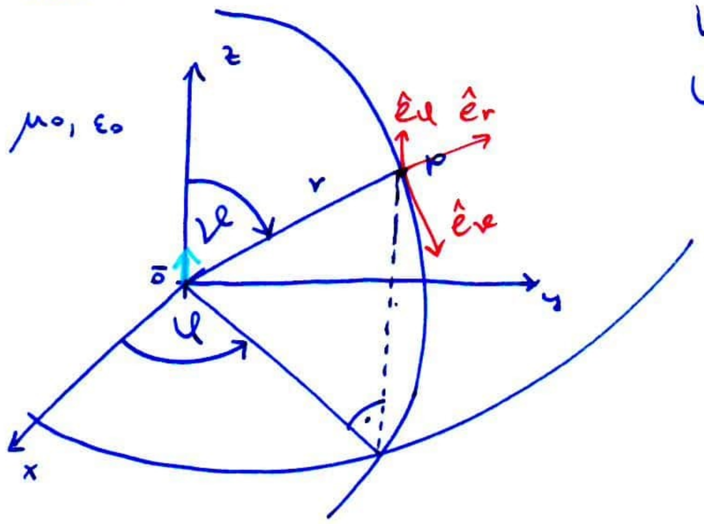
$$l \ll \lambda = \frac{c}{f} \qquad i(t) = \hat{I}\cdot\cos(\omega t)$$

Hertz-dipólussal $\Rightarrow$

$$\bar{p} = \hat{e}_z l\cdot\hat{I}$$

A Hertz-dipólus elektromágneses tere

- gömbkoordináta-rendszer



ϑ: elevációs szög

φ: azimutális szög

$$E_r(r,\vartheta,t) = \frac{\hat{I}l}{4\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{2}{r^2}\left[\cos(\omega t-\beta r)+\frac{1}{\beta r}\cdot\sin(\omega t-\beta r)\right]\cdot\cos(\vartheta)$$

$$E_\vartheta(r,\vartheta,t) = \frac{\hat{I}l}{4\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\beta}{r}\left[-\left(1-\frac{1}{\beta^2 r^2}\right)\sin(\omega t-\beta r)+\frac{1}{\beta r}\cdot\cos(\omega t-\beta r)\right]\cdot\sin(\vartheta)$$

ez tűnik el leglassabban

$$E_\varphi \equiv 0$$

$$H_r = H_\vartheta \equiv 0$$

$$H_\varphi(r,\vartheta,t) = \frac{\hat{I} l}{4\pi}\cdot\frac{\beta}{r}\left[\boxed{-\sin(\omega t-\beta r)} + \frac{1}{\beta r}\cdot\cos(\omega t-\beta r)\right]\cdot\sin(\vartheta)$$

ez tűnik el. ~~[illegible]~~ leglassabban

Hullámtér tulajdonságai:

- forgásszimmetrikus : $\frac{\partial}{\partial\varphi} \equiv 0$
- $\left.\begin{matrix}\cos(\omega t-\beta r)\\ \sin(\omega t-\beta r)\end{matrix}\right\}$ => haladó hullámok „r" irányba (gömbhullámok)

  hullámhossz: $\lambda = \frac{2\pi}{\beta}$
- Amplitúdók távolságfüggése:

  $E_r \sim \frac{1}{r^2}$ és $\sim\frac{1}{r^3}$

  $E_\vartheta \sim \frac{1}{r^2}$ és $\sim\frac{1}{r^3}$

$\sim\frac{1}{r^3}$: „sztatikus" tér (E_r, E_ϑ) } közeltér

$\sim\frac{1}{r^2}$: „indukciós" tér (E_r, H_φ) } közeltér

$\sim\frac{1}{r}$: „sugárzó" tér (E_ϑ, H_φ) } távoltér

A távoltér:

$$E_\vartheta(r,\vartheta,t) = -\frac{\hat{I}}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{l}{\lambda}\cdot\frac{1}{r}\cdot\sin(\vartheta)\cdot\overbrace{\sin(\omega t-\beta r)}^{\cos(\omega t-\beta r-\frac{\pi}{2})}$$

$$H_\varphi(r,\vartheta,t) = -\frac{\hat{I}}{2}\cdot\frac{l}{\lambda}\cdot\frac{1}{r}\cdot\sin(\vartheta)\cdot\sin(\omega t-\beta r)$$

l: antenna hossza

$\frac{l}{\lambda}$: dimenzió nélküli

Komplex amplitúdókkal: $i(t) = \hat{I}\cdot\cos(\omega t) \rightarrow \bar{I}$

$$\tilde{E}_\vartheta(r,\vartheta) = j\,\frac{\bar{I}}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{l}{\lambda}\cdot\frac{\sin\vartheta}{r}\cdot e^{-j\beta r}$$

$$\tilde{H}_\varphi(r,\vartheta) = j\,\frac{\bar{I}}{2}\cdot\frac{l}{\lambda}\cdot\frac{\sin\vartheta}{r}\cdot e^{-j\beta r}$$

$|\tilde{E}_\vartheta| \sim \sin\vartheta / r$

$|\tilde{H}_\varphi| \sim \sin\vartheta / r$

I. $\bar{E} \perp \bar{H}$ $\quad (\hat{e}_\vartheta \perp \hat{e}_\varphi)$

II. Azonos fázisban vannak

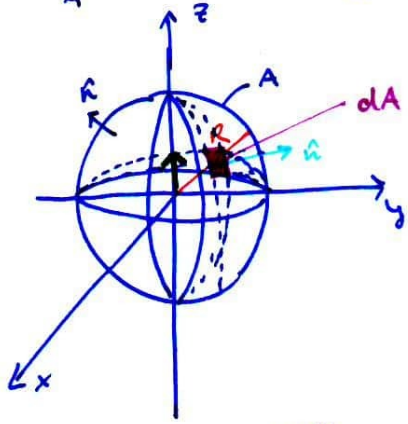
$$\left.\begin{matrix} \text{I.} \\ \text{II.} \end{matrix}\right\} \bar{S} = \frac{1}{2}\bar{E}\times\bar{H}^*$$

$$\bar{S} = \hat{e}_r S_r \,, \quad S_r \in \mathbb{R}$$ => nincs meddő teljesítményáramlás

=> Teljesítményáramlás a távoltérben:

$$\bar{S} = \frac{1}{2}\left(\hat{e}_\vartheta \tilde{E}_\vartheta(r,\vartheta)\right)\times\left(\hat{e}_\varphi \tilde{H}_\varphi(r,\vartheta)\right) = \frac{1}{2}\hat{e}_r \frac{|\bar{I}|^2}{4}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{\ell}{\lambda}\right)^2\left(\frac{\sin\vartheta}{r}\right)^2 \cdot \underbrace{1}_{e^{-j\beta r}\cdot(e^{-j\beta r})^*} \in \mathbb{R}$$

$$P = \oint_A \mathrm{Re}\{\bar{S}\}\, d\bar{A} = \oint_A \hat{e}_r S_r (r,\vartheta)\cdot(d\bar{A}\cdot\hat{n}) \ominus$$

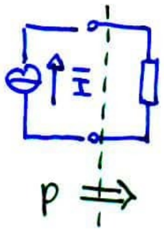


$$dA = (R\cdot d\vartheta)\cdot(\overset{R\cdot}{\vee}d\varphi\cdot\sin\vartheta) = R^2\sin\vartheta\, d\vartheta\, d\varphi$$

$$\ominus \int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi} S_r(\underset{r\equiv R}{r},\vartheta)\cdot R^2 \sin\vartheta\, d\vartheta\, d\varphi = \ldots = \frac{1}{2}|\bar{I}|^2\Big(\underbrace{80\pi^2\left(\frac{\ell}{\lambda}\right)^2\,\Omega}_{R_s}\Big)$$

R_s : sugárzási ellenállás

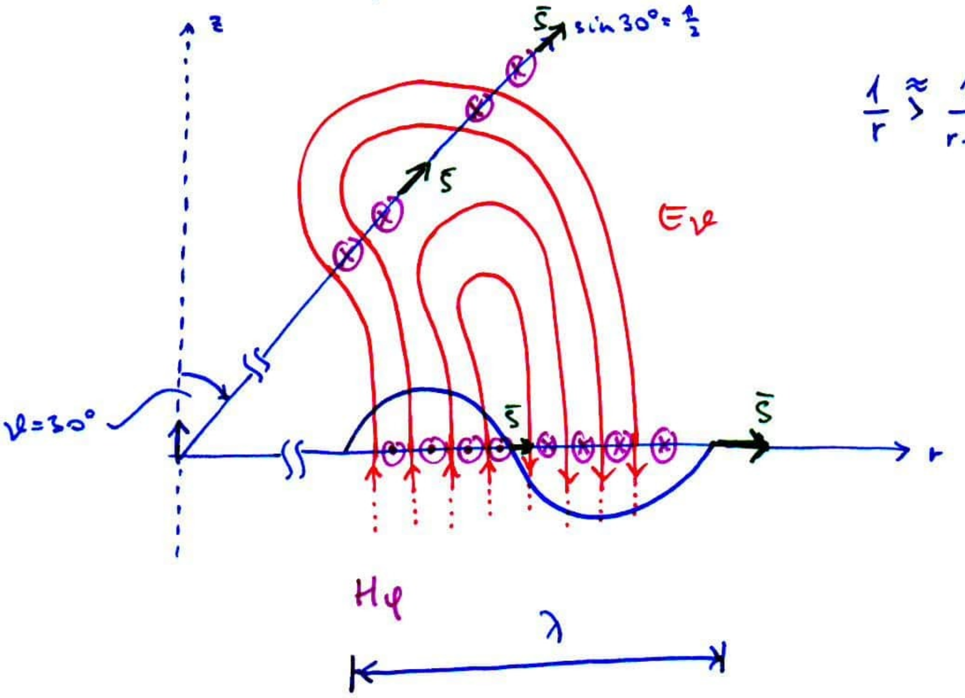
Áramköri modell:



$\bar{Z}_s = R_s + jX_s$

$$P = \frac{1}{2}|\bar{I}|^2\cdot R_s$$ => szétsugárzó teljesítmény

Vázlatos ábra a távoltérről:



$$\frac{1}{r} \approx \frac{1}{r+\lambda} \quad (r \gg \lambda)$$

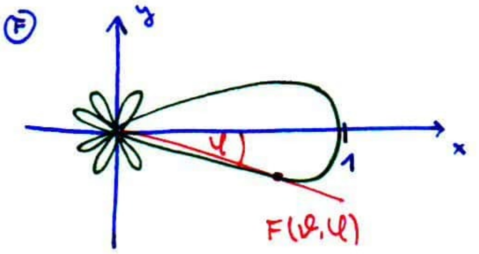
Antennajellemzők

I. Sugárzási karakterisztika (amplitúdó irányban) távoltérben:

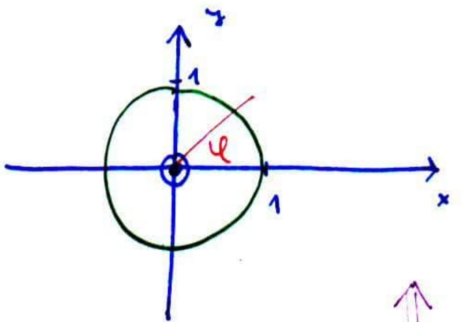
$$F(\vartheta,\varphi) \triangleq \frac{|E(r=R,\vartheta,\varphi)|}{|E(r=R,\vartheta,\varphi)|_{max}}, \quad 0 \le F \le 1$$

R-től független

pl.: (F)

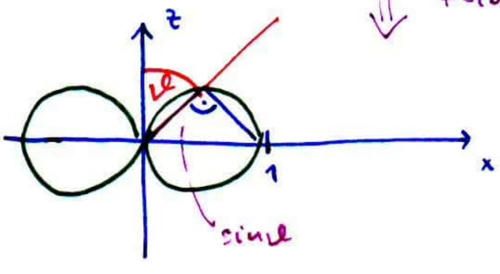


: irányított antenna



: irányítatlan antenna

⇕ fölülről – alulról



: Tórusz

$$F(\vartheta,\varphi) \equiv F(\vartheta) = \frac{\left| K \cdot \frac{\sin\vartheta}{r} \right|_{r=R}}{\left| K \cdot \frac{\sin\vartheta}{r} \right|_{\substack{max \\ r=R}}} = \frac{|\sin\vartheta|}{1} = |\sin\vartheta|$$

$\boxed{\text{II.}}$ Irányhatás:

$$D \triangleq \frac{S_r(\vartheta,\varphi,r=R)\big|_{max}}{S_r(\vartheta,\varphi,r=R)\big|_{átl}} \circledequal$$

$átl \rightarrow \vartheta,\varphi$ szerinti térbeli átlag

$$\circledequal \frac{S_r(\vartheta,\varphi,r=R)\big|_{max}}{\dfrac{P_{sug}}{4R^2\pi}} \quad : \quad 1 < D < \infty$$

$P_{sug} \leftarrow$ átlag

$4R^2\pi \leftarrow$ gömbfelszínen sugárzik át

- Hertz-dipólusnál:

$$S_r(\vartheta,r) = \frac{1}{8}|I|^2\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{\ell}{\lambda}\right)^2 \cdot \frac{\sin^2\vartheta}{r^2}$$

$$S_r(\vartheta,r=R)\big|_{max} \underset{\vartheta=90^\circ}{=} \frac{1}{8}|I|^2\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{\ell}{\lambda}\right)^2 \cdot \frac{1}{R^2}$$

$$P_{sug} = \frac{1}{2}|I|^2 \cdot R_s = \frac{1}{2}|I|^2\left(80\pi^2\left(\frac{\ell}{\lambda}\right)^2 \Omega\right)$$

$$\boxed{D = 1{,}5}$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = Z_0 = 377\,\Omega \triangleq 120\pi\,\Omega$$

Tereh E.
15. 12. 03.

Antennák

- irányhatás:

$$D = \frac{S_r(r=R,\vartheta,\varphi)_{max}}{S_r(r=R,\vartheta,\varphi)\big|_{átlag}} = \frac{S_r(r=R,\vartheta,\varphi)\big|_{max}}{P_s/4R^2\pi}$$

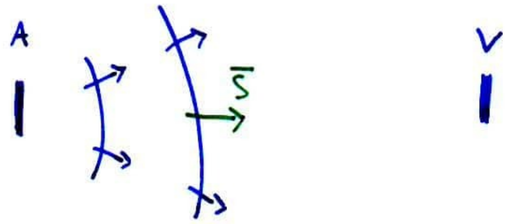
sugárzott teljesítmény

Hertz-dipólus: 1.5

- Nyereség:

$$G = \frac{S_r(r=R,\vartheta,\varphi)}{\frac{P_{be}}{4R^2\pi}}, \quad P_{be} = P_s + P_{veszt} \Rightarrow G < D$$

Hatásos felület:



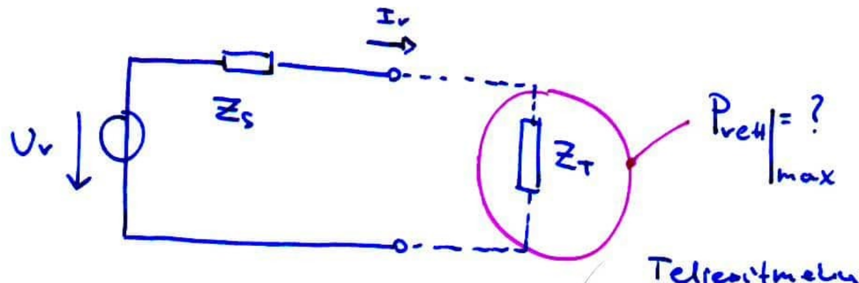
A vevő antenna helyén:

E_0 amplitúdó

$$\bar{S}_0 = \frac{1}{2}\bar{E}_0 \times \bar{H}_0^* \Rightarrow$$

$$\Rightarrow \boxed{S_0 = \frac{1}{2}\frac{|E_0|^2}{Z_0}}$$

A vevőből kivehető hatásos teljesítmény:



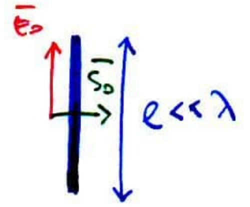
Teljesítményillesztés:

$$Z_T \triangleq Z_s^*$$

$$I_v = \frac{U_v}{\underbrace{2R_s}_{Re\{Z_s + Z_s^*\}}} \Rightarrow \boxed{P_{vett}^{max} = \frac{1}{2}R_s \cdot |I_v|^2}$$

$$\boxed{\underline{\text{Def.}}: A_h: P_{vett}^{max} = S_0 \cdot A_h}$$ Hatásos felület

Rövid dipólus hatásos felülete:



$$U_v = \ell \cdot E_0$$
$$R_s = 80\pi^2 \left(\frac{\ell}{\lambda}\right)^2 \Omega$$

$$P_{vill}^{max} = \frac{1}{2} R_s \frac{|U_v|^2}{4 R_s^2} = \frac{|U_v|^2}{8 R_s} = \frac{\ell^2 E_0^2}{8 \cdot 80\pi^2 \left(\frac{\ell}{\lambda}\right)^2}$$

$$S_0 = \frac{1}{2} \frac{|E_0|^2}{Z_0} \qquad A_h = \frac{P_{vill}^{max}}{S_0} = \frac{2 Z_0 \lambda^2}{8 \cdot 80\pi^2} = \frac{240\pi \cdot \lambda^2}{80 \cdot 8 \cdot \pi^2} = \frac{3\lambda^2}{8\pi} \rightarrow$$

$\leftarrow$ antenna hosszától nem függ

Átviteli egyenlet

1. (A) az adó : P_A . (B) a vevő : P_B-t vesz.
2. (A) a vevő : P_A' . (B) az adó : P_B'-t ad.

1) $P_B = \underbrace{\left(\frac{P_A}{4\pi d^2}\right)}_{S_{max}} D_A \cdot A_B$

2) $P_A' = \left(\frac{P_B'}{4\pi d^2}\right) D_B \cdot A_A$

Reciprocitás: Ha $P_A = P_B' \Rightarrow P_B = P_A'$

$$\Rightarrow \frac{D_A \cdot A_B}{4\pi d^2} = \frac{D_B \cdot A_A}{4\pi d^2} \Rightarrow \boxed{\frac{A_A}{D_A} = \frac{A_B}{D_B}} = \text{const.}$$

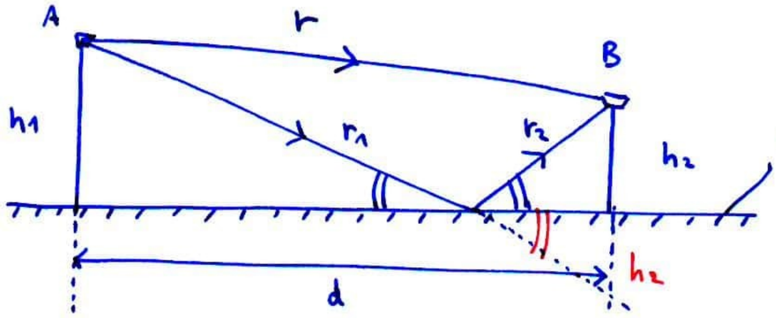
Rövid dipólus antennára:

$$A_h = \frac{3\lambda^2}{8\pi}, \quad D = 1.5 \quad \Rightarrow \quad \boxed{\frac{A_h}{D} = \frac{\lambda^2}{4\pi}}$$

$$\Leftarrow \boxed{P_B = P_A \cdot \frac{D_A D_B \lambda^2}{(4\pi d)^2}}$$ Átviteli egyenlet

Kétutas földfelszíni terjedés

$d \gg h_1, h_2$



$\Gamma = -1$: minden visszaverődik

Úthossz különbség : $\Delta = (r_1 + r_2) - r = \sqrt{\underbrace{d^2 + (h_1+h_2)^2}_{d^2+h_1^2+2h_1h_2+h_2^2}} - \sqrt{\underbrace{d^2+(h_1-h_2)^2}_{(d^2+h_1^2+h_2^2)-2h_1h_2}}$

$d \gg h_1, h_2$

$\sqrt{1+x} \approx 1 + \frac{x}{2}$, ha $|x| \ll 1$

$$\Rightarrow \boxed{\Delta = \frac{2h_1h_2}{d}}$$

fázis változás: $e^{-i\beta z}$

fázis különbség: $\Delta \cdot \beta = \Delta \frac{2\pi}{\lambda}$

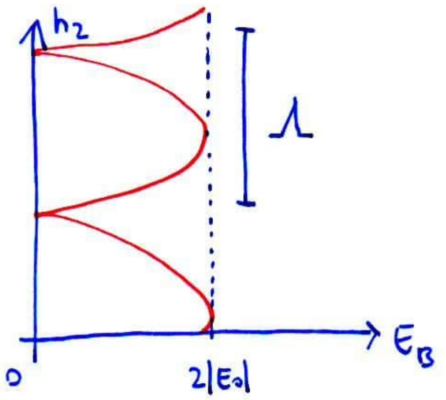
$E_B = E_0 + \Gamma \cdot E_0 \cdot e^{-i\frac{2\pi\Delta}{\lambda}} \longrightarrow |E_B| = |E_0| \left| 1 - e^{-i\frac{2\pi\Delta}{\lambda}} \right| \Leftrightarrow \gamma$

az elsőnek nagyobb az amplitúdója, de $r_1 + r_2 \simeq r \Rightarrow$
$\Rightarrow$ elhanyagolható

$\Leftarrow (=) \; |E_0|\cdot\left|e^{-j\Delta\frac{2\pi}{\lambda}}\right|\cdot\left|e^{j\Delta\frac{\pi}{\lambda}} - e^{-j\Delta\frac{\pi}{\lambda}}\right| =$

$= |E_0|\cdot\left|2\cdot\sin\left(\Delta\frac{\pi}{\lambda}\right)\right| =$

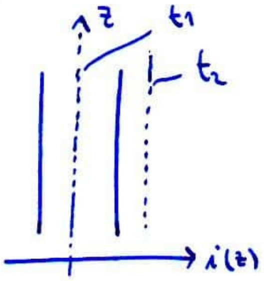
$$\boxed{= |E_0|\cdot\left|2\cdot\sin\left(\frac{h_1 h_2 \beta}{d}\right)\right|}$$



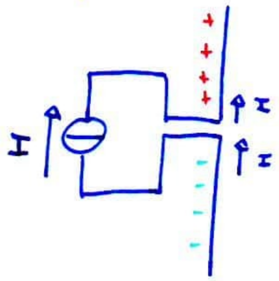
Valóságos rövid dipólantenna

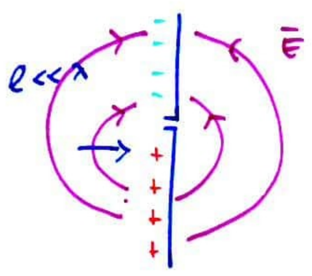
gond:

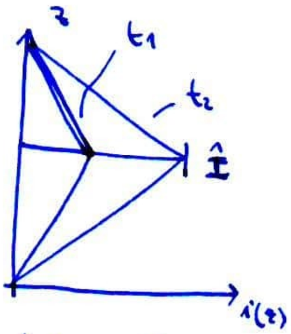
$l \ll \lambda$



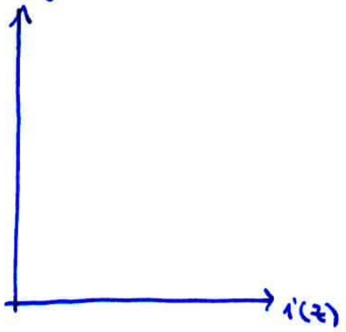
valóság:







háromszög áram



$i(t) = \hat{I}\cdot\cos(\omega t)$

Távoltér $\approx$ Távoltér
(Δ áramú) (egyenletes áram eloszlású)

$\hat{I} = \frac{1}{2}\hat{I}_\Delta$

$P_s = \frac{1}{2}R_s|\hat{I}|^2 = \frac{1}{2}80\pi^2\left(\frac{l}{\lambda}\right)\cdot\left(\frac{\hat{I}_\Delta}{2}\right)^2 =$

$= \frac{1}{2}\underbrace{\left(20\pi^2\left(\frac{l}{\lambda}\right)^2\right)}_{R_{s\Delta}}\cdot|\hat{I}_\Delta|^2$