A2 1. ZH javítókulcs

① a)

$$\int_{-\infty}^{\infty} \frac{dx}{2+3x^2} \underset{\substack{\uparrow \\ \text{integrandus} \\ \text{páros}}}{=} 2\int_0^{\infty} \frac{dx}{2+3x^2} = 2\lim_{A\to\infty} \int_0^A \frac{dx}{2+3x^2} = \lim_{A\to\infty} \int_0^A \frac{1}{1+\left(\frac{\sqrt{3}x}{\sqrt{2}}\right)^2}\,dx$$

$\boxed{3}$

$$= \lim_{A\to\infty} \left[\sqrt{\frac{2}{3}} \operatorname{arctg} \frac{\sqrt{3}x}{\sqrt{2}}\right]_0^A = \sqrt{\frac{2}{3}} \lim_{A\to\infty} \operatorname{arctg} \frac{\sqrt{3}A}{\sqrt{2}} = \sqrt{\frac{2}{3}}\,\frac{\pi}{2} = \underline{\underline{\frac{\pi}{\sqrt{6}}}}$$

$\boxed{3}$ $\boxed{4}$

b)

$$\int_0^2 \frac{dx}{\sqrt[3]{(x-1)^2}} = \lim_{\varepsilon\to 0+} \left(\int_0^{1-\varepsilon} \cdot + \int_{1+\varepsilon}^{2} \cdot\right) \underset{\uparrow}{=} 3\lim_{\varepsilon\to 0+} \left(\left[\sqrt[3]{x-1}\right]_0^{1-\varepsilon} + \left[\sqrt[3]{x-1}\right]_{1+\varepsilon}^{2}\right) =$$

$\boxed{3}$ $\boxed{3}$

$$\int (x-1)^{-2/3}\,dx = \frac{(x-1)^{1/3}}{1/3} + c$$

$$= 3\lim_{\varepsilon\to 0+} \left(-\sqrt[3]{\varepsilon} - (-1) + 1 - \sqrt[3]{\varepsilon}\right) = \underline{\underline{6}}$$

$\boxed{4}$

② $\boxed{5}$ $\boxed{5}$

$$V = \pi \int_1^3 \left(x - \frac{1}{x}\right)^2 dx = \pi \int_1^3 \left(x^2 - 2 + \frac{1}{x^2}\right) dx = \pi \left[\frac{x^3}{3} - 2x - \frac{1}{x}\right]_1^3 =$$

$$= \pi\left[9 - 6 - \frac{1}{3} - \frac{1}{3} + 2 + 1\right] = \underline{\underline{\frac{16\pi}{3}}}$$

$\boxed{5}$

3.

$$f(x,y)=\begin{cases}\dfrac{3xy^3}{x^4+y^4}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

a) $f(x,y)$ folytonos, ha $(x,y)\neq(0,0)$ $\boxed{3}$

egyébként

$$\lim_{(x,y)\to(0,0)}\frac{3xy^3}{x^4+y^4} = \lim_{r\to 0}\frac{3r^4\cos\varphi\sin^3\varphi}{r^4(\cos^4\varphi+\sin^4\varphi)} = \frac{3\cos\varphi\sin^3\varphi}{\cos^4\varphi+\sin^4\varphi}$$

$x=r\cos\varphi$

$y=r\sin\varphi$

ha $\exists$ a limesz

függ φ-től

$\Downarrow$

$\nexists \lim_{(x,y)\to(0,0)} f(x,y)$

$\Rightarrow$ $(0,0)$-ban nem folytonos $\boxed{7}$

b) $(x,y)\neq(0,0)$

$$f'_x(x,y)=\frac{3y^3(x^4+y^4)-3xy^3\cdot 4x^3}{(x^4+y^4)^2}=\frac{3y^7-9x^4y^3}{(x^4+y^4)^2} \quad \boxed{3}$$

$$f'_y(x,y)=\frac{9xy^2(x^4+y^4)-3xy^3\cdot 4y^3}{(x^4+y^4)^2}=\frac{9x^5y^2-3xy^6}{(x^4+y^4)^2} \quad \boxed{3}$$

egyébként:

$$f'_x(0,0)=\lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{0-0}{h}=0 \quad \boxed{2}$$

$$f'_y(0,0)=\lim_{h\to 0}\frac{f(0,h)-f(0,0)}{h}=\lim_{h\to 0}\frac{0-0}{h}=0 \quad \boxed{2}$$

c) ha $(x,y) \neq (0,0)$

f'_x és f'_y folytonos $\Rightarrow$ deriválható $\boxed{3}$

$$\operatorname{grad} f(x,y) = \frac{1}{(x^6+y^6)^2}\left(3y^7 - 5x^4y^3,\ 3x^5y^2 - 9xy^6\right)$$

$(0,0)$-ban nem deriválható, mert a fv nem folytonos. $\boxed{2}$

(4) $f(x,y) = 3y + e^{xy^2} - 2y \operatorname{arctg} \frac{x}{y}$ $\quad P_0(0,1)$

$f'_x(x,y) = y^2 e^{xy^2} - 2y \frac{1}{1+\frac{x^2}{y^2}} \cdot \frac{1}{y} \rightsquigarrow f'_x(0,1) = -1$ $\quad \boxed{3}$

$f'_y(x,y) = 3 + 2xy\, e^{xy^2} - 2 \operatorname{arctg} \frac{x}{y} - 2y \cdot \frac{1}{1+\frac{x^2}{y^2}} \cdot \left(-\frac{x}{y^2}\right)$

$\rightsquigarrow f'_y(0,1) = 3$ $\quad \boxed{3}$

a) az érintősík egyenlete: $\quad f(0,1) = 4$

$z = 4 - 1(x-0) + 3(y-1)$ $\quad \boxed{4}$

$\hookrightarrow \boxed{x - 3y + z = 1}$

b) $f(x,y)$ P_0 egy környezetében deriválható, mert a parciális deriváltak folytonosak

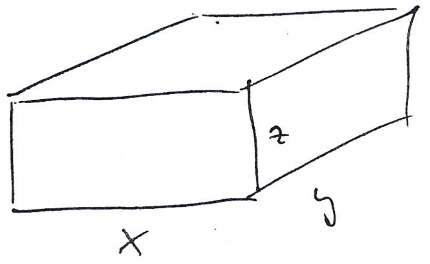
$\Downarrow$

$\operatorname{grad} f(P_0) = (-1, 3)$ $\quad \boxed{3}$

$\underline{v} = 2\underline{i} - 7\underline{j} \Rightarrow \|\underline{v}\| = \sqrt{4+49} = \sqrt{53}$

$\hookrightarrow \underline{e} := \frac{\underline{v}}{\|\underline{v}\|} = \frac{1}{\sqrt{53}}(2, -7)$ $\quad \boxed{3}$

$\left.\frac{df}{d\underline{e}}\right|_{P_0} = \langle \operatorname{grad} f(P_0), \underline{e} \rangle = \frac{1}{\sqrt{53}}(2\cdot(-1) - 7\cdot 3) = \underline{\underline{-\frac{23}{\sqrt{53}}}}$ $\quad \boxed{4}$



$V = xyz = 1000$

a költségek:

$F(x,y,z) = xy\cdot 1 + (2xz + 2yz)\cdot 2 + xy \cdot 9$ E.Ft.

$z = \dfrac{1000}{xy}$

$G(x,y) := F\left(x, y, \dfrac{1000}{xy}\right) = 10xy + \dfrac{4000}{xy}(x+y) = 10xy + \dfrac{4000}{y} + \dfrac{4000}{x}$

(1.) $\dfrac{\partial G}{\partial x}(x,y) = 10y - \dfrac{4000}{x^2} = 0 \quad \rightsquigarrow \quad y = \dfrac{400}{x^2}$

(2) $\dfrac{\partial G}{\partial y}(x,y) = 10x - \dfrac{4000}{y^2} = 0$

$\Downarrow$ (2.)

$x - 400 \cdot \dfrac{x^4}{400^2} = 0$

$x\left(1 - \dfrac{x^3}{400}\right) = 0 \quad \to x = 0 \ \lightning \qquad \to x = \sqrt[3]{400}$

szimmetria miatt $\Rightarrow y = \sqrt[3]{400}$

$G''_{xx}(x,y) = \dfrac{8000}{x^3} \quad \rightsquigarrow \quad G''_{xx}(\sqrt[3]{400}, \sqrt[3]{400}) = \dfrac{8000}{400} = 20$

$G''_{xy} = 10$

$G''_{yy}(x,y) = \dfrac{8000}{y^3} \quad \rightsquigarrow \quad G''_{yy}(\sqrt[3]{400}, \sqrt[3]{400}) = 20$

$H = \begin{pmatrix} 20 & 10 \\ 10 & 20 \end{pmatrix} \quad \rightsquigarrow \quad \det H = 400 - 100 = 300 > 0$

$G''_{xx}(\sqrt[3]{400}, \sqrt[3]{400}) = 20 > 0$

$\Rightarrow$ MINIMUM

minimális költség:

$$x = y = \sqrt[3]{400}$$

$$\boxed{2}$$

$$z = \frac{1000}{\sqrt[3]{400^2}}$$

a költségeknek $\nexists$ maximuma, mert, ha

$$z \to 0 \quad \Rightarrow \quad xy \to \infty \quad \Rightarrow \quad F \to \infty$$

$$\boxed{3}$$