1, $\boxed{17}$

$$\underbrace{\int y\, e^{5y^2}\, dy}_{I_1} = \underbrace{\int \frac{1}{x^2-1}\, dx}_{I_2} \quad (3) \text{ Szeparált dif.}$$

$$I_1 = \frac{1}{10} e^{5y^2} + c \quad (4) \qquad I_2 = \int \left(\frac{-1/2}{x+1} + \frac{+1/2}{x-1} \right) dx \ (4) = -\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + c \quad (4)$$

$x > 1$ esetén implicit mo.:

$$\underline{\underline{\frac{1}{10} e^{5y^2} = -\frac{1}{2}\ln(x+1) + \frac{1}{2}\ln(x-1) + c}} \quad (2)$$

2, $\boxed{20}$ Homogén: $y' = -4xy \Rightarrow \int \frac{dy}{y} = -4\int x\, dx \Rightarrow \ln|y| = -2x^2 + c$

$$\Rightarrow \underline{y_{H,ált}(x) = A \cdot e^{-2x^2}, \ A \in \mathbb{R}} \quad (7)$$

$y_{I,p}(x) = A(x)\, e^{-2x^2}$, beírva:

$$A'(x)\, \cancel{e^{-2x^2}} + \cancel{A(x) \cdot (-4x)\, e^{-2x^2}} + \cancel{4x\, A(x)\, e^{-2x^2}} = x \cdot \cancel{e^{-2x^2}}$$

$$A' = x \Rightarrow A = \int x\, dx = \frac{x^2}{2} \Rightarrow y_{I,p}(x) = \frac{x^2}{2} e^{-2x^2}$$

$$\underline{y_{I,ált}(x) = \left(A + \frac{x^2}{2}\right) e^{-2x^2}, \ A \in \mathbb{R}} \quad (10)$$

Illesztés: $y(1) = 5 \Rightarrow 5 = \left(A + \frac{1}{2}\right) e^{-2} \Rightarrow \underline{A = 5e^2 - \frac{1}{2}}$

$$\underline{\underline{y(x) = \left(5e^2 - \frac{1}{2} + \frac{x^2}{2}\right) e^{-2x^2}}} \quad (3)$$

3, a, $\boxed{8}$

$$\left.\begin{array}{l} x \Rightarrow \lambda_{1,2} = 0 \\ \sin(2x) \Rightarrow \lambda_{3,4} = \pm 2i \end{array}\right\} (4) \quad \lambda^2 \underbrace{(\lambda+2i)(\lambda-2i)}_{\lambda^2+4} = \lambda^4 + 4\lambda^2$$

$$\Rightarrow \underline{\underline{y^{(4)} + 4y'' = 0}} \quad (4)$$

3/b, $y'' - 5y' + 6y = 3\sin(2x)$

$\boxed{8}$ $\lambda^2 - 5\lambda + 6 = (\lambda-2)(\lambda-3) = 0 \implies \lambda_1 = +2,\ \lambda_2 = +3$

$\implies$ Nincs külső rezonancia (4p)

$\underline{\underline{y_{I,p}(x) = A\sin(2x) + B\cos(2x)}}$ (4p)

4, a, $\sum_{n=1}^{\infty} a_n = \lim_{k\to\infty} \sum_{n=1}^{k} a_n$ (4)

b, $\underline{T.:}$ Ha $\sum_{n=1}^{\infty} a_n$ konvergens, akkor $a_n \xrightarrow{n\to\infty} 0$ (4)

(6) $\underline{B.:}$ $S_{n+1} = \sum_{k=1}^{n+1} a_k = S_n + a_{n+1} = \sum_{k=1}^{n} a_k + a_{n+1} \Big/ {n\to\infty}$

$$\begin{array}{ccccc} S_{n+1} & = & S_n & + & a_{n+1} \\ \downarrow & & \downarrow & & \downarrow \\ S & = & S & + & \lim_{n\to\infty} a_n \end{array} \implies \lim_{n\to\infty} a_n = 0$$

5, a, $\sum_{n=1}^{\infty} \underbrace{(-1)^n (2n+3)^{-1/2}}_{a_n}$ $\underline{\text{konvergens}}$, mert Leibniz, mivel

(7) • alternál; • $\frac{1}{\sqrt{2n+3}} \to 0$; • $|a_n|$ mon. csökken.

b, $\underline{\text{Divergens}}$, mert $\frac{n-4}{n^2} \underset{\uparrow}{\geq} \frac{n/2}{n^2} = \frac{1}{2n}$, és $\sum_n \frac{1}{2n} = \infty$

(7) ha $n \geq 9$ (minoráns krit.)

c, $\frac{a_{n+1}}{a_n} = \frac{(n+4)!}{\sqrt{(2n+2)!}} \cdot \frac{\sqrt{(2n)!}}{(n+3)!} = \frac{n+4}{\sqrt{(2n+1)(2n+2)}} = \frac{1+4/n}{\sqrt{(2+1/n)(2+2/n)}}$

(7) $\xrightarrow{n\to\infty} \frac{1}{\sqrt{2\cdot 2}} = \frac{1}{2} < 1 \xrightarrow[\text{krit.}]{\text{Hányados}}$ $\underline{\text{konvergens}}$

6, a, β (det. szerint.) c, γ $\left(\begin{array}{c} q^2 - q - 2 = (q-2)(q+1) \\ q_1 = +2;\ q_2 = -1 \end{array}\right)$

$\boxed{12}$ b, γ (Pl.: $y' = \frac{y}{x}$) d, β Leibniz, de $\frac{2}{3} < 1$.