I. Robotok kinematikája és geometriai modellje

2008. 02. 10

IIT: Lantos
Kiss Bálint
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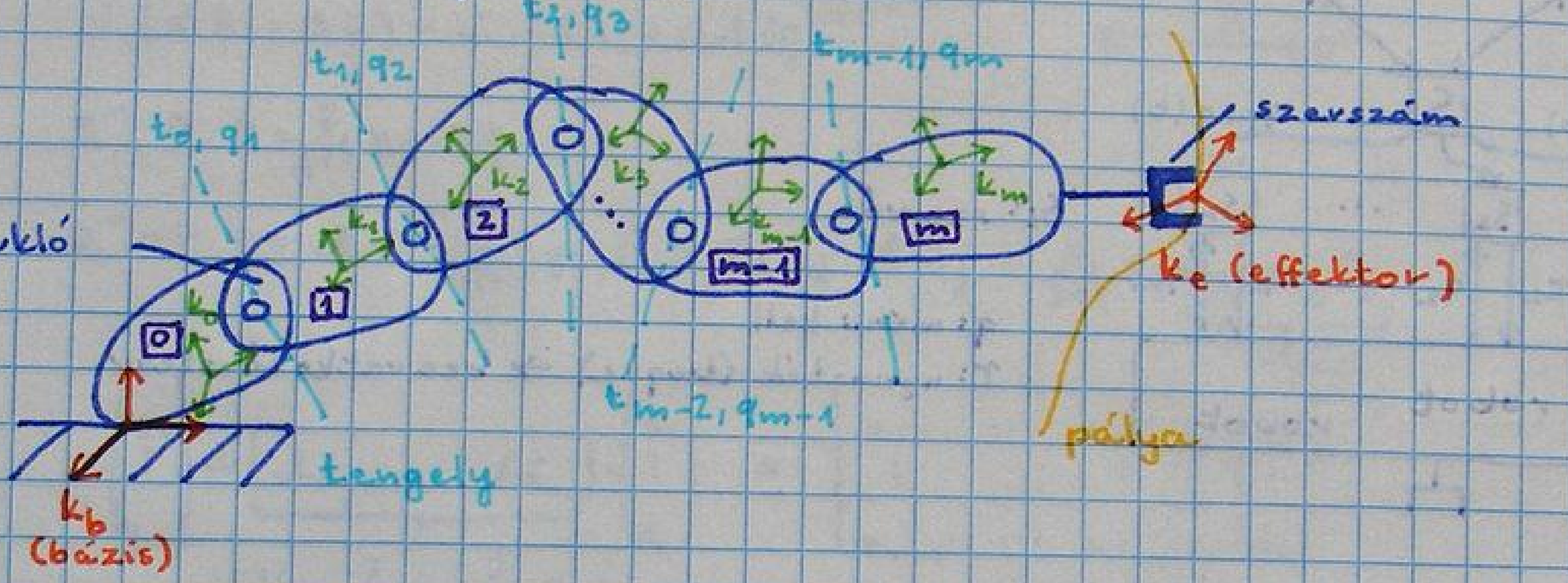
1 ZH (vizsgába 20 p)
edu.iit.bme.hu
6 részből áll
1-4 L, 5 H, 6 K
könyv: Robotok irányítása

Jegyzetben: Robotok programozása

1.) Robotikai alapfogalmak:

robot:
- irányított mechanizmus
- előírható pályán mozog (mobilis robotok, rögzített robot)
- előírható feladatok



csukló: egy szabadságfokkal rendelkezik

q_m: indexe annak, amit a csukló mozgat

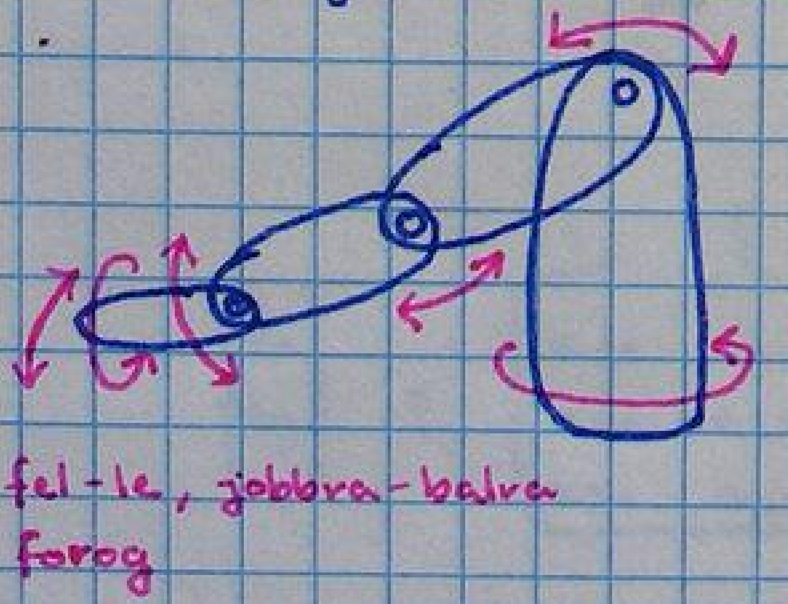
tengely: forgatható (fogaskerék)
mozgatható (fogasléc)

pozíció { TCP: True Center Point k_b szerint megadva
orientáció k_e szerint

effektor: munkavégző eszköz (E)

mechanikai szabadságfok: összes szegmens térbeli helyzetét le tudja írni (csuklók számától függ)

merevség: két koordináta rendszer (kerek)



Emberi kar:
- 6 forgás → 6 csukló → 6 szabadságfok
- orientáció, 3 szabad
- elágazás nélküli (ember modellezésénél már lenne elágazás)
- nyílt láncú
- merev testű

k_i: csuklóknál a tengelyre helyezzük az origót → gömbi geometria
dinamikánál a tömegközéppontba jó helyezni (TMK változhat 8)

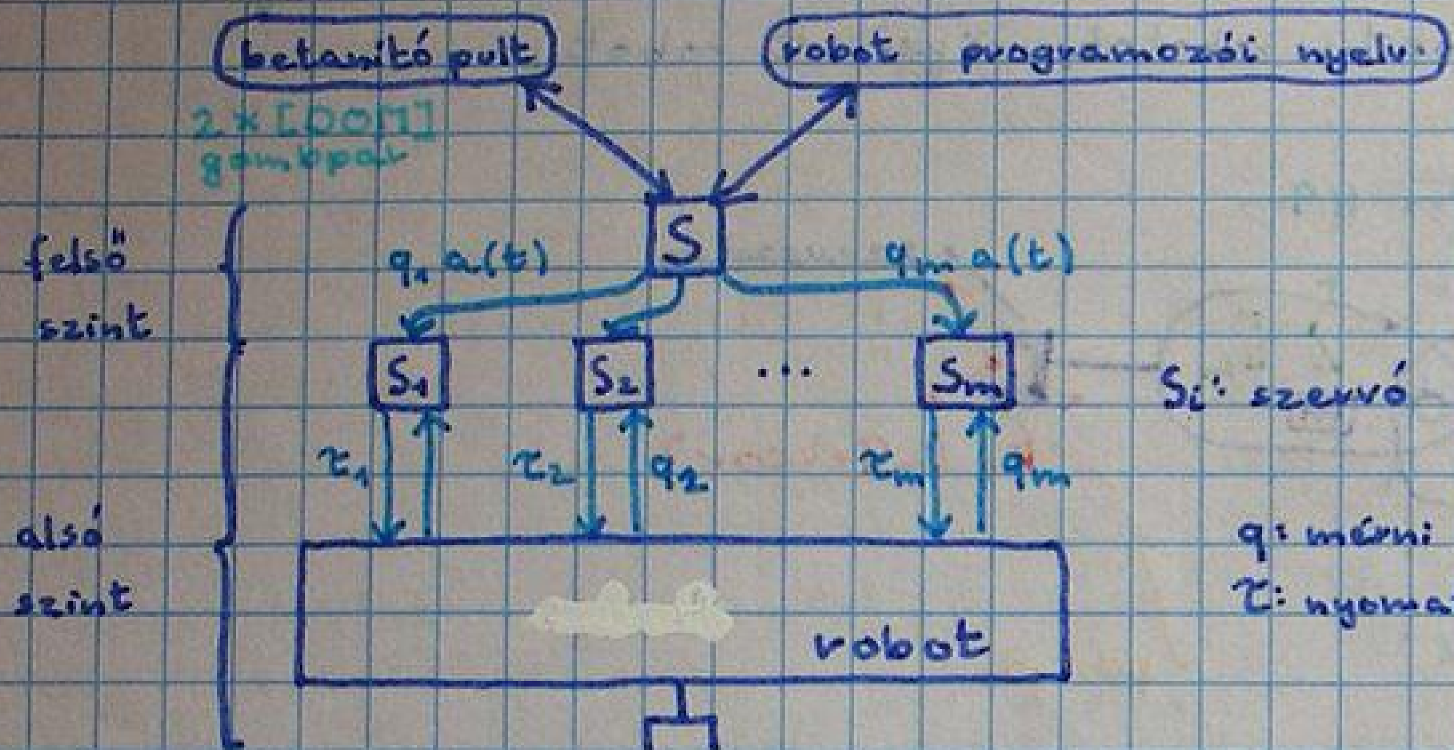
Szegmens (link):	0, 1, ..., m
Csukló (joint):	$t_0, t_1, \ldots, t_{m-1}$
Csukló koordináta (joint variable):	$q_1, q_2, \ldots, q_m$ (1 szabadságfok)
Rotációs csukló (revolute joint):	R
Transzlációs csukló (prismatic / sliding joint):	T
Csuklóképlet:	RRTRRR (6 szabadságfokú)
Szabadságfok (dimension of freedom):	6-DOM

2.) Végberendezés (end effector):

- megfogó (gripper) ⇒ anyagmozgató robot; mechanikus vagy szervós (1-DOM)
- szerszám (tool) ⇒ szerelő robot (reassembly)
- ponthegesztő berendezés (spot welding)
- ívhegesztő berendezés (arc welding)
- festékszóró pisztoly (painting gun)

} ⇒ technológiai robot

3.) Hierarchia:



S_i: szervó

q: mérni kell

τ: nyomaték (torque) → beavatkozó szerv

S_i rendszer:
- motor, áttétel
- érzékelő (q_i)
- alapjel ($q_i a$)
- hiba: $q_{ri} = q_i a - q_i$
- szabályozási algoritmus
- teljesítményi elektronika (PWM → τ_i)

4.) Irányítási módok:
- pont-pont irányítás (Point to Point)
  - $q_i a(t)$ → csak a célhelyzet
  - ütközésveszély
- folytonos pálya irányítás (Continuous Path Control)
  - $q_i a(t)$
  - kisebb az ütközésveszély
- koordinált mozgás

5.) Érzékelők:

belső:
- q_i
- $\dot{q}_i$ (csuklóváltozó sebessége)

külső:
- pozíció / orientáció
- lézertechnika
- sztereó technika
- erő / nyomaték érzékelő (6-komponensű)

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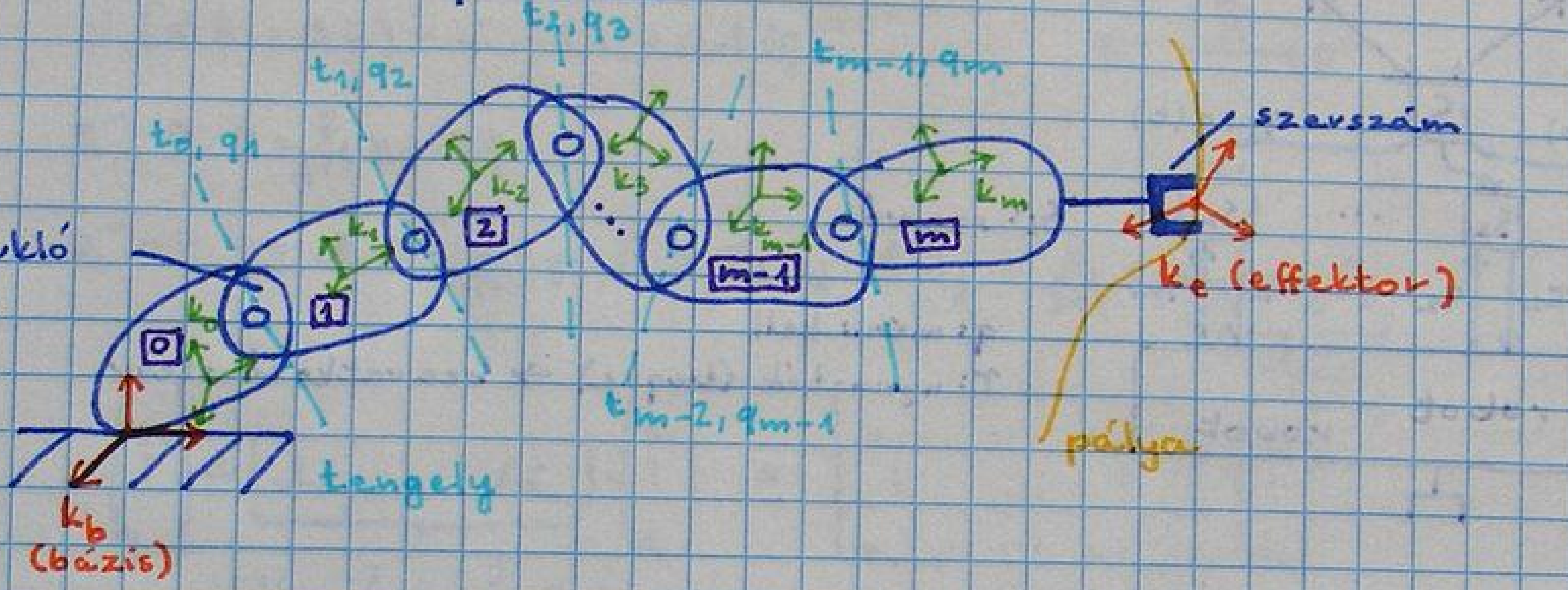

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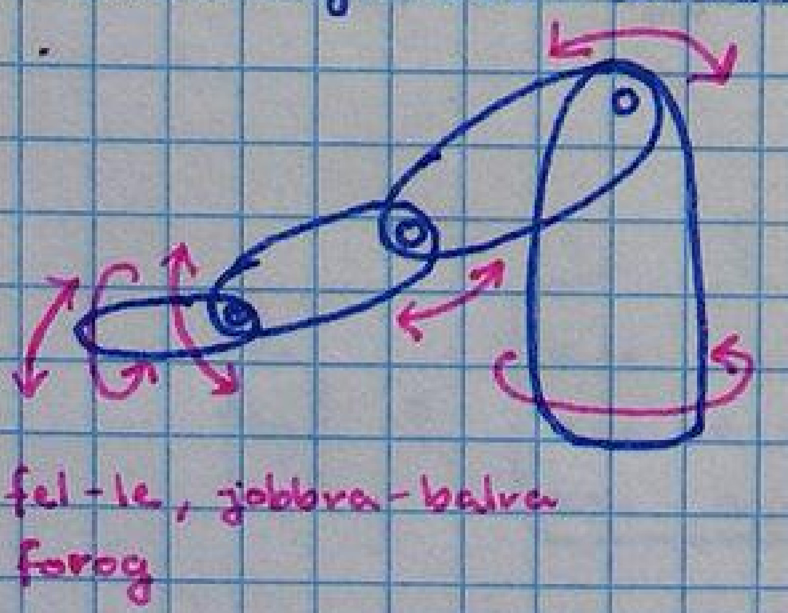

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(spot welding, arc welding, painting gun) ⇒ technológiai robot

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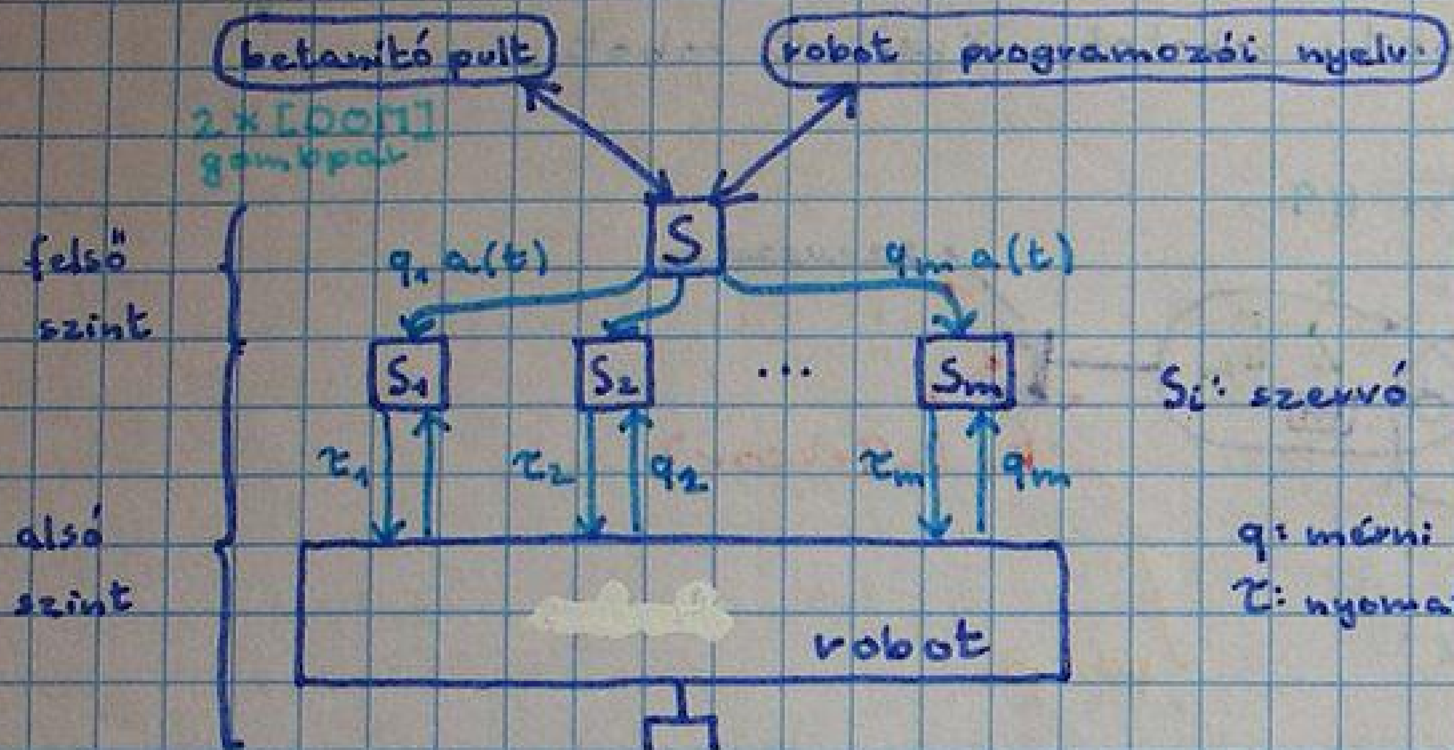

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1.) Robotok orientációja

Vektortér: R^n

$\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n$ bázis

$\bar{x} = \sum_{j=1}^{n} x_j \cdot \bar{e}_j$

Lineáris leképezés:

$$\begin{array}{ccc} R^n & \xrightarrow{\mathcal{A}} & R^m \\ \bar{e}_1 & & \bar{f}_1 \\ \vdots & & \vdots \\ \bar{e}_n & \overset{=}{A} & \bar{f}_n \\ R^n & \longrightarrow & R^m \end{array}$$

$\mathcal{A}: R^n \longrightarrow R^n$

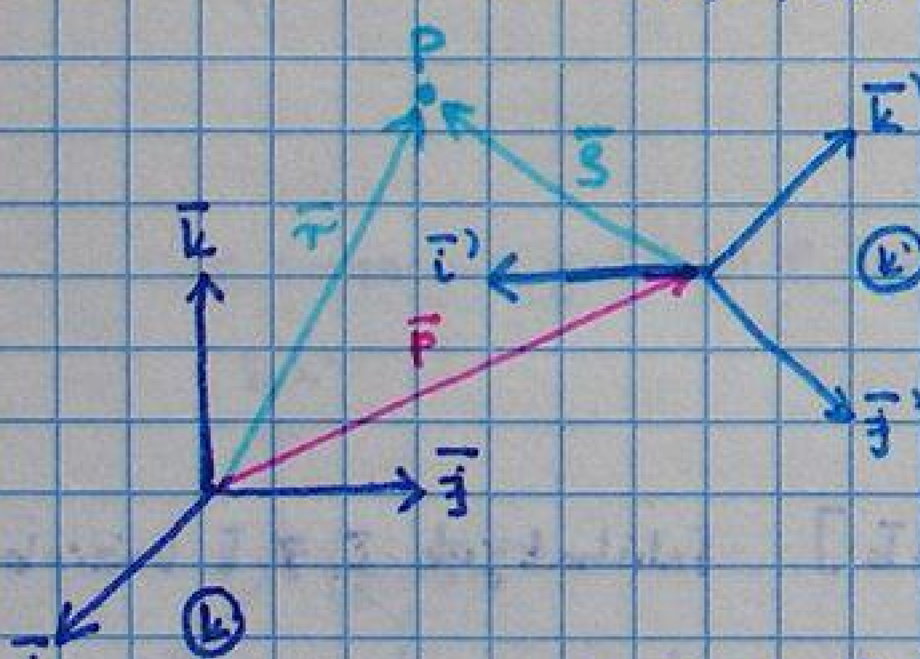
$\mathcal{A}(\alpha \bar{x} + \beta \bar{y}) = \alpha \mathcal{A}(\bar{x}) + \beta \mathcal{A}(\bar{y})$

$R^n \rightarrow \bar{x} = \sum_{j=1}^{n} x_j \cdot \bar{e}_j$

$R^m \rightarrow \bar{y} = \mathcal{A}\bar{x} = \sum_{i=1}^{m} y_i \cdot \bar{f}_i = \sum_{j=1}^{n} x_j \cdot \mathcal{A}\bar{e}_j = \sum_{j=1}^{n} x_j \underbrace{\sum_{i=1}^{m} a_{ij} \bar{f}_j}_{\mathcal{A}\bar{e}_i}$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = \underbrace{\left[\mathcal{A}\bar{e}_1 \; \dots \; \mathcal{A}\bar{e}_n\right]}_{} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

felírva $\bar{f}_1, \dots, \bar{f}_m$ bázisban



$\bar{r} = \bar{s} + \bar{p}$

$r = r_x \bar{i} + r_y \bar{j} + r_z \bar{k}$

$p = p_x \bar{i} + p_y \bar{j} + p_z \bar{k}$

$s = s_x \bar{i} + s_y \bar{j} + s_z \bar{k}$

$\bar{i}' = a_{11} \bar{i} + a_{21} \bar{j} + a_{31} \bar{k}$

$\bar{j}' = a_{12} \bar{i} + a_{22} \bar{j} + a_{32} \bar{k}$

$\bar{k}' = a_{13} \bar{i} + a_{23} \bar{j} + a_{33} \bar{k}$

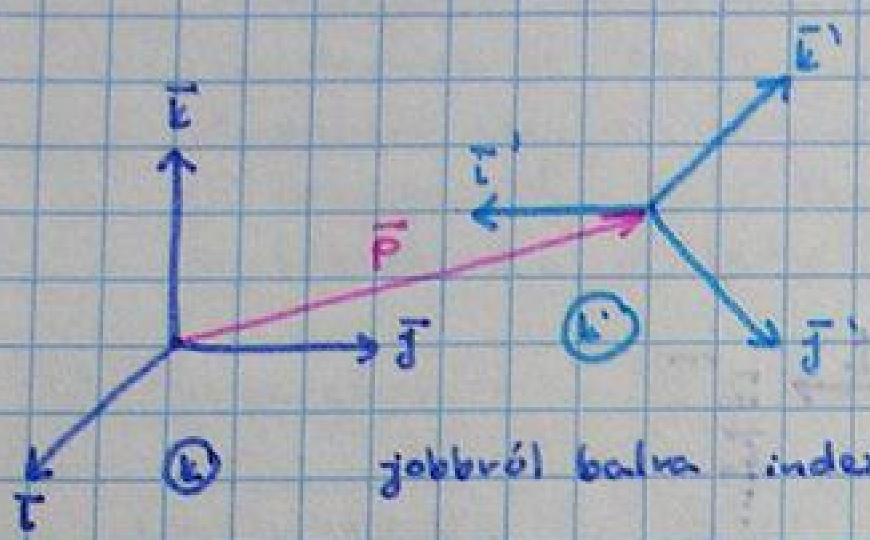
$$\begin{aligned} r_x \bar{i} + r_y \bar{j} + r_z \bar{k} = {} & s_x (a_{11} \bar{i} + a_{21} \bar{j} + a_{31} \bar{k}) + \\ & s_y (a_{12} \bar{i} + a_{22} \bar{j} + a_{32} \bar{k}) + \\ & s_z (a_{13} \bar{i} + a_{23} \bar{j} + a_{33} \bar{k}) + \\ & p_x \bar{i} + p_y \bar{j} + p_z \bar{k} \end{aligned}$$

$$\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \underbrace{\begin{array}{ccc} \overbrace{}^{\bar{i}} & \overbrace{}^{\bar{j}} & \overbrace{}^{\bar{k}} \\ \end{array} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{\text{felírva } \bar{i}, \bar{j}, \bar{k} \text{ bázisban} \; \overset{=}{A}} \cdot \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} + \underbrace{\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}}_{\bar{p}}$$

$$\begin{pmatrix} r_1\\ r_2\\ r_3\end{pmatrix} = \overline{\overline{A}} \cdot \begin{pmatrix} s_x\\ s_y\\ s_z\end{pmatrix} + \begin{pmatrix} p_x\\ p_y\\ p_z\end{pmatrix} \Rightarrow \begin{matrix} \bar{r} = \overline{\overline{A}}\cdot\bar{s} + \bar{p}\\ 1 = 1 \end{matrix}$$

homogén transzformáció lineáris

$$\begin{pmatrix} \bar{r}\\ \\ 1\end{pmatrix} = \underbrace{\left(\begin{array}{c:c} \overline{\overline{A}} & \bar{p}\\ \hdashline \bar{0}^T & 1 \end{array}\right)}_{\overline{\overline{T}}_{K,K'}} \cdot \begin{pmatrix} \bar{s}\\ \\ 1\end{pmatrix}$$



jobbról balra indexel

$$\overline{\overline{T}}_1 \cdot \overline{\overline{T}}_2 = \left[\begin{array}{c:c} \overline{\overline{A}}_1 & \bar{p}_1\\ \hdashline \bar{0}^T & 1\end{array}\right] \cdot \left[\begin{array}{c:c} \overline{\overline{A}}_2 & \bar{p}_2\\ \hdashline \bar{0}^T & 1\end{array}\right] = \left[\begin{array}{c:c} \overline{\overline{A}}_1 \cdot \overline{\overline{A}}_2 & \overline{\overline{A}}_1 \cdot \bar{p}_2 + \bar{p}_1\\ \hdashline \bar{0}^T & 1\end{array}\right]$$

$$\overline{\overline{T}} \cdot \overline{\overline{T}}^{-1} = \underbrace{\overline{\overline{I}}_4}_{\text{egységmátrix}} = \left[\begin{array}{c:c} \overline{\overline{A}} & \bar{p}\\ \hdashline \bar{0}^T & 1\end{array}\right] \cdot \left[\begin{array}{c:c} \overline{\overline{A}}^{-1} & -\overline{\overline{A}}^{-1}\cdot\bar{p}\\ \hdashline \bar{0}^T & 1\end{array}\right] = \left[\begin{array}{c:c} \overline{\overline{I}}_3 & 0\\ \hdashline \bar{0}^T & 1\end{array}\right]$$

$$\overline{\overline{T}}^{-1} = \left[\begin{array}{c:c} \overline{\overline{A}}^{-1} & -\overline{\overline{A}}^{-1}\cdot\bar{p}\\ \hdashline \bar{0}^T & 1\end{array}\right]$$

Vektorszorzás: $\bar{a}$ fix, $\mathcal{A}\bar{x} = \bar{a} \times \bar{x}$

$$\begin{array}{ccc} \mathbb{R}^3 & \xrightarrow{\mathcal{A}} & \mathbb{R}^3\\ \bar{i} & & \bar{i}\\ \bar{j} & & \bar{j}\\ \bar{k} & & \bar{k}\\ \mathbb{R}^3 & \xrightarrow{[\bar{a}\times]} & \mathbb{R}^3 \end{array}$$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z \end{vmatrix}$$

$[\bar{a}\times] = [\bar{a}\times\bar{i} \quad \bar{a}\times\bar{j} \quad \bar{a}\times\bar{k}]$ felírhatjuk $\bar{i}, \bar{j}, \bar{k}$ bázisban

$$\bar{a}\times\bar{i} = \begin{vmatrix} i & j & k\\ a_x & a_y & a_z\\ 1 & 0 & 0 \end{vmatrix} = \bar{i}\cdot\underbrace{\begin{vmatrix} a_y & a_z\\ 0 & 0\end{vmatrix}}_{0} - \bar{j}\cdot\underbrace{\begin{vmatrix} a_x & a_z\\ 1 & 0\end{vmatrix}}_{-a_z} + \bar{k}\cdot\underbrace{\begin{vmatrix} a_x & a_y\\ 1 & 0\end{vmatrix}}_{-a_y} = 0\cdot\bar{i} + a_z\cdot\bar{j} - a_y\bar{k}$$

$$[\bar{a}\times] = \begin{bmatrix} 0 & -a_z & a_y\\ a_z & 0 & -a_x\\ -a_y & a_x & 0 \end{bmatrix}$$

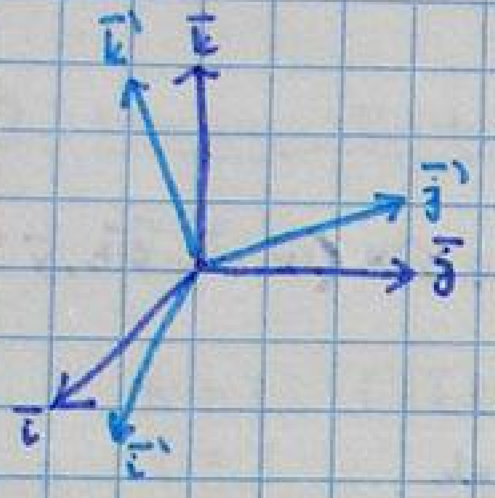
Diadikus szorzat: $\bar{a}, \bar{b}$ fix, $\mathcal{A}\bar{x} = \bar{a}\langle\bar{b}, \bar{x}\rangle$

$$\begin{array}{ccc} \mathbb{R}^3 & \xrightarrow{\mathcal{A}} & \mathbb{R}^3\\ \bar{i} & & \bar{i}\\ \bar{j} & & \bar{j}\\ \bar{k} & [a\circ b] & \bar{k}\\ \mathbb{R}^3 & \xrightarrow{\hspace{2em}} & \mathbb{R}^3 \end{array}$$

$$[\bar{a}\circ\bar{b}] = [\bar{a}\overbrace{\langle\bar{b},\bar{i}\rangle}^{b_x} \quad \bar{a}\overbrace{\langle\bar{b},\bar{j}\rangle}^{b_y} \quad \bar{a}\overbrace{\langle\bar{b},\bar{k}\rangle}^{b_z}] = \begin{bmatrix} a_x b_x & a_x b_y & a_x b_z\\ a_y b_x & a_y b_y & a_y b_z\\ a_z b_x & a_z b_y & a_z b_z \end{bmatrix}$$

2.) Orientáció jellemzői

a.) Direkt módszer:

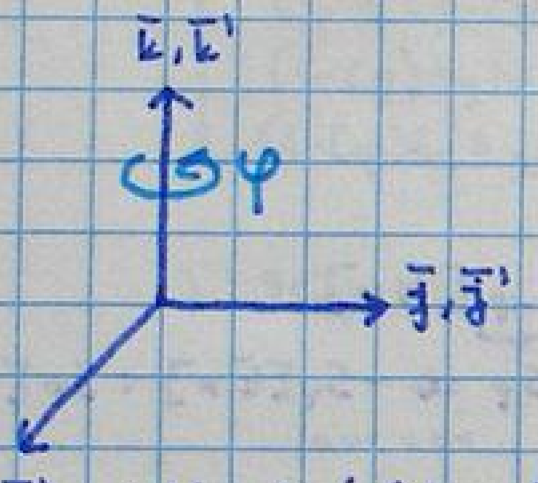


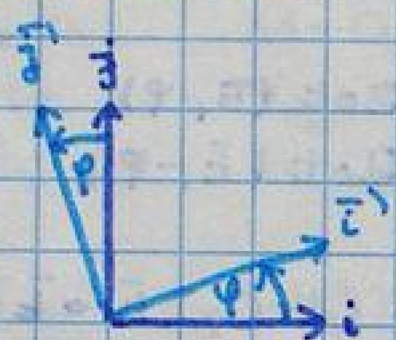
$$\overline{\overline{A}} = \begin{bmatrix} \bar{i}' & \bar{j}' & \bar{k}' \end{bmatrix}$$

felírva az $\bar{i}, \bar{j}, \bar{k}$ bázisban

b.) Elemi forgatás:

- forgatás z körül φ szöggel





$\bar{i}, \bar{i}'$ jobbsodrású koordináta rendszer

$$\begin{bmatrix} \bar{i}_{elf} & \bar{j}_{elf} & \bar{k}_{elf} \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

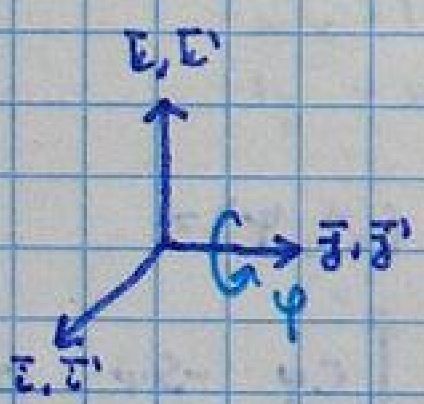
felírva $\bar{i}, \bar{j}, \bar{k}$ bázisban

$\cos\varphi = C\varphi, \quad \cos q_1 = C_1$

$\cos(\varphi + \psi) = C\varphi\psi, \quad \cos(q_1 + q_2) = C_{12}$

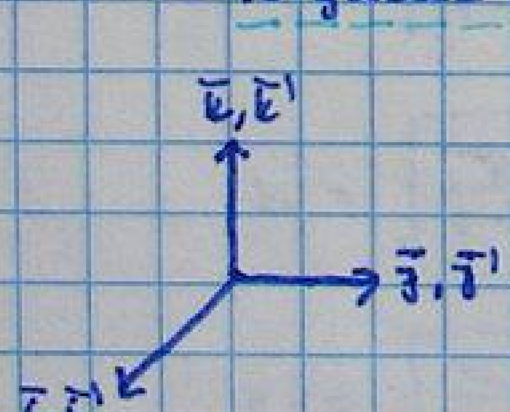
$$\overline{\overline{Rot}}(z, \varphi) = \begin{bmatrix} C\varphi & -S\varphi & 0 \\ S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- forgatás y körül φ szöggel



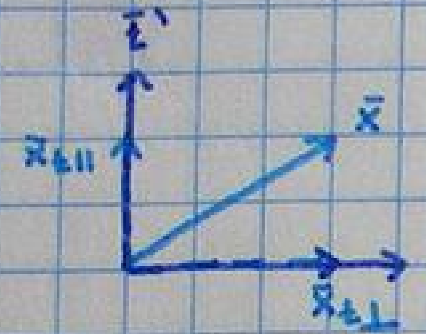
$$\overline{\overline{Rot}}(y, \varphi) = \begin{bmatrix} C\varphi & 0 & S\varphi \\ 0 & 1 & 0 \\ -S\varphi & 0 & C\varphi \end{bmatrix}$$

- forgatás x körül φ szöggel



$$\overline{\overline{Rot}}(x, \varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\varphi & -S\varphi \\ 0 & S\varphi & C\varphi \end{bmatrix}$$

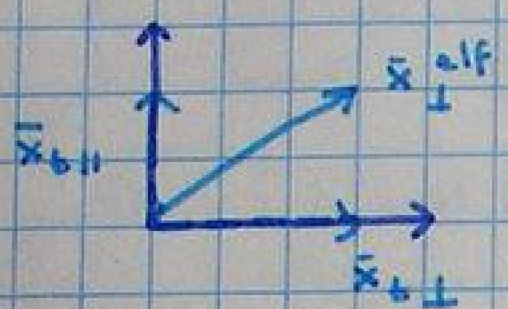
c.) Forgatás általános tengely körül $\quad \bar t \; (|\bar t| = 1), \; \varphi$



$$\bar x \to \bar x_{t\parallel} = \underbrace{\bar t}_{\text{irány}} \underbrace{\langle \bar t, \bar x \rangle}_{\text{hossz}}$$

$$\bar x_t \cdot \bar x_{t\perp} = \bar x - \bar x_{t\parallel} = \bar x - \bar t \langle \bar t, \bar x \rangle$$

$$\bar x^{elf} = \bar x_{t\parallel} + \bar x_{t\perp}^{elf}$$



$$\bar x_{t\perp}^{elf} = C\varphi\, \bar x_{t\perp} + S\varphi\, \bar t \times \bar x_{t\perp} = C\varphi (\bar x - \bar t \langle \bar t, \bar x \rangle) + S\varphi\, \bar t \times (\bar x - \bar t \langle \bar t, \bar x \rangle)$$

$$\bar x^{elf} = \bar t \langle \bar t, \bar x \rangle + C\varphi (\bar x - \bar t \underbrace{\langle \bar t, \bar x \rangle}_{\text{skalárja } \bar t\text{-nek és } \bar x\text{-nek}}) + S\varphi\, \bar t \times \bar x$$

$$\bar x^{elf} = C\varphi\, \bar x + (1 - C\varphi)\, \bar t \langle \bar t, \bar x \rangle + S\varphi\, \bar t \times \bar x$$

$$\begin{array}{ccc} \mathbb{R}^3 & \xrightarrow{\mathrm{Rot}(\bar t, \varphi)} & \mathbb{R}^3 \\ \bar i & & \bar i \\ \bar j & & \bar j \\ \bar k & \xrightarrow{\overline{\overline{\mathrm{Rot}}}(\bar t, \varphi)} & \bar k \\ \mathbb{R}^3 & & \mathbb{R}^3 \end{array}$$

$$\overline{\overline{\mathrm{Rot}}}(\bar t, \varphi) = C\varphi\, \overline{\overline{I}} + (1 - C\varphi)\, \bar t \circ \bar t + S\varphi [\bar t \times]$$ Rodriquez-képlet

Forgatás: $\mathrm{Rot}(\bar t, \varphi)$

Forgatás inverze: $\mathrm{Rot}(\bar t, -\varphi)$

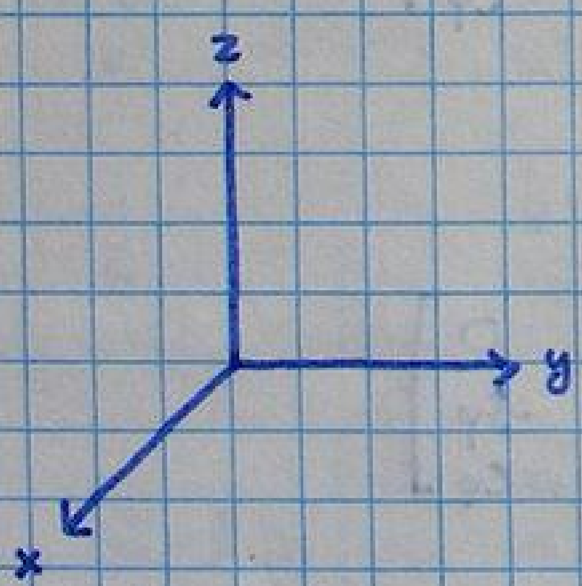
$$\mathrm{Rot}(\bar t, -\varphi) = C\varphi\, \overline{\overline{I}} + (1 - C\varphi)\, \bar t \circ \bar t + \underbrace{S_{-\varphi}}_{-S\varphi} [\bar t \times]$$

$$\Rightarrow -S\varphi [\bar t \times] = S\varphi [\bar t \times]^T$$

$$\overline{\overline{\mathrm{Rot}}}(\bar t, \varphi)^{-1} = \overline{\overline{\mathrm{Rot}}}(\bar t, \varphi)^{-1}$$

ortonormált mátrix esetén: $\overline{\overline{A}}^{-1} = \overline{\overline{A}}^T \Rightarrow \begin{bmatrix} \overline{\overline{A}} & \bar p \\ \bar 0^T & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \overline{\overline{A}}^T & -\overline{\overline{A}}^T \bar p \\ \bar 0^T & 1 \end{bmatrix}$

d.) Euler-szögek



$$\overline{\overline{\mathrm{Euler}}}(\varphi, \vartheta, \psi) = \overline{\overline{\mathrm{Rot}}}(z, \varphi)\, \overline{\overline{\mathrm{Rot}}}(y', \vartheta)\, \overline{\overline{\mathrm{Rot}}}(z'', \psi) =$$

$$= \begin{bmatrix} -C\varphi & -S\varphi & 0 \\ S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\vartheta & 0 & S\vartheta \\ 0 & 1 & 0 \\ -S\vartheta & 0 & C\vartheta \end{bmatrix} \cdot \begin{bmatrix} C\psi & -S\psi & 0 \\ S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \Big[$$

e.) RPY szögek:

R: roll
P: pitch
Y: yaw

$$\mathrm{RPY}(\varphi, \vartheta, \psi) = \overline{\overline{\mathrm{Rot}}}(z, \varphi)\, \overline{\overline{\mathrm{Rot}}}(y', \vartheta)\, \overline{\overline{\mathrm{Rot}}}(x'', \psi) =$$

$$= \begin{bmatrix} C\varphi & -S\varphi & 0 \\ S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\vartheta & 0 & S\vartheta \\ 0 & 1 & 0 \\ -S\vartheta & 0 & C\vartheta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & -S\psi \\ 0 & S\psi & C\psi \end{bmatrix} =$$

$$= \Big[$$

f.) Kvaternió: $\bar q = (\sin\frac{\varphi}{2} \cdot \bar t, \cos\frac{\varphi}{2} \cdot 1)$

g.) $e^{[\bar t, \times]\varphi} \Leftrightarrow$ Rodriquez-képlet

2009.02.18.

Inverz orientáció probléma

1. Inverz Rodrigez-feladat

$$\overline{\overline{Rot}}(\bar{t},\varphi) = C\varphi \bar{\bar{I}} + (1-C\varphi)[\bar{t}\circ\bar{t}] + S\varphi[\bar{t}\times] = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}$$

$\bar{t} = ?$ $\quad (|\bar{t}| = 1)$

$\varphi = ?$

Alkalmazás: $\bar{\bar{A}}_\alpha = \overline{\overline{Rot}}(\bar{t},\varphi)\bar{\bar{A}} \Rightarrow \overline{\overline{Rot}}(\bar{t},\varphi) = \bar{\bar{A}}_\alpha \bar{\bar{A}}^T = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix} \xrightarrow[\text{Rodrigez}]{\text{inverz}} \bar{t},\varphi$

$\downarrow$

$\bar{e}_{orient} = (\bar{t},\varphi)$

$$\begin{bmatrix} C\varphi + (1-C\varphi)t_x t_x & (1-C\varphi)t_x t_y - S\varphi t_z & (1-C\varphi)t_x t_z + S\varphi t_y \\ (1-C\varphi)t_y t_x + S\varphi t_z & C\varphi + (1-C\varphi)t_y t_y & (1-C\varphi)t_z t_z - S\varphi t_x \\ (1-C\varphi)t_z t_x - S\varphi t_y & (1-C\varphi)t_z t_y + S\varphi t_x & C\varphi + (1-C\varphi)t_z t_z \end{bmatrix} = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}$$

1.) $3C\varphi + (1-C\varphi)\underbrace{(t_x^2 + t_y^2 + t_z^2)}_{1} = l_x + m_y + n_z \quad \Rightarrow \quad C\varphi = \dfrac{l_x + m_y + n_z - 1}{2}$

arccos helyett célszerűbb arctant használni (meredekebben indul, így pontosabb)

~~arccos~~ $\quad atan(T\varphi) \rightarrow \varphi$

$(S\varphi, C\varphi) \xrightarrow{atan2} \varphi$

2.) $$\left.\begin{aligned} m_z - n_y &= 2S\varphi t_x \\ n_x - l_z &= 2S\varphi t_y \\ l_y - m_x &= 2S\varphi t_z \end{aligned}\right\} \Rightarrow S\varphi = \frac{\sqrt{(m_z - n_y)^2 + (n_x - l_z)^2 + (l_y - m_x)^2}}{2}$$

$\Leftrightarrow \varphi \in [0, \pi]$

$(S\varphi, C\varphi) \xrightarrow{atan2} \varphi \ (1x)$

↳ megoldások száma

$$t_x = \sqrt{\frac{l_x - C\varphi}{1 - C\varphi}} \cdot sign(m_z - n_y)$$

$$t_y = \sqrt{\frac{m_y - C\varphi}{1 - C\varphi}} \cdot sign(\overset{n_x - l_z}{\cancel{l_y - m_x}})$$

$$t_z = \sqrt{\frac{n_z - C\varphi}{1 - C\varphi}} \cdot sign(\overset{l_y - m_x}{\cancel{n_x - l_z}})$$

Szinguláris konfiguráció: $1 - C\varphi = 0$

nincs forgatás, akármilyen $\bar{t}$ jó

$\bar{e}_{orient} = \bar{t}\cdot 0 = 0$

2 Inverz Euler feladat

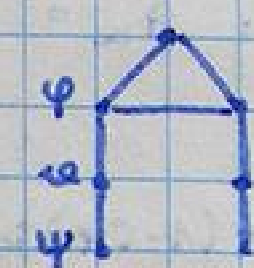
$$\overline{\overline{\text{Euler}}}(\varphi,\vartheta,\psi) = \begin{bmatrix} C_\varphi C_\vartheta C_\psi - S_\varphi S_\psi & -C_\varphi C_\vartheta S_\psi - S_\varphi C_\psi & C_\varphi S_\vartheta \\ S_\varphi C_\vartheta C_\psi + C_\varphi S_\psi & -S_\varphi C_\vartheta S_\psi + C_\varphi C_\psi & S_\varphi S_\vartheta \\ -S_\vartheta C_\psi & S_\vartheta S_\psi & C_\vartheta \end{bmatrix} = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}$$

Alkalmazás: - nevezetes robotok utolsó 3 csuklója Euler filozófiájú: $q_4 = \varphi,\ q_5 = \vartheta,\ q_6 = \psi$ } Stanford, Puma

- nevezetes robot programozási nyelvek az orientációt Euler-szögben fejezik ki: $o = \varphi,\ a = \vartheta,\ t = \psi$; o, a, t: szögek } VAL, ARPS

1.) $T_\varphi = \frac{n_y}{n_x} \xrightarrow{atan} \varphi$ (2×)

2.) $\left.\begin{aligned} C_\varphi n_x + S_\varphi n_y &= S_\vartheta \\ n_z &= C_\vartheta \end{aligned}\right\} \rightarrow atan2 \rightarrow \vartheta$ (1x)



3.) $\left.\begin{aligned} -S_\varphi l_x + C_\varphi l_y &= S_\psi \\ -S_\varphi m_x + C_\varphi m_y &= C_\psi \end{aligned}\right\} \rightarrow atan2 \rightarrow \psi$ (1x)

Szinguláris konfig.: $n_x = n_y = 0$

$n_z = +1 \Rightarrow \vartheta = 0 \Rightarrow \left.\begin{aligned} S_{\varphi+\psi} &= l_y \\ C_{\varphi+\psi} &= l_x \end{aligned}\right\} \rightarrow atan2 \rightarrow \varphi+\psi$

$n_z = 1 \Rightarrow \vartheta = \pi \Rightarrow \left.\begin{aligned} S_{\psi-\varphi} &= m_x \\ C_{\psi-\varphi} &= m_y \end{aligned}\right\} \rightarrow atan2 \rightarrow \psi-\varphi$

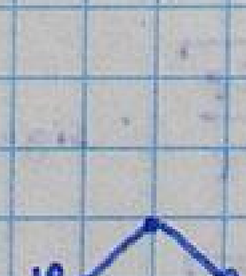
3 Inverz RPY-szögek

$$\overline{\overline{RPY}}(\varphi,\vartheta,\psi) = \begin{bmatrix} C_\varphi C_\vartheta & C_\varphi S_\vartheta C_\psi - S_\varphi C_\psi & C_\varphi S_\vartheta C_\psi + S_\varphi S_\psi \\ S_\varphi C_\vartheta & S_\varphi S_\vartheta S_\psi + C_\varphi C_\psi & S_\varphi S_\vartheta C_\psi - C_\varphi S_\psi \\ -S_\vartheta & C_\vartheta S_\psi & C_\vartheta C_\psi \end{bmatrix} = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}$$

Alkalmazás: - két csukló modellje

1.) $T_\varphi = \frac{l_y}{l_x} \xrightarrow{atan} \varphi$ (2x)

2.) $\left.\begin{aligned} S_\vartheta &= -l_z \\ C_\vartheta &= C_\varphi l_x + S_\varphi l_y \end{aligned}\right\} \rightarrow atan2 \rightarrow \vartheta$ (1x)

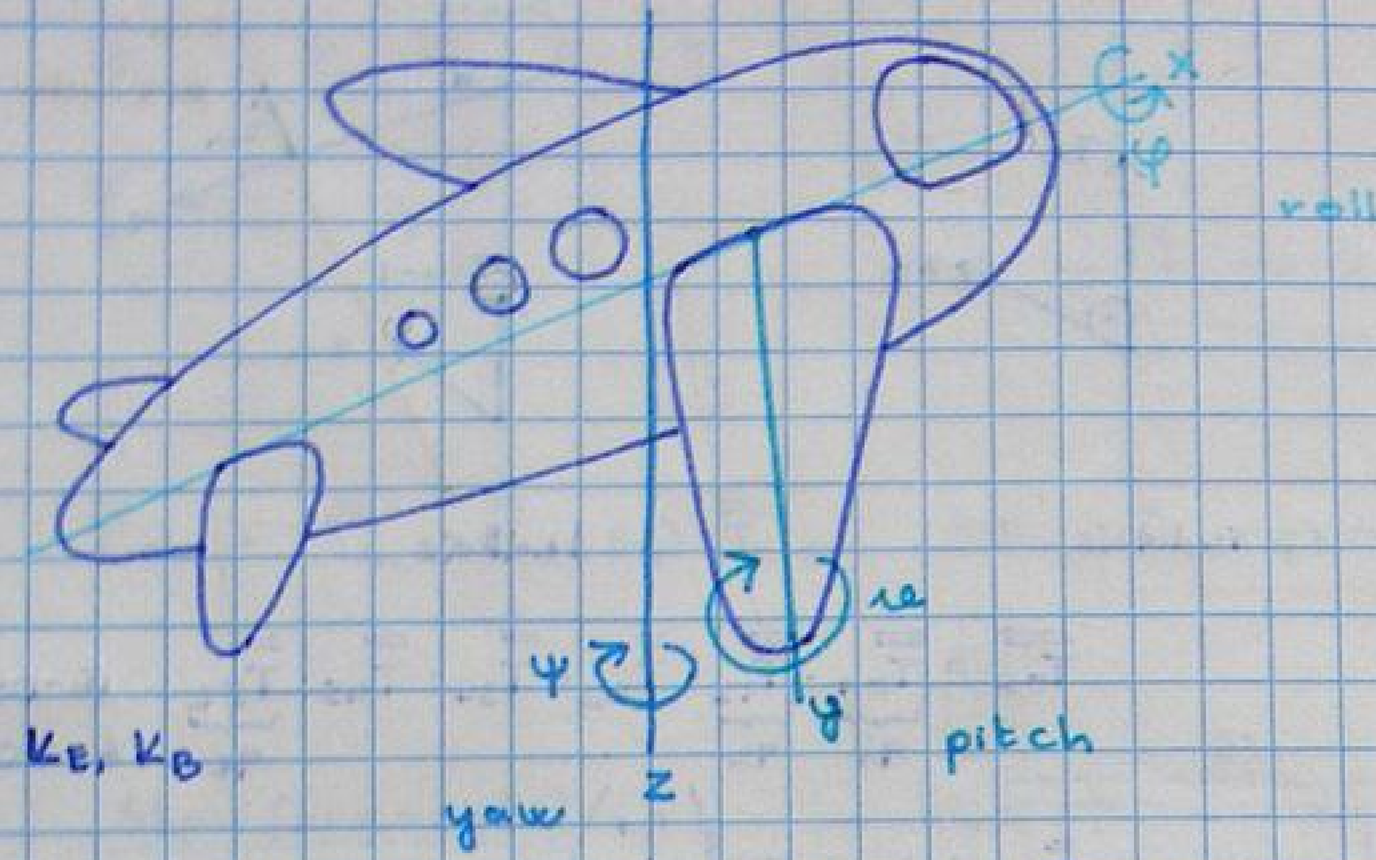


3.) $\left.\begin{aligned} S_\psi &= S_\varphi n_x - C_\varphi n_y \\ C_\psi &= -S_\varphi m_x + C_\varphi m_y \end{aligned}\right\} \rightarrow atan2 \rightarrow \psi$ (1x)

Szinguláris konfig.: $l_x = l_y = 0$

$l_z = +1 \Rightarrow \vartheta = -\frac{\pi}{2} \Rightarrow \left.\begin{aligned} S_{\varphi+\psi} &= -n_y \\ C_{\varphi+\psi} &= -n_x \end{aligned}\right\} \rightarrow atan2 \rightarrow \varphi+\psi$

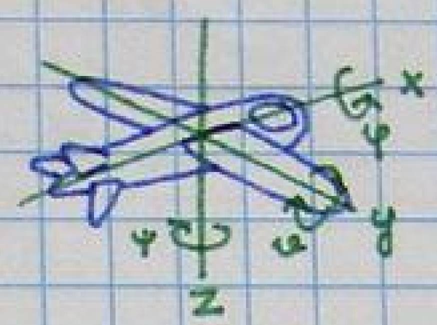
$l_z = -1 \Rightarrow \vartheta = \frac{\pi}{2} \Rightarrow \left.\begin{aligned} S_{\psi-\varphi} &= m_x \\ C_{\psi-\varphi} &= m_y \end{aligned}\right\} \rightarrow atan2 \rightarrow \psi-\varphi$



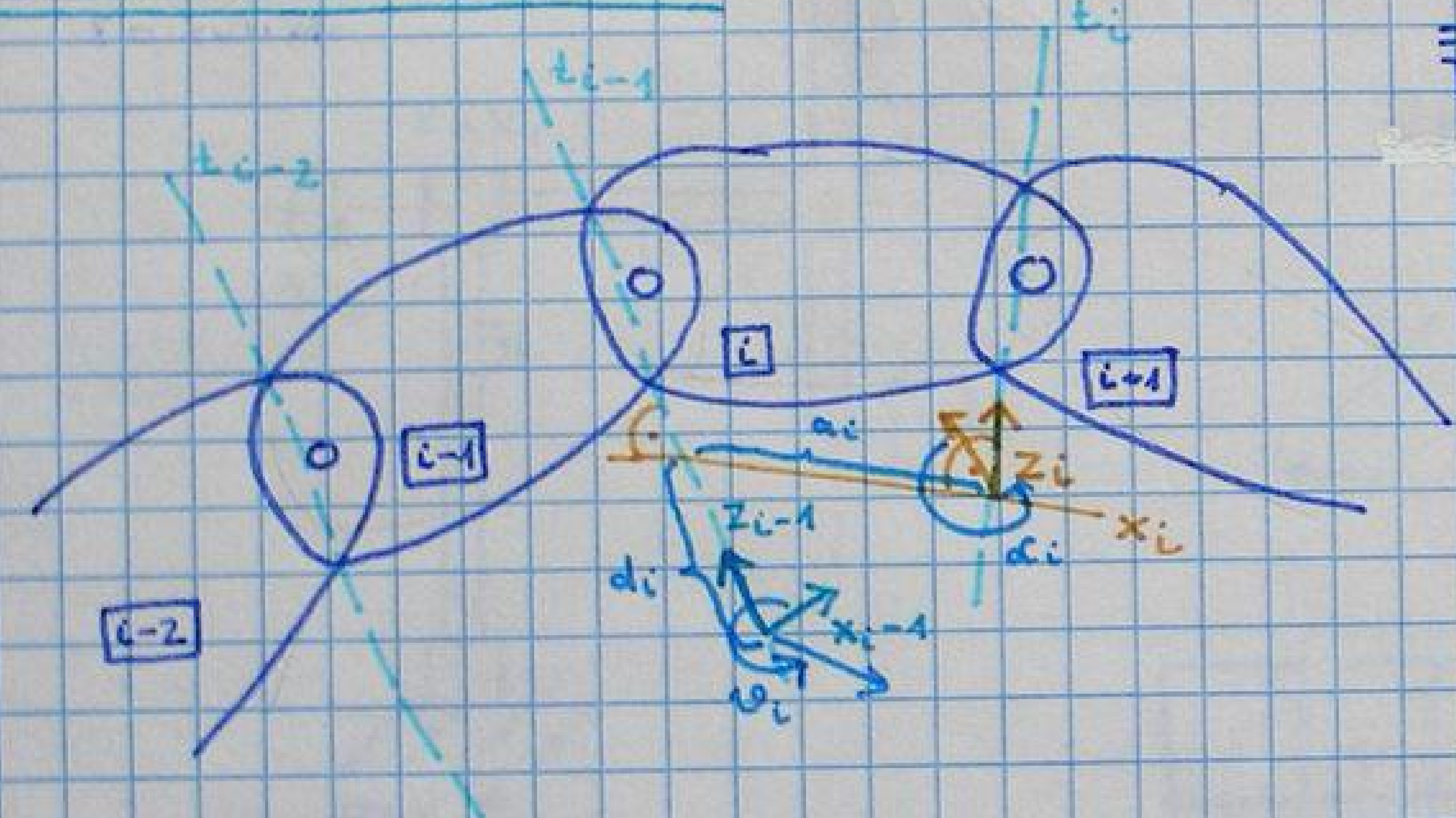
K_E, K_B

$$\overline{\overline{A}}_{K_E,K_B} = \overline{\overline{Rot}}(z,\psi)\,\overline{\overline{Rot}}(y,\vartheta)\,\overline{\overline{Rot}}(x,\varphi)$$

$RPY(\psi, \vartheta, \varphi)$



Denavit - Hartenberg alak

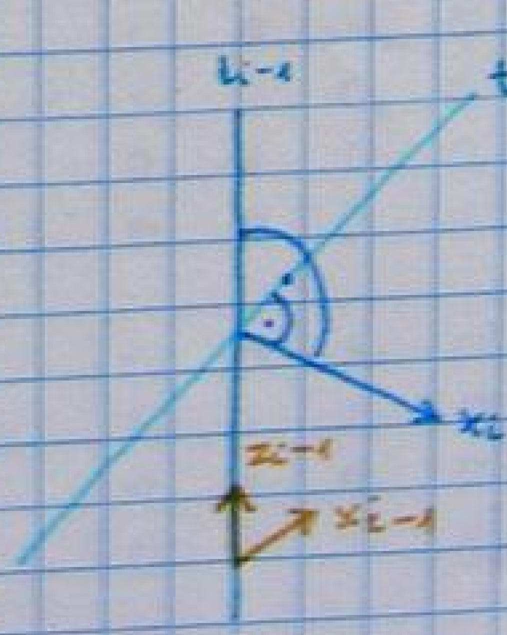


$$\overline{\overline{T}}_{i-1,i} = \overline{\overline{Rot}}(z_{i-1},\vartheta_i)\,\underbrace{\overline{\overline{Trans}}(z_{i-1},d_i)\cdot\overline{\overline{Trans}}(x_i,a_i)}_{\text{transzláció}}\,\overline{\overline{Rot}}(x_i,\alpha_i)$$

$$T_{i-1,i} = \overline{\overline{Rot}}(z_{i-1},\vartheta_i)\,\overline{\overline{Trans}}(z_{i-1},d_i)\,\overline{\overline{Trans}}(x_i,a_i)\,\overline{\overline{Rot}}(x_i,\alpha_i) =$$

$$= \begin{bmatrix} C_{\vartheta_i} & -S_{\vartheta_i} & 0 \\ S_{\vartheta_i} & C_{\vartheta_i} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{\alpha_i} & -S_{\alpha_i} & 0 \\ 0 & S_{\alpha_i} & C_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{i-1,i} = \begin{bmatrix} C_{\vartheta_i} & -S_{\vartheta_i}C_{\alpha_i} & S_{\vartheta_i}S_{\alpha_i} & a_i C_{\vartheta_i} \\ S_{\vartheta_i} & C_{\vartheta_i}C_{\alpha_i} & -C_{\vartheta_i}S_{\alpha_i} & a_i S_{\vartheta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



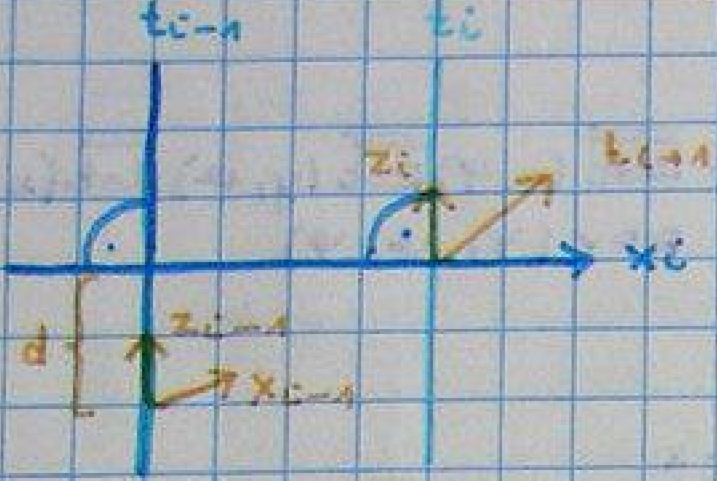
metsző egyenesek

$a_i = 0$

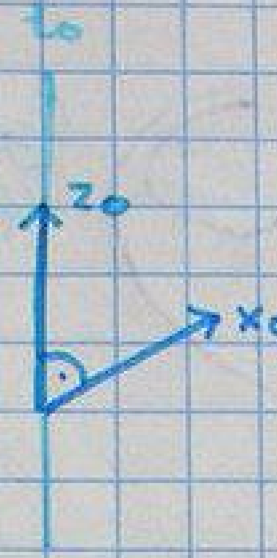
$R \rightarrow q_i = \vartheta_i$ $\quad \vartheta_i, d_i, a_i, \alpha_i$

$T \rightarrow q_i = d_i$ $\quad \vartheta_i, d_i, a_i, \alpha_i$

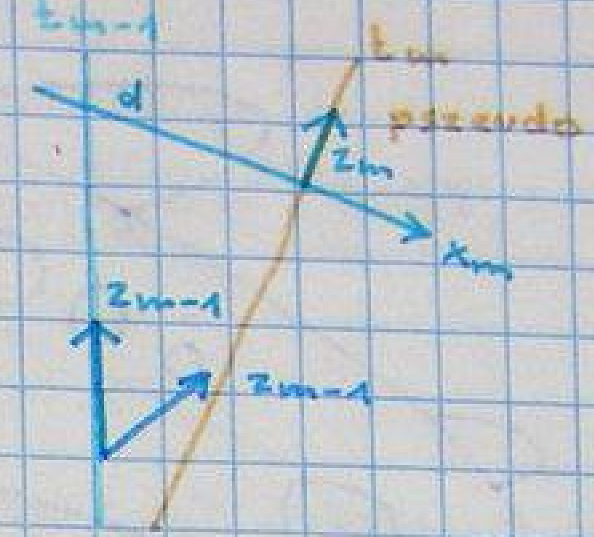
párhuzamos egyenesek



$a_i = 0°$



indulás



leállás

$$\overline{\overline{T}}_{06} = \underbrace{\overline{\overline{T}}_{01}}_{q_1} \cdot \underbrace{\overline{\overline{T}}_{12}}_{q_2} \cdot \underset{\cdots}{\overline{\overline{T}}_{23}} \cdot \overline{\overline{T}}_{34} \cdot \overline{\overline{T}}_{45} \cdot \underbrace{\overline{\overline{T}}_{56}}_{q_6} \quad \text{(6-DOF)}$$

$$\overline{\overline{T}}_{06}(\overline{q}), \quad \overline{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_6 \end{pmatrix}$$

Direkt geometriai feladat

Inverz = ?

2008. 02. 19.

$$\bar{\bar{T}}_{0,6} = \bar{\bar{T}}_{0,1}\,\bar{\bar{T}}_{1,2}\,\bar{\bar{T}}_{2,3}\,\bar{\bar{T}}_{3,4}\,\bar{\bar{T}}_{4,5}\,\bar{\bar{T}}_{5,6}$$

$$\bar{\bar{T}}_{0,6}(\bar{q}), \quad \bar{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_6 \end{pmatrix}$$

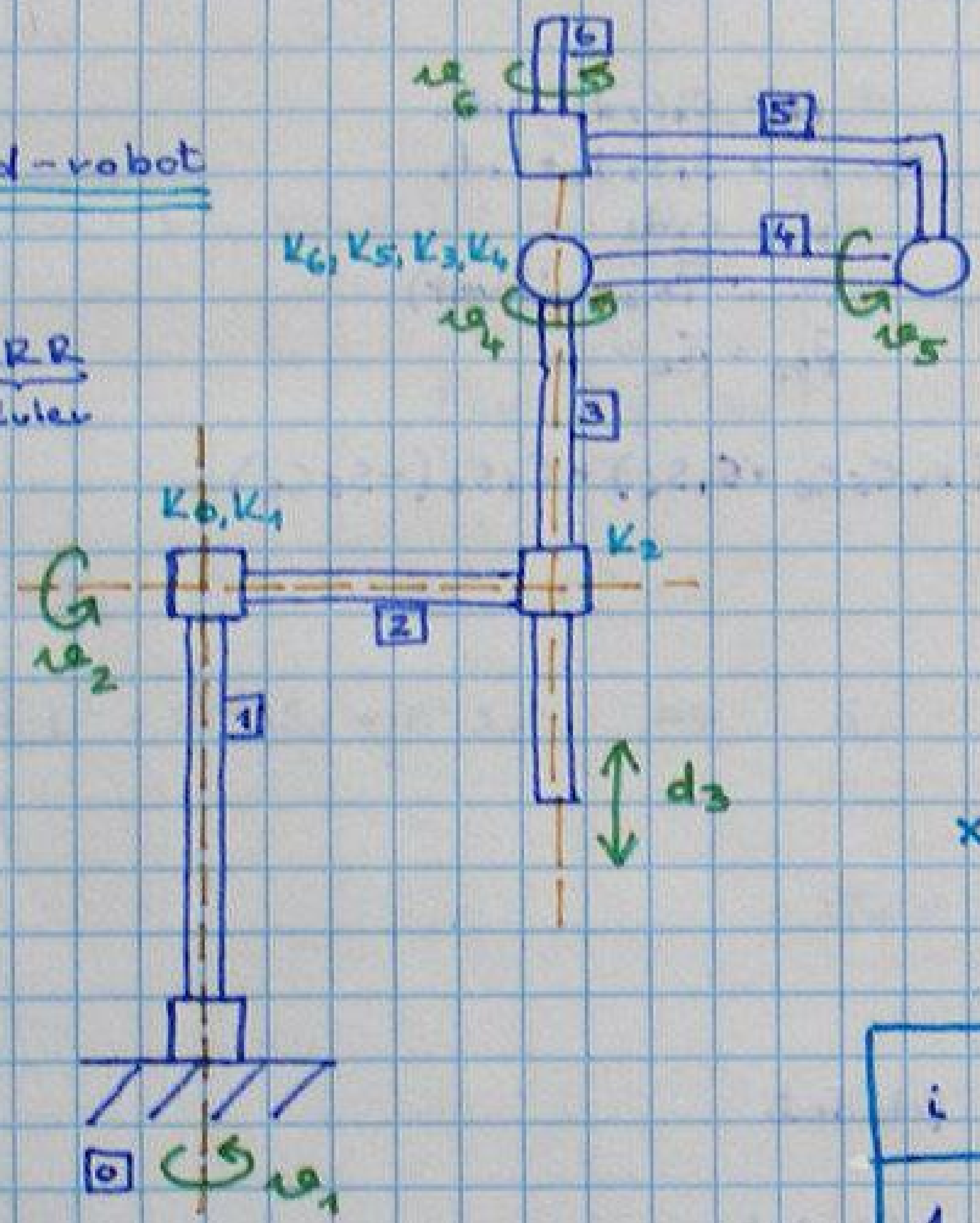
$$\bar{\bar{T}}_{i-1,i}$$

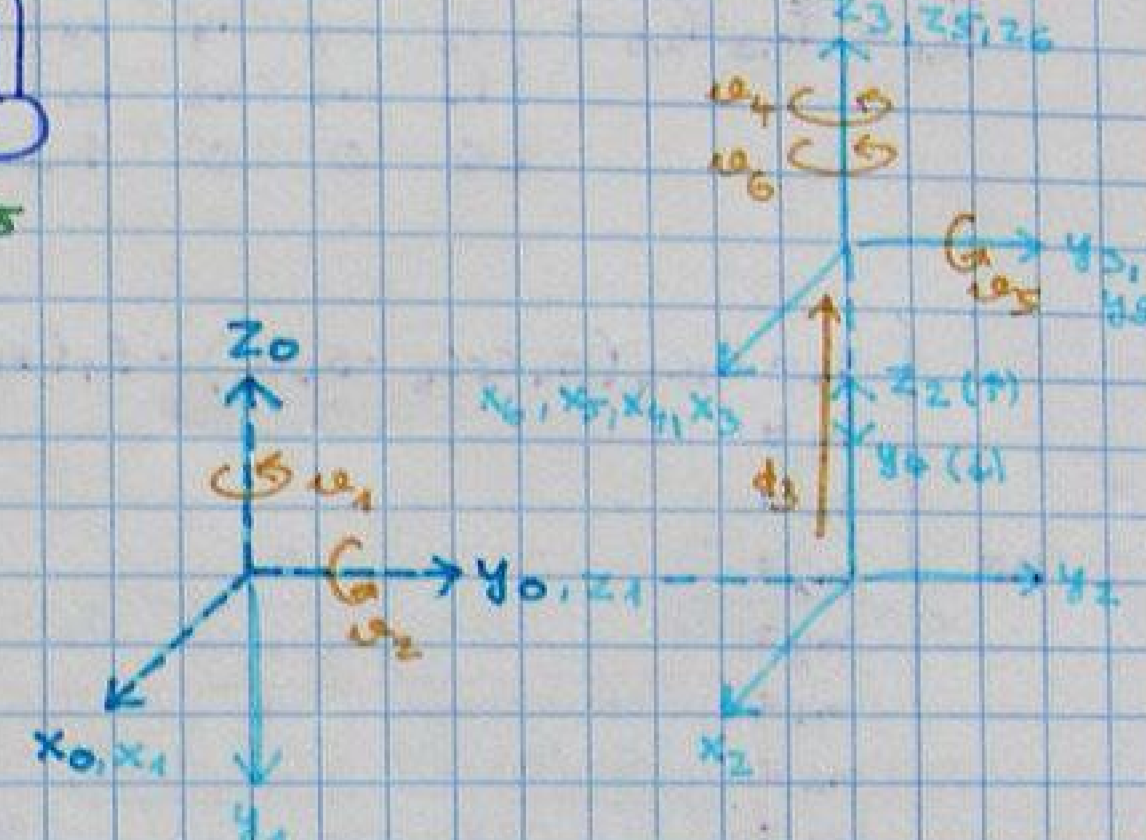
$$\underbrace{K_B \xrightarrow{\bar{\bar{T}}_{B,0}} K_0 \xrightarrow{\bar{\bar{T}}_{0,6}(\bar{q})} K_6 \xrightarrow{\bar{\bar{T}}_{6,E}} K_E}_{\bar{\bar{T}}_{B,E}} \quad \text{ROBOTGRÁF}$$

$$\bar{\bar{T}}_{B,E} = \bar{\bar{T}}_{B,0} \cdot \bar{\bar{T}}_{0,6}(\bar{q}) \cdot \bar{\bar{T}}_{6,E} \Rightarrow \boxed{\bar{\bar{T}}_{0,6}(\bar{q}) = \bar{\bar{T}}_{B,0}^{-1} \cdot \bar{\bar{T}}_{B,E} \cdot \bar{\bar{T}}_{6,E}^{-1}}$$

Stanford-robot

- RRT $\underbrace{RRR}_{\text{Euler}}$





i	q_i	ϑ_i	d_i	a_i	α_i	$S_{\alpha i}$	$C_{\alpha i}$
1	ϑ_1	ϑ_1	0	0	$-90°$	-1	0
2	ϑ_2	ϑ_2	d_2	0	$90°$	1	0
3	d_3	$0°$	d_3	0	$0°$	0	1
4	ϑ_4	ϑ_4	0	0	$-90°$	-1	0
5	ϑ_5	ϑ_5	0	0	$0°$	1	0
6	ϑ_6	ϑ_6	0	0	$0°$	0	1

$$\bar{\bar{T}}_{i-1,i} = \begin{bmatrix} C_{\vartheta i} & -S_{\vartheta i}C_{\alpha i} & S_{\vartheta i}S_{\alpha i} & a_i C_{\vartheta i} \\ S_{\vartheta i} & C_{\vartheta i}C_{\alpha i} & -C_{\vartheta i}S_{\alpha i} & a_i S_{\vartheta i} \\ 0 & S_{\alpha i} & C_{\alpha i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{1,2} = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \bar{\bar{T}}_{2,3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \bar{\bar{T}}_{3,4} = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{4,5} = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \bar{\bar{T}}_{5,6} = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \bar{\bar{T}}_{0,1} = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{3,6} = \underbrace{\bar{\bar{T}}_{34}\,\bar{\bar{T}}_{45}\,\bar{\bar{T}}_{56}}_{HF} = \overline{\text{Euler}}\,(\underbrace{\varphi}_{q_4}, \underbrace{\vartheta}_{q_5}, \underbrace{\psi}_{q_6}) \qquad \bar{\bar{T}}_{0,3} = \underbrace{\bar{\bar{T}}_{0,1}\,\bar{\bar{T}}_{1,2}\,\bar{\bar{T}}_{2,3}}_{HF}$$

$$\bar{\bar{T}}_{3,6} = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 & 0 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{0,3} = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & C_1S_2d_3 - S_1d_2 \\ S_1C_2 & C_1 & S_1S_2 & S_1S_2d_3 + C_1d_2 \\ -S_2 & 0 & C_2 & C_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{0,6} = \bar{\bar{T}}_{0,3}\,\bar{\bar{T}}_{3,6} = \begin{bmatrix} l_x & m_x & n_x & p_x \\ l_y & m_y & n_y & p_y \\ l_z & m_z & n_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{array}{l} p_x = C_1S_2d_3 - S_1d_2 \\ p_y = S_1S_2d_3 + C_1d_2 \\ p_z = C_2d_3 \end{array}$$

(ami $\bar{\bar{T}}_{0,3}$-nál volt)

$\bar{p}_{03} = \bar{p}_{06}$

$$\text{HF} \left\{ \begin{array}{l} l_x = C_1C_2(C_4C_5C_6 - S_4S_6) - S_1(S_4C_5C_6 + C_4S_6) + C_1S_2(-S_5C_6) \\ l_y = \\ l_z = \\ m_x = \\ m_y = \\ m_z = \\ n_x = \\ n_y = \\ n_z = -S_2(C_4S_5) + C_2(C_5) \end{array} \right.$$

$\bar{q} \longmapsto \bar{\bar{T}}_{06}(\bar{q})$ direkt geometriai feladat

$\bar{\bar{T}}_{06}\,\text{num} \longrightarrow \bar{q}$ inverz geometriai feladat

$$\bar{\bar{A}}_{06} = \bar{\bar{A}}_{03}\,\bar{\bar{A}}_{36} \Rightarrow \bar{\bar{A}}_{36} = \bar{\bar{A}}_{03}^{-1}\,\bar{\bar{A}}_{06} = \bar{\bar{A}}_{03}^{T}\,\bar{\bar{A}}_{06}$$

$$\uparrow$$

$\bar{\bar{A}}_{36}\,\text{num} \xrightarrow{\text{inv. Euler}} q_4, q_5, q_6 \qquad q_1, q_2, q_3$

1 Inverz pozícionáló feladat

$$\left.\begin{aligned} p_x &= C_1 S_2 d_3 - S_1 d_2 \\ p_y &= S_1 S_2 d_3 + C_1 d_2 \end{aligned}\right\} \overset{(1)}{\;} -S_1 p_x + C_1 p_y = d_2 \Rightarrow A C_\alpha + B S_\alpha = D \quad (2x)$$

$$p_z = C_2 d_3$$

$$\overset{(2)}{\;}(\underbrace{C_1 p_x + S_1 p_y}_{S_2 d_3}) C_2 - \underbrace{p_z}_{C_2 d_3} S_2 = 0 \Rightarrow T_2 = \frac{C_1 p_x + S_1 p_y}{p_z}$$

$$\overset{(3)}{\;}(\underbrace{C_1 p_x + S_1 p_y}_{S_2 d_3}) S_2 + \underbrace{p_z}_{C_2 d_3} C_2 = d_3 = q_3$$

$$q_1, q_2, q_3$$

$$\bar{\bar{A}}_{36} = \overline{\overline{\text{Euler}}}(q_4, q_5, q_6) = \underbrace{\bar{\bar{A}}_{03}^T \, \bar{\bar{A}}_{06}}_{\begin{bmatrix} L_x & M_x & N_x \\ L_y & M_y & N_y \\ L_z & M_z & N_z \end{bmatrix}} \xrightarrow{\text{inv. Euler}} \begin{aligned} \varphi &= q_4 \\ \vartheta &= q_5 \\ \psi &= q_6 \end{aligned}$$

$$A C_\alpha + B S_\alpha = D \qquad S_\alpha^2 + C_\alpha^2 = 1$$

$$A^2 \underbrace{C_\alpha^2}_{1 - S_\alpha^2} = D^2 - 2DBS_\alpha + B^2 S_\alpha^2 \Rightarrow S_\alpha^2 - 2\frac{DB}{A^2+B^2} \cdot S_\alpha + \frac{D^2 - A^2}{A^2+B^2} = 0$$

$$\boxed{S_\alpha = \frac{DB + \delta_\alpha A \sqrt{A^2 + B^2 - D^2}}{A^2 + B^2}}$$

$$C_\alpha^2 - 2\frac{DA}{A^2+B^2} \cdot C_\alpha + \frac{D^2 - B^2}{A^2+B^2} = 0$$

$$\boxed{C_\alpha = \frac{DA + \gamma_\alpha B \sqrt{A^2 + B^2 - D^2}}{A^2 + B^2}}$$

ha nincs megoldás: $D < 0$ (csak valós lehet)
elhagytuk a munkateret

ha van megoldás: 2 megoldás van

$$\boxed{(\delta_\alpha, \gamma_\alpha) = \begin{cases} (+1, -1) \\ (-1, +1) \end{cases}}$$

hiányzás: 3óra

$\boxed{2009.03.05.}$

m-DOF, $m<6$ (hiányzó szabadsági fokok)

1.) Választunk m feltételt → ezt precízen betartjuk

$RR \to v_{x0}, v_{y0} \leftrightarrow \dot{q}_1, \dot{q}_2$

RRTR
(SCARA) $\to v_{x0}, v_{y0}, v_{z0}, w_{z0} \leftrightarrow \dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4$

2.) Ha nincs értelmes altér amelyben mozog a robot, akkor minimalizáljuk az egyenlethibák négyzetösszegét (L.S)
Ilyenkor nincs megoldás.

Séma: $\bar{\bar{A}}_{n\times m},\quad n>m,\quad \bar{x}\in R^m,\ \bar{y}\in R^n \to \|\bar{\bar{A}}\bar{x}-\bar{y}\|^2 \to \min$

$$F = \langle \bar{\bar{A}}\bar{x}-\bar{y}, \bar{\bar{A}}\bar{x}-\bar{y}\rangle = \langle \bar{\bar{A}}\bar{x}, \bar{\bar{A}}\bar{x}\rangle - 2\langle \bar{\bar{A}}\bar{x}, \bar{y}\rangle + \langle \bar{y}, \bar{y}\rangle =$$

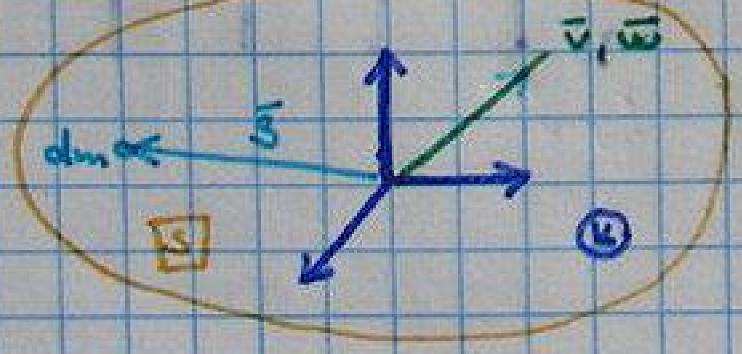
$$= \langle \bar{\bar{A}}^T\bar{\bar{A}}\bar{x}, \bar{x}\rangle - 2\langle \bar{x}, \bar{\bar{A}}^T\bar{y}\rangle + \langle \bar{y}, \bar{y}\rangle$$

$$F'_{\bar{x}} = \bar{0} = \cancel{2}\bar{\bar{A}}^T\bar{\bar{A}}\bar{x} - \cancel{2}\bar{\bar{A}}^T\bar{y} \Rightarrow \bar{x} = (\bar{\bar{A}}^T\bar{\bar{A}})^{-1}\bar{\bar{A}}^T\bar{y}$$

$$\begin{pmatrix}\bar{v}_m\\ \bar{\omega}_m\end{pmatrix} = \bar{\bar{J}}_m\dot{\bar{q}} \Rightarrow \dot{\bar{q}} = (\bar{\bar{J}}_m^T\bar{\bar{J}}_m)^{-1}\bar{\bar{J}}_m^T\begin{pmatrix}\bar{v}_m\\ \bar{\omega}_m\end{pmatrix}$$

$$\boxed{\bar{\bar{J}}_m^T}\ \boxed{\bar{\bar{J}}_m} = \boxed{\bar{\bar{J}}_m^T\bar{\bar{J}}_m}$$

$K = \frac{1}{2}mv^2$



$\bar{v}_s = \bar{v} + \bar{\omega}\times\bar{s}$

$K = \int_{S} \frac{1}{2}\bar{v}_s^2\, dm$

$$\bar{v}_s^2 = \langle \bar{v}+\bar{\omega}\times\bar{s}, \bar{v}+\bar{\omega}\times\bar{s}\rangle = \langle \bar{v}, \bar{v}\rangle + \underbrace{2\langle \bar{v}, \bar{\omega}\times\bar{s}\rangle}_{2\langle \bar{s}, \bar{v}\times\bar{\omega}\rangle} + \langle \underbrace{\bar{\omega}\times\bar{s}}_{-\bar{s}\times\bar{\omega}}, \bar{\omega}\times\bar{s}\rangle$$

$\langle \bar{s}\times\bar{\omega}, \bar{s}\times\bar{\omega}\rangle$

$[\bar{s}\times]^T \quad [\bar{s}\times]$

⇓

$\langle [\bar{s}\times]^T[\bar{s}\times]\bar{\omega}, \bar{\omega}\rangle$

$$K = \frac{1}{2}\langle \bar{v}, \bar{v}\rangle \int dm + \langle \int \bar{s}\, dm, \bar{v}\times\bar{\omega}\rangle +$$

$$+ \frac{1}{2}\langle \int [\bar{s}\times]^T[s\times]\, dm\, \bar{\omega}, \bar{\omega}\rangle$$

$m = \int dm,\qquad m\bar{s}_c = \int \bar{s}\, dm$

$\bar{\bar{K}} = \int [s\times]^T[s\times]\, dm$

$$\begin{bmatrix} 0 & s_z & -s_y \\ -s_z & 0 & s_x \\ s_y & -s_x & 0 \end{bmatrix}\begin{bmatrix} 0 & -s_z & s_y \\ s_z & 0 & s_x \\ -s_y & s_x & 0 \end{bmatrix} = \begin{bmatrix} s_y^2+s_z^2 & -s_x s_y & -s_x s_z \\ -s_x s_y & s_x^2+s_z^2 & -s_y s_z \\ -s_x s_z & -s_y s_z & s_x^2+s_y^2 \end{bmatrix}$$

identitás mátrix ?

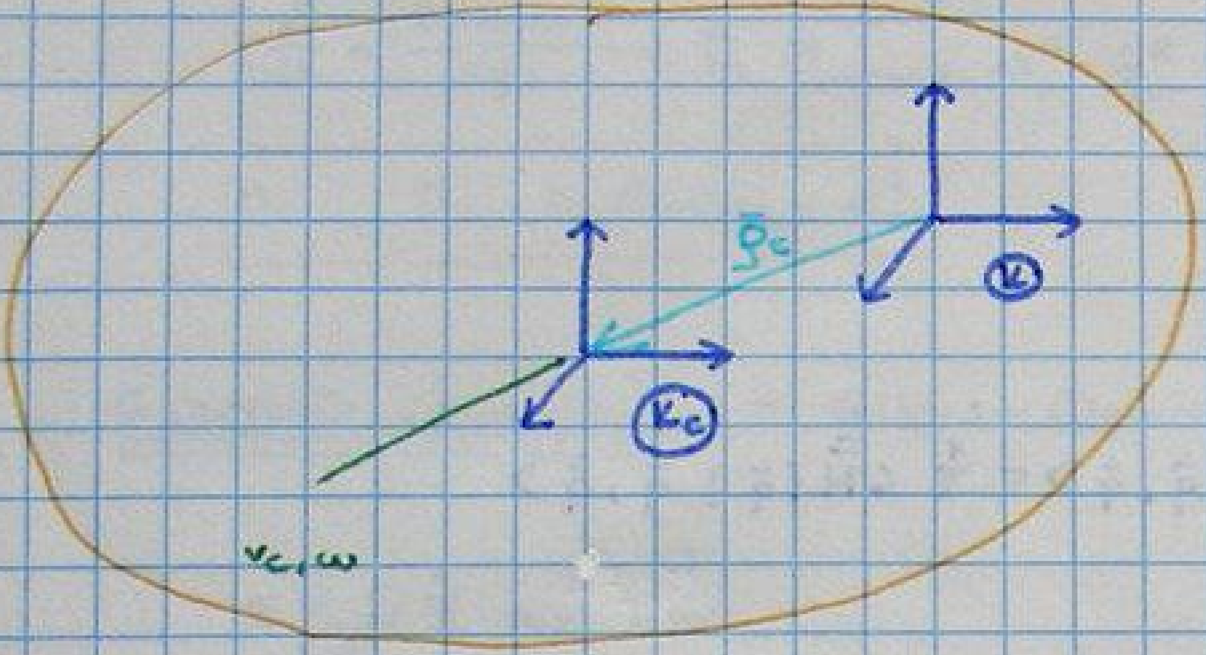
$\Theta = ml^2$

$$\bar{\bar{K}} = \int [\bar{s}\times]^T[\bar{s}\times]\,dm = \begin{bmatrix} K_x & K_{xy} & K_{xz}\\ * & K_y & K_{yz} \\ * & * & K_z\end{bmatrix} \qquad K_z = \int (s_x^2 + s_y^2)\,dm \qquad K_{xz} = -\int s_x s_z\,dm$$

*: értéke az átlós párja

kinetikus energia: $K = \frac{1}{2} \langle \bar v,\bar v\rangle m + \langle m\bar s_c, \bar v\times\bar\omega\rangle + \frac{1}{2}\langle \bar{\bar K}\bar\omega,\bar\omega\rangle$

tömegközéppont



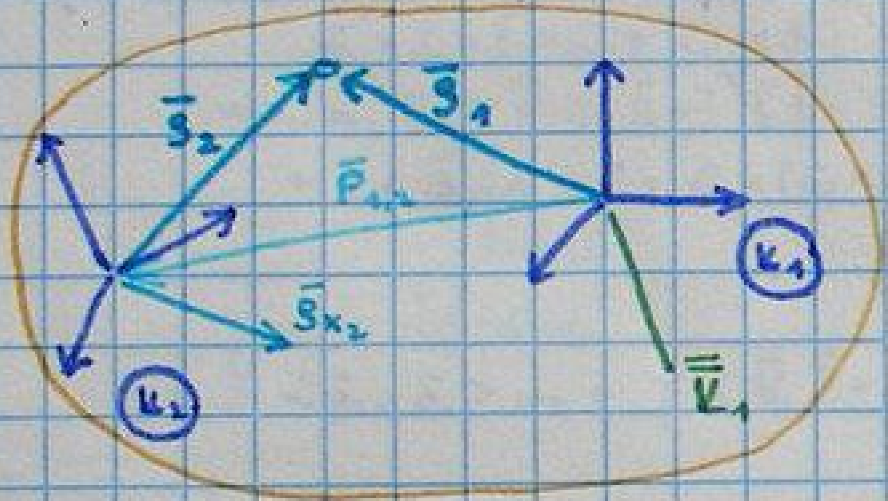
$$K = \frac{1}{2}\langle \bar v_c,\bar v_c\rangle m + \frac{1}{2}\langle \bar{\bar K}_c\bar\omega,\bar\omega\rangle$$

$\bar v = \bar{\bar\Omega}\dot{\bar q}$ $\qquad \bar v_c = \bar v + \bar\omega\times\bar s_c = \bar v - \bar s_c\times\bar\omega = \bar{\bar\Omega}_c\dot{\bar q}$

$\bar\omega = \bar{\bar\Gamma}\dot{\bar q}$ $\qquad \bar{\bar\Omega}_c = \bar{\bar\Omega} - [\bar s_c\times]\bar{\bar\Gamma}$

$$\bar a_c = \bar a + \bar\varepsilon\times\bar s_c + \bar\omega\times(\bar\omega\times\bar s_c)$$

$$\left.\begin{aligned}\bar a &= \bar{\bar\Omega}\ddot{\bar q} + \bar\Theta\\ \bar\varepsilon &= \bar{\bar\Gamma}\ddot{\bar q} + \bar\Phi\end{aligned}\right\} \bar a_c = \bar{\bar\Omega}_c\ddot{\bar q} + \bar\Theta_c,\quad \bar\Theta_c = \Theta + \bar\Phi\times\bar s_c + \bar\omega\times(\bar\omega\times\bar s_c)$$

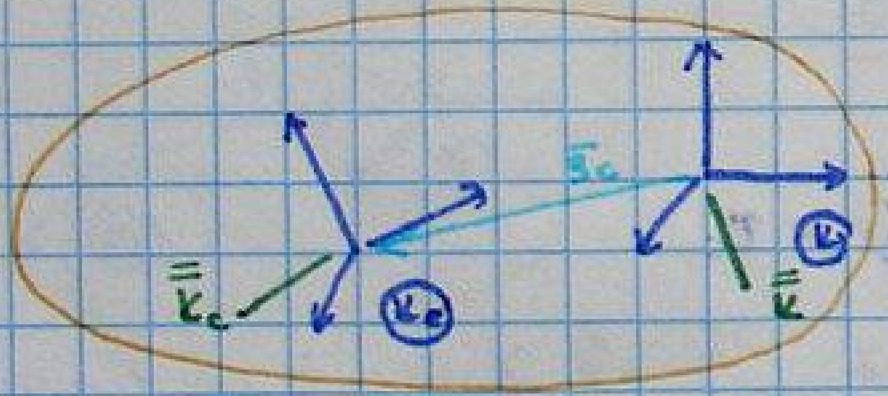


$$\bar s_1 = \bar p_{1,2} + \bar s_2$$

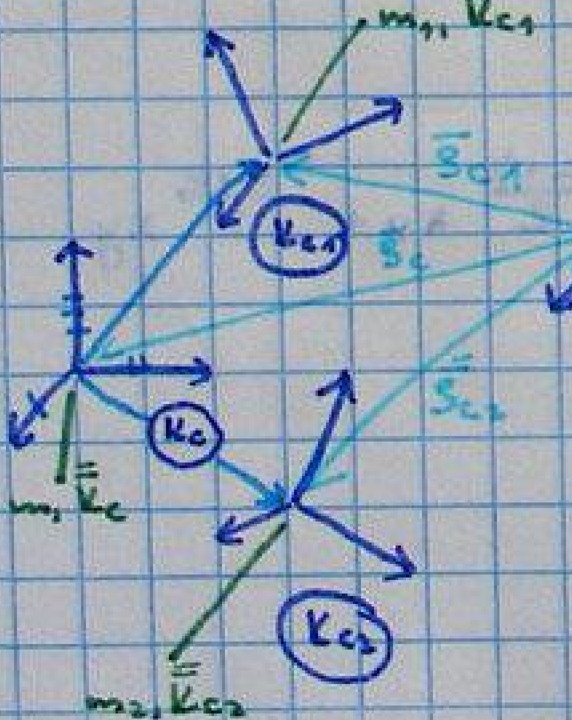
Huygens-elv:

$$\bar{\bar K}_1 = \bar{\bar A}_{12}\bar{\bar K}_2\bar{\bar A}_{12}^T - [(\bar{\bar A}_{12}\, m\bar q_{2c})\times][\bar p_{12}\times] - [\bar p_{12}\times][(\bar{\bar A}_{12}\, m\bar q_{2c})\times] - m[\bar p_{12}\times][\bar p_{12}\times]$$

$$\bar a\times(\bar b\times\bar c) = \bar b\langle\bar a,\bar c\rangle - \bar c\langle\bar a,\bar b\rangle$$



$$\bar{\bar K}_c = \bar{\bar A}_{K,K_c}\bar{\bar K}\bar{\bar A}_{K,K_c}^T + m\|\bar s_c\|^2\bar{\bar I} - m[\bar s_c\circ\bar s_c]$$



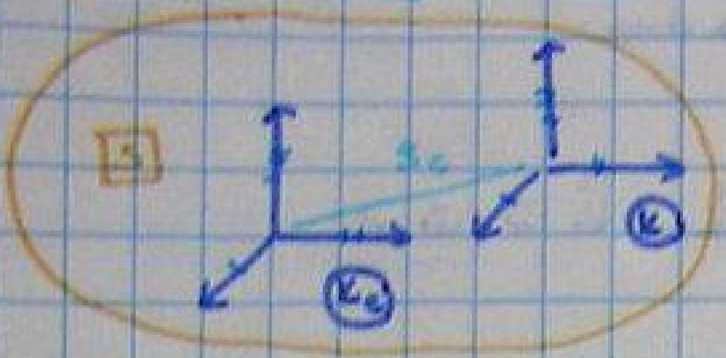
$m = m_1 + m_2$

$m\bar s_c = m_1\bar s_{c1} + m_2\bar s_{c2} \Rightarrow \bar s_c = \dfrac{m_1\bar s_{c1}+m_2\bar s_{c2}}{m_1+m_2}$

$$\bar{\bar K}_c = \bar{\bar A}_{K,K_{c1}}\bar{\bar K}_{c1}\bar{\bar A}^T_{K,K_{c1}} + m_1\|\bar s_{c1}-\bar s_{c2}\|^2\bar{\bar I} - m_1[(\bar s_{c1}-\bar s_c)\circ(\bar s_{c1}-\bar s_c)] +$$

$$+ \bar{\bar A}_{K,K_{c2}}\bar{\bar K}_{c2}\bar{\bar A}^T_{K,K_{c2}} + m_2\|\bar s_{c2}-\bar s_{c1}\|^2\bar{\bar I} - m_2[(\bar s_{c2}-\bar s_c)\circ(\bar s_{c2}-\bar s_c)]$$

1 merev test:



$$K = \frac{1}{2}\langle \bar v_c, v_c\rangle m + \frac{1}{2}\langle \bar{\bar K}_c \bar\omega, \bar\omega\rangle = \frac{1}{2}\langle \bar{\bar\Omega}_c \dot{\bar q}, \bar{\bar\Omega}_c \dot{\bar q}\rangle m + \frac{1}{2}\langle \bar{\bar K}_c \bar{\bar\Gamma}\dot{\bar q}, \bar{\bar\Gamma}\dot{\bar q}\rangle$$

$$K = \frac{1}{2}\langle (\bar{\bar\Omega}_c^T \bar{\bar\Omega}_c m + \bar{\bar\Gamma}^T \bar{\bar K}_c \bar{\bar\Gamma})\dot{\bar q}, \dot{\bar q}\rangle$$

$$\bar v_c = \bar{\bar\Omega}_c \dot{\bar q}$$
$$\bar\omega = \bar{\bar\Gamma}\dot{\bar q}$$
$$\bar{\bar\Omega}_c = \bar{\bar\Omega} - [s_c\times]\bar{\bar\Gamma}$$

m-DOF robot:

$$K = \frac{1}{2}\langle \underbrace{\sum_{s=1}^{m} \{\Omega_{cs}^T \Omega_{cs} m_s + \bar{\bar\Gamma}_s^T \bar{\bar K}_{cs} \bar{\bar\Gamma}_s\}}_{\bar{\bar H}(\bar q)} \dot{\bar q}, \dot{\bar q}\rangle = \frac{1}{2}\langle \bar{\bar H}(\bar q)\dot{\bar q}, \dot{\bar q}\rangle$$

$\bar{\bar H}(\bar q) = [D_{jk}(\bar q)]_{m\times n}$ $\quad \bar{\bar H}$ szimmetrikus és pozitív definit ($\Rightarrow$ invertálható)

$$K = \frac{1}{2}\sum_j \sum_k D_{jk}(\bar q)\dot q_j \dot q_k$$

Lagrange-egyenletek

$L = K - P$

P → potenciális energia

K → kinetikus energia

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q_i} - \frac{\partial L}{\partial q_i} = \tau_i \quad \text{vagy} \quad \frac{d}{dt}\frac{\partial L}{\partial \dot q_i} - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} = \tau_i$$

$$K = \frac{1}{2}\sum_j \sum_k D_{jk}\dot q_j \dot q_k$$

$$\frac{\partial K}{\partial \dot q_i} = \frac{1}{2}\sum_k D_{ik}\dot q_k + \frac{1}{2}\sum_j D_{ji}\dot q_i = \sum_j D_{ij}\dot q_j$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot q_i} = \sum_j D_{ij}\ddot q_j + \sum_j\sum_k \frac{\partial D_{ij}}{\partial q_k}\dot q_j \dot q_k = \qquad a\,xy + b\,xy = \frac{a+b}{2}xy + \frac{a+b}{2}xy$$

$$= \sum_j D_{ij}\ddot q_j + \sum_j\sum_k \underbrace{\frac{1}{2}\left(\frac{\partial D_{ij}}{\partial q_k} + \frac{\partial D_{ik}}{\partial q_j}\right)}_{\text{szimmetrizált}}\dot q_j \dot q_k$$

$$\frac{\partial K}{\partial q_i} = \frac{1}{2}\sum_j\sum_k \frac{\partial D_{jk}}{\partial q_i}$$

$$\frac{\partial P}{\partial q_i} =: D_i \qquad \Rightarrow \qquad \boxed{\sum_j D_{ij}\ddot q_j + \sum_j\sum_k D_{ijk}\dot q_j \dot q_k + D_i = \tau}$$

$$\bar{\bar H}(\bar q) = [D_{jk}(\bar q)], \qquad D_{ijk} = \frac{1}{2}\left(\frac{\partial D_{ij}}{\partial q_k} + \frac{\partial D_{ik}}{\partial q_j} - \frac{\partial D_{jk}}{\partial q_i}\right)$$

Hatások:
- D_{ii} – effektív inercia
- D_{ij}, $i \neq j$ – csatoló inercia
- D_{ijj} – centripetális hatás
- D_{ijk}, $j \neq k$ – Coriolis hatás
- D_i – gravitációs hatás

$\boxed{2009.02.11.}$

Meghajtó motor figyelembevétele

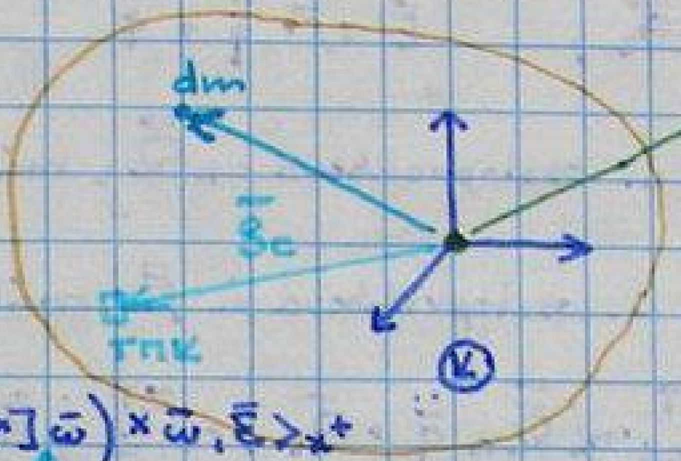
$$K_{rotor,i} = \frac{1}{2}\Theta_{rotor,i}\cdot(\nu_i\cdot\dot{q}_i)^2$$

$$\frac{d}{dt} = \frac{\partial K_{rotor,i}}{\partial \dot{q}_i} = \frac{d}{dt}\Theta_{rotor,i}\,\nu_i^2\cdot\dot{q}_i = \Theta_{rotor,i}\,\nu_i^2\,\ddot{q}_i$$

$$\boxed{D_{ii} := D_{ii} + \Theta_{rotor,i}\,\nu_i^2}$$

Gyorsulás „energia"

$= $ Gibbs-függvény, $\frac{1}{2}ma^2$



$$G = \int_{[S]} \frac{1}{2}\bar{a}_s^2\,dm$$

$$\bar{a}_s = \bar{a} + \bar{\varepsilon}\times\bar{s} + \bar{\omega}\times(\bar{\omega}\times\bar{s})$$

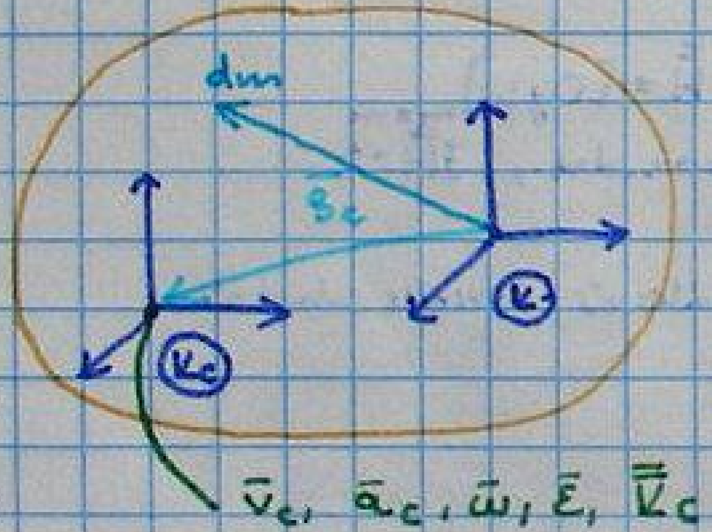
$$\bar{a}_s^2 = \langle \bar{a}+\bar{\varepsilon}\times\bar{s}+\bar{\omega}\times(\bar{\omega}\times\bar{s}),\ \bar{a}+\bar{\varepsilon}\times\bar{s}+\bar{\omega}\times(\bar{\omega}\times\bar{s})\rangle =$$

$$= \langle\bar{a},\bar{a}\rangle + \langle[\bar{s}\times]^T[\bar{s}\times]\bar{\varepsilon} - 2([s\times]^T[s\times]\bar{\omega})\times\bar{\omega},\bar{\varepsilon}\rangle + $$

$$+ 2\langle\bar{s},\ \bar{a}\times\bar{\varepsilon}+\bar{\omega}\times(\bar{\omega}\times\bar{a})\rangle + \ldots$$

$\cdot dm$ $\quad\cdot dm$ $\quad\cdot dm$ $\quad\cdot dm$

$$G = \frac{1}{2}\langle\bar{a},\bar{a}\rangle m + \frac{1}{2}\langle\bar{\bar{K}}\bar{\varepsilon} - 2(\bar{\bar{K}}\bar{\omega})\times\bar{\omega},\bar{\varepsilon}\rangle + 2\langle m\bar{s}_c,\ \bar{a}\times\bar{\varepsilon}+\bar{\omega}\times(\bar{\omega}\times\bar{a})\rangle + \ldots$$



$$\boxed{G = \frac{1}{2}\langle\bar{a}_c,\bar{a}_c\rangle m + \frac{1}{2}\langle\bar{\bar{K}}_c\bar{\varepsilon} - 2(\bar{\bar{K}}_c\bar{\omega})\times\bar{\omega},\bar{\varepsilon}\rangle}$$

$$\bar{v}_c = \bar{v} + \bar{\omega}\times\bar{s}_c = \bar{v} - [\bar{s}_c\times]\bar{\omega} = \bar{\bar{\Omega}} - [\bar{s}_c\times]\bar{\bar{\Pi}} = \bar{\bar{\Omega}}_c\cdot\dot{q}$$

$$\bar{a}_c = \bar{a} + \bar{\varepsilon}\times\bar{s}_c + \bar{\omega}\times(\bar{\omega}\times\bar{s}_c) = \bar{\bar{\Omega}}_c\,\ddot{q} + \bar{\Theta}_c$$

$$\bar{a} = \bar{\bar{\Omega}}\,\ddot{q} + \bar{\Theta}$$

$$\bar{\varepsilon} = \bar{\bar{\Pi}}\,\ddot{q} + \bar{\Theta}$$

$$\boxed{\begin{aligned}\bar{\bar{\Theta}}_c &= \bar{\Theta} + \bar{\Phi}\times\bar{s}_c + \bar{\omega}\times\cancel{(\bar{s}_c)}(\bar{\omega}\times\bar{s}_c)\\ \bar{\bar{\Omega}}_c &= \bar{\bar{\Omega}} - [\bar{s}_c\times]\bar{\bar{\Pi}}\end{aligned}}$$

$$\boxed{G = \frac{1}{2}\langle\bar{a}_c,\bar{a}_c\rangle m + \frac{1}{2}\langle\bar{\bar{K}}_c\bar{\varepsilon} - 2(\bar{\bar{K}}_c\bar{\omega})\times\bar{\omega},\bar{\varepsilon}\rangle}$$

Appell-egyenletek: m-DOF robot

$$\frac{\partial G}{\partial \ddot{q}_i} + \frac{\partial D}{\partial q_i} = \tau_i, \quad i = 1, \dots, m$$

$$\frac{\partial G}{\partial \ddot{q}_i} = \frac{1}{2} \left\langle \frac{\partial \bar{a}_c}{\partial \ddot{q}_i}, \bar{a}_c \right\rangle m + \frac{1}{2} \left\langle \bar{a}_c, \frac{\partial \bar{a}_c}{\partial \ddot{q}_i} \right\rangle m + \frac{1}{2} \left\langle \bar{\bar{K}}_c \frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i}, \bar{\varepsilon} \right\rangle^{*} + \frac{1}{2} \left\langle \bar{\bar{K}}_c \bar{\varepsilon} - 2(\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega}, \frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i} \right\rangle$$

$$\boxed{\frac{\partial G}{\partial \ddot{q}_i} = \left\langle \frac{\partial \bar{a}_c}{\partial \ddot{q}_i}, \bar{a}_c \right\rangle m + \left\langle \frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i}, \bar{\bar{K}}_c \bar{\varepsilon} - (\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega} \right\rangle}$$

$$\bar{\bar{A}} \bar{x} = [\bar{a}_1 \ \bar{a}_2 \ \dots] \begin{pmatrix} x_1 \\ x_2 \\ \vdots \end{pmatrix} = \bar{a}_1 x_1 + \bar{a}_2 x_2 + \dots$$

$$\bar{a}_c = \bar{\bar{\Omega}}_c \ddot{\bar{q}} + \bar{\Theta}_c, \qquad \bar{\varepsilon} = \bar{\bar{\Gamma}} \ddot{\bar{q}} + \bar{\Phi}$$

$\frac{\partial \bar{a}_c}{\partial \ddot{q}_i} = \bar{\Omega}_{c,i}$ $\quad$ i. oszlopvektora az $\bar{\bar{\Omega}}_c$ mátrixnak

$\frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i} = \bar{\Gamma}_i$ $\quad$ i. oszlopvektora a $\bar{\bar{\Gamma}}$ mátrixnak

$$\frac{\partial G}{\partial \ddot{q}_i} + \frac{\partial P}{\partial q_i} = \bar{\Omega}_{c,i}^T \cdot m (\bar{\bar{\Omega}}_c \ddot{\bar{q}} + \bar{\Theta}_c) + \bar{\Gamma}_i^T \{ \bar{\bar{K}}_c (\bar{\bar{\Gamma}} \ddot{\bar{q}} + \bar{\Phi}) - (\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega} \}$$

$$\frac{\partial G}{\partial \ddot{q}_i} + \frac{\partial P}{\partial \bar{q}} = \bar{\tau}; \quad (\bar{\bar{\Omega}}_c^T \bar{\bar{\Omega}}_c m + \bar{\bar{\Gamma}}^T \bar{\bar{K}}_c \bar{\bar{\Gamma}}) \ddot{\bar{q}} + \bar{\bar{\Omega}}_c^T \bar{\Theta}_c m + \bar{\bar{\Gamma}}^T [\bar{\bar{K}}_c \bar{\Phi} - (\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega}] + \frac{\partial P}{\partial \bar{q}} = \bar{\tau}$$

$$\bar{\bar{H}}(\bar{q}) = \sum_{s=1}^{m} \{ \bar{\bar{\Omega}}_{cs}^T \bar{\bar{\Omega}}_{cs} m_s + \bar{\bar{\Gamma}}_s^T \bar{\bar{K}}_{cs} \bar{\bar{\Gamma}} \}$$ ált. inerciamátrix, $\bar{\bar{H}} = [D_{jk}]_{m \times m}$

szimmetrikus és poz.def., $\exists \bar{\bar{H}}^{-1}$

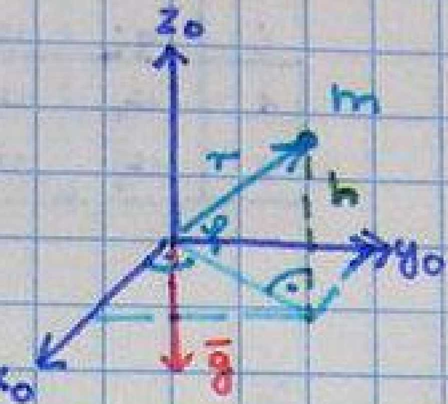
$$\bar{h}_{cc} = \sum_{s=1}^{m} \{ \bar{\bar{\Omega}}_{cs}^T \bar{\Theta}_{cs} m_s + \bar{\bar{\Gamma}}_s^T [\bar{\bar{K}}_{cs} \bar{\Phi}_s - (\bar{\bar{K}}_{cs} \bar{\omega}_s) \times \bar{\omega}_s] \}$$ centripetális és Coriolis-hatás

$$\bar{h}_g = \frac{\partial P}{\partial \bar{q}}, \quad h_{g_i} = D_i = \frac{\partial P}{\partial q_i}$$

Mozgásegyenlet: $\boxed{\bar{\bar{H}}(\bar{q}) \ddot{\bar{q}} + \bar{h}_{cc}(\bar{q}, \dot{\bar{q}}) + \bar{h}_g(\bar{q}) = \bar{\tau}}$

Potenciális energia:

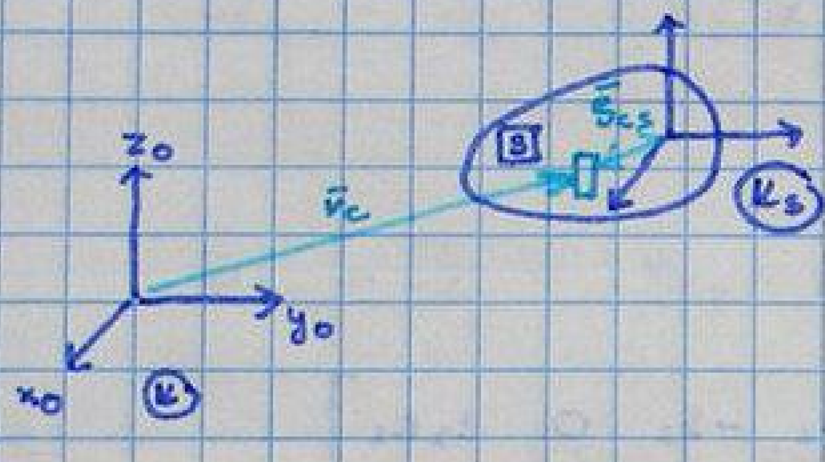
$$P = mgh$$



$$h = |\bar r| \sin\varphi$$

$$\langle \bar r, \bar g\rangle = |\bar r|\cdot|\bar g|\cdot \underbrace{\cos(90^\circ+\varphi)}_{-\sin(\varphi)}$$

$$P = -m\langle \bar g, \bar r\rangle$$



$$P = -\sum_{s=1}^{m} m_s (\bar g^T\ 0)\,\underbrace{\bar{\bar T}_{0,s}}\begin{pmatrix}\bar s_{cs}\\ 0\end{pmatrix}$$

$$\bar{\bar T}_{s,i-1},\ \underbrace{\bar{\bar T}_{i-1,i}}_{q_i}\ \bar{\bar T}_{i,s}$$

$$\frac{\partial P}{\partial q_i} = D_i = -\sum_{s=i}^{m} (\bar g^T\ 0)\,\bar{\bar T}_{0,i-1}\,\underbrace{\frac{\partial \bar{\bar T}_{i-1,i}}{\partial q_i}}_{\bar{\bar\Delta}_{i-1}\bar{\bar T}_{i-1,i}}\,\bar{\bar T}_{i,s}\begin{pmatrix}\bar s_{cs}\\ 1\end{pmatrix} m_s =$$

$$= \underbrace{-(\bar g^T\ 0)\,\bar{\bar T}_{0,i-1}\,\bar{\bar\Delta}_{i-1}}_{\bar G_i^T}\ \underbrace{\sum_{s=1}^{m} \bar{\bar T}_{i-1,s}\begin{pmatrix}m_s\bar s_{cs}\\ m_s\end{pmatrix}}_{\begin{pmatrix}\bar R_i\\ M_i\end{pmatrix}=?}$$

$\bar{\bar\Delta}_{i-1}$: rotációs $\rightarrow \begin{bmatrix}0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$

transzlációs $\rightarrow \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}$

$\bar G_i^T$: rotációs $\rightarrow (-\bar g\cdot \bar m_{0,i-1},\ \bar g\cdot \bar l_{0,i-1}, 0, 0)$

transzlációs $\rightarrow (0, 0, 0, -\bar g\cdot \bar m_{0,i-1})$

$$\bar R_i = \sum_{s=1}^{m} \left\{\bar{\bar A}_{i-1,s}\, m_s \bar s_{cs} + \bar p_{i-1,s}\, m_s\right\}$$

$$M_i = \sum_{s=1}^{m} m_s$$

$$\bar R_i = \sum_{s=i+1}^{m} \left\{\underbrace{\bar{\bar A}_{i-1,s}}_{\bar{\bar A}_{i-1,i}\bar{\bar A}_{i,s}}\, m_s \bar s_{cs} + \underbrace{\bar p_{i-1,s}}_{\bar p_{i-1,i}+\bar{\bar A}_{i,s}\bar p_{i,s}}\, m_s\right\} + \bar{\bar A}_{i-1,i}\, m_i \bar s_{ci} + \bar p_{i-1,i}\, m_i$$

$$\bar R_i = \bar{\bar A}_{i-1,i}\left\{\bar R_{i+1} + m_i \bar s_{ci}\right\} + M_i\, \bar p_{i-1,i}$$

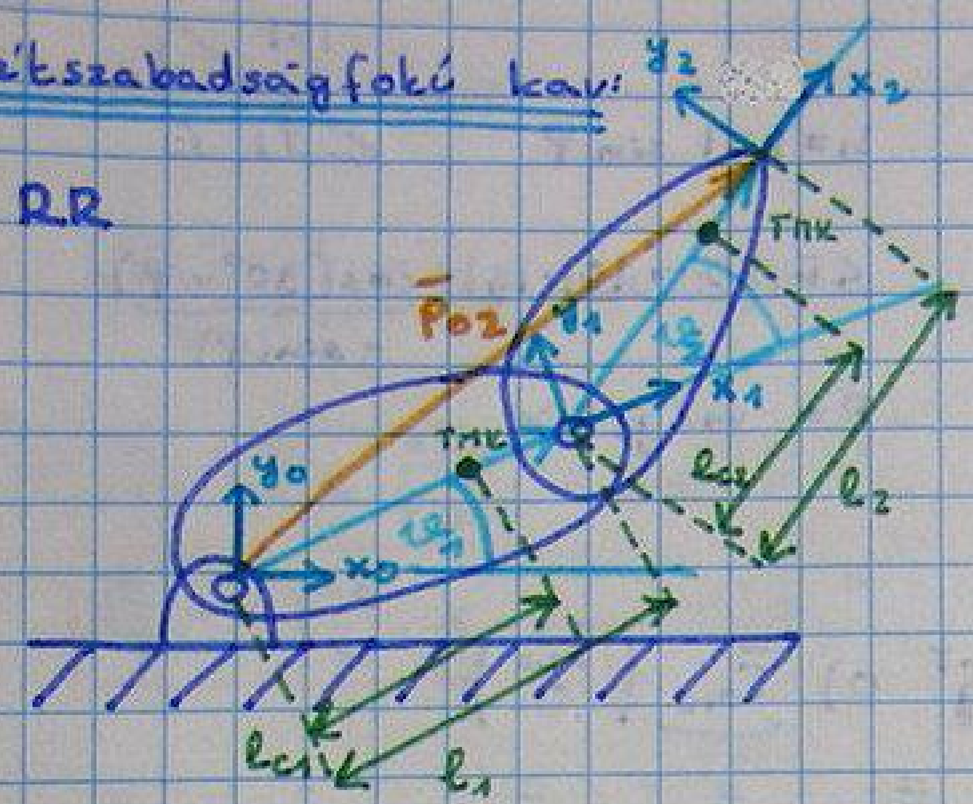
$$M_i = M_{i+1} + m_i$$

Hátratartó rekurzió: $\bar R_{m+1} = \bar 0,\quad M_{m+1} = 0$

$$\bar{\bar T}_{0,0} = \bar{\bar I}_4,\quad h_{g_i}(\bar q) = D_i(\bar q) = \bar G_i^T\begin{pmatrix}\bar R_i\\ M_i\end{pmatrix}$$

Kétszabadságfokú kar:

RR



i	q_i	ϑ_i	d_i	a_i	α_i
1	ϑ_1	ϑ_1	0	l_1	0°
2	ϑ_2	ϑ_2	0	l_2	0°

$$\bar{\bar{T}}_{0,1} = \begin{bmatrix} C_1 & -S_1 & 0 & l_1C_1 \\ S_1 & C_1 & 0 & l_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \bar{\bar{T}}_{1,2} = \begin{bmatrix} C_2 & -S_2 & 0 & l_2C_2 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{02} = \bar{\bar{T}}_{01} \cdot \bar{\bar{T}}_{12} = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1C_1 + l_2C_{12} \\ S_{12} & C_{12} & 0 & l_1S_1 + l_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0\bar{\omega}_1 = {}^1\bar{\omega}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \dot{q}_1 = \bar{\bar{\Gamma}}_1 \dot{\bar{q}}$$

$${}^0\bar{\omega}_2 = {}^2\bar{\omega}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (\dot{q}_1 + \dot{q}_2) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \bar{\bar{\Gamma}}_2 \dot{\bar{q}}$$

$$\bar{p}_{01} = \begin{pmatrix} l_1C_1 \\ l_1S_1 \\ 0 \end{pmatrix} \rightarrow {}^0\bar{v}_1 = \frac{d}{dt}\bar{p}_{01} = \begin{pmatrix} -l_1S_1\dot{q}_1 \\ l_1C_1\dot{q}_1 \\ 0 \end{pmatrix} = \begin{pmatrix} -l_1S_1 \\ l_1C_1 \\ 0 \end{pmatrix} \dot{q}_1$$

$$\rightarrow {}^1\bar{v}_1 = \bar{\bar{A}}_{01}^T\, {}^0\bar{v}_1 = \begin{bmatrix} C_1 & S_1 & 0 \\ -S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} -l_1S_1 \\ l_1C_1 \\ 0 \end{pmatrix} \dot{q}_1 = \begin{pmatrix} 0 \\ l_1 \\ 0 \end{pmatrix} \dot{q}_1 = \bar{\bar{\Omega}}_1 \dot{\bar{q}}, \quad \bar{\bar{\Omega}}_1 = \begin{pmatrix} 0 \\ l_1 \\ 0 \end{pmatrix}$$

$$\rightarrow {}^0\bar{v}_2 = \frac{d}{dt}\bar{p}_{02} = \begin{pmatrix} -l_1S_1\dot{q}_1 - l_2S_{12}(\dot{q}_1 + \dot{q}_2) \\ l_1C_1\dot{q}_1 + l_2C_{12}(\dot{q}_1 + \dot{q}_2) \\ 0 \end{pmatrix}$$

$$\rightarrow {}^2\bar{v}_2 = \bar{\bar{A}}_{02}^T\, {}^0\bar{v}_2 = \begin{bmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -l_1S_1 - l_2S_{12} & -l_2S_{12} \\ l_1C_1 + l_2C_2 & l_2C_{12} \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} =$$

$$= \underbrace{\begin{bmatrix} l_1S_2 & 0 \\ l_1C_2 + l_2 & l_2 \\ 0 & 0 \end{bmatrix}}_{\bar{\bar{\Omega}}_2} \dot{\bar{q}} = \bar{\bar{\Omega}}_2 \dot{\bar{q}}$$

2009.03.12.

TMK-ra: $${}^{0}\bar{v}_1 = \begin{bmatrix}0\\ l_{c1}\\ 0\end{bmatrix}\dot{q}_1 = \bar{\bar{\Omega}}_{c1}\cdot\dot{\bar{q}} \quad \text{és} \quad {}^{0}\bar{v}_2 = \begin{bmatrix} l_1 S_2 & 0\\ l_1 C_2 + l_{c2} & l_{c2}\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = \bar{\bar{\Omega}}_{c2}\cdot\dot{\bar{q}}$$

$$\bar{\bar{\Omega}}_{c1} = \bar{\bar{\Omega}}_1 - [\bar{s}_{c1}\times]\bar{\bar{\Gamma}}_1; \quad \bar{s}_{c1} = \begin{bmatrix}-(l_1 - l_{c1})\\ 0\\ 0\end{bmatrix} \leftrightarrow \bar{\bar{\Omega}}_{c1} = \begin{bmatrix}0\\ l_{c1}\\ 0\end{bmatrix}$$

$$\bar{\bar{\Omega}}_{c2} = \bar{\bar{\Omega}}_2 - [\bar{s}_{c2}\times]\bar{\bar{\Gamma}}_2; \quad \bar{s}_{c2} = \begin{bmatrix}-(l_2 - l_{c2})\\ 0\\ 0\end{bmatrix} \leftrightarrow \bar{\bar{\Omega}}_{c2} = \begin{bmatrix} l_1 S_2 & 0\\ l_1 C_2 + l_{c2} & l_{c2}\\ 0 & 0\end{bmatrix}$$

Jakobinus: $${}^{2}\bar{\bar{J}}_2 = \begin{bmatrix} {}^{2}\bar{d}_0 & {}^{2}\bar{d}_1\\ {}^{2}\bar{t}_0 & {}^{2}\bar{t}_1\end{bmatrix} = \begin{bmatrix}\bar{\bar{\Omega}}_2\\ \bar{\bar{\Gamma}}_2\end{bmatrix} \qquad {}^{2}\bar{\bar{J}}_0 = \begin{bmatrix}\bar{\bar{A}}_{02} & \bar{\bar{0}}\\ \bar{\bar{0}} & \bar{\bar{A}}_{02}\end{bmatrix}\cdot {}^{2}\bar{\bar{J}}_2 = \text{HF}$$

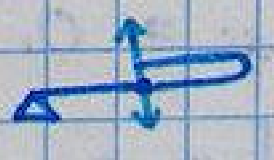
$${}^{2}\bar{\bar{J}}_2 = \left[\begin{array}{cc} l_1 S_2 & 0\\ l_1 C_2 + l_2 & l_2\\ 0 & 0\\ \hline 0 & 0\\ 0 & 0\\ 1 & 1\end{array}\right] \qquad {}^{0}\bar{\bar{J}}_2 = \left[\begin{array}{cc} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12}\\ l_1 C_1 + l_2 C_{12} & l_2 C_{12}\\ 0 & 0\\ \hline 0 & 0\\ 0 & 0\\ 1 & 1\end{array}\right]$$

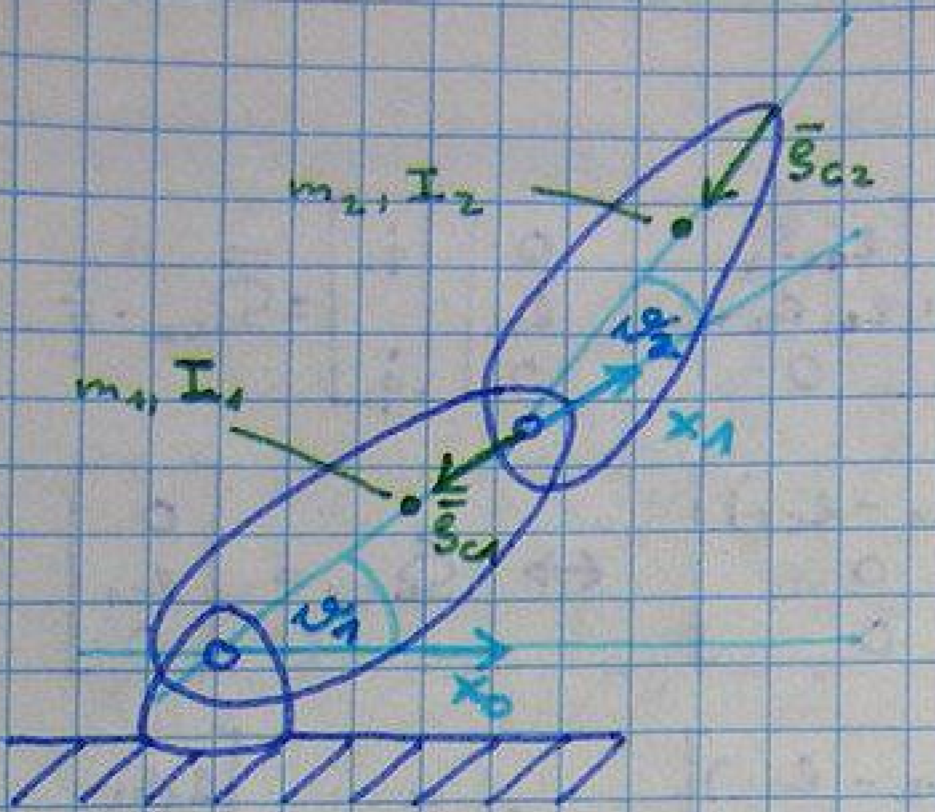
$$\begin{bmatrix}v_{2,x_0}\\ v_{2,y_0}\end{bmatrix} = \begin{bmatrix}-l_1 S_1 - l_2 S_{12} & -l_2 C_{12}\\ l_1 C_1 + l_2 C_{12} & l_2 C_{12}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}$$

mj.: $$\begin{bmatrix}a & b\\ c & d\end{bmatrix}^{-1} = \frac{1}{\underbrace{ad - bc}_{\text{det.}}}\cdot\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

$$\begin{vmatrix}v_{2,x_0}\\ v_{2,y_0}\end{vmatrix} = (-l_1 S_1 - l_2 S_{12})\, l_2 C_{12} - (-l_2 C_{12})(l_1 C_1 + l_2 C_{12}) = l_1 l_2 S_2$$

$$\det = 0 \quad \begin{cases} \vartheta_2 = 0^\circ \\ \vartheta_2 = 180^\circ \end{cases}$$





$$\bar{\bar{H}} = \begin{bmatrix} D_{11} & D_{12} \\ * & D_{22} \end{bmatrix} = \bar{\bar{\Omega}}_{c1}^T \cdot \bar{\bar{\Omega}}_{c1} m_1 + \bar{\bar{\Pi}}_1^T \bar{\bar{K}}_{c1} \bar{\bar{\Pi}}_1 + $$

$$+ \bar{\bar{\Omega}}_{c2}^T \cdot \bar{\bar{\Omega}}_{c2} m_2 + \bar{\bar{\Pi}}_2^T \bar{\bar{K}}_{c2} \bar{\bar{\Pi}}_2 =$$

$$= \begin{bmatrix} 0 & l_{c1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ l_{c1} & 0 \\ 0 & 0 \end{bmatrix} m_1 + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & I_1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} l_1 S_2 & l_1 C_2 + l_{c2} & 0 \\ 0 & l_{c2} & 0 \end{bmatrix} \begin{bmatrix} l_1 S_2 & 0 \\ l_1 C_2 + l_{c2} & l_{c2} \\ 0 & 0 \end{bmatrix} m_2 + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & I_2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} l_{c1}^2 & 0 \\ 0 & 0 \end{bmatrix} m_1 + \begin{bmatrix} I_1 & 0 \\ 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} l_1 S_2^2 + l_1^2 C_2^2 + 2 l_1 C_2 l_{c2} + l_{c2}^2 & l_{c2}(l_1 l_2 + l_{c2}) \\ l_{c2}(l_1 C_2 + l_{12}) & l_{c2}^2 \end{bmatrix} m_2 + \begin{bmatrix} I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

$$\Rightarrow \bar{\bar{H}} = \begin{bmatrix} l_{c1}^2 & 0 \\ 0 & 0 \end{bmatrix} m_1 + \begin{bmatrix} I_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} C_2 & (l_1 C_2 + l_{c2}) l_{c2} \\ * & l_{c2}^2 \end{bmatrix} m_2 + \begin{bmatrix} I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

$$\boxed{\begin{aligned} D_{11} &= m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} C_2) + I + I_2 \\ D_{12} &= m_2 (l_1 C_2 + l_{c2}) l_{c2} + I_2 = D_{21} \\ D_{22} &= m_2 l_{c2}^2 + I_2 \\ P &= m_1 g l_{c1} S_1 + m_2 g (l_1 S_1 + l_{c2} S_{12}) \\ D_{ijk} &= \frac{1}{2} \left(\frac{\partial D_{ij}}{\partial q_k} + \frac{\partial D_{ik}}{\partial q_j} + \frac{\partial D_{jk}}{\partial q_i} \right) \end{aligned}}$$

Elmélet (nyíltláncú): 1.) $D_{ijk} = -D_{kji}$, ha $i,k \geq j$

2.) $D_{iji} = 0$, ha $i \geq j$

$$\bar{\bar{D}}^1 = \begin{bmatrix} D_{111} & D_{112} \\ * & D_{122} \end{bmatrix} = \begin{bmatrix} 0 & D_{112} \\ * & D_{122} \end{bmatrix}, \qquad \bar{\bar{D}}^2 = \begin{bmatrix} D_{211} & D_{212} \\ * & D_{222} \end{bmatrix} = \begin{bmatrix} -D_{112} & 0 \\ * & 0 \end{bmatrix}$$

$$\underset{ijk}{D_{112}} = \frac{1}{2} \left(\frac{\partial D_{11}}{\partial q_2} + \frac{\partial D_{12}}{\partial q_1} - \frac{\partial D_{12}}{\partial q_1} \right) = -m_2 l_1 l_{c2} S_2$$

$$\underset{ijk}{D_{122}} = \frac{1}{2} \left(\frac{\partial D_{12}}{\partial q_2} + \frac{\partial D_{12}}{\partial q_2} - \frac{\partial D_{22}}{\partial q_1} \right) = -m_2 l_1 l_{c2} S_2$$

$$\Rightarrow \bar{\bar{D}}^1 = \begin{bmatrix} 0 & D_{112} \\ D_{112} & D_{122} \end{bmatrix}, \qquad \bar{\bar{D}}^2 = \begin{bmatrix} -D_{112} & 0 \\ 0 & 0 \end{bmatrix}$$

$$D_1 = \frac{\partial P}{\partial q_1} = m_1 g\, l_{c1} C_1 + m_2 g \left(l_1 C_1 + l_{c2} C_{12} \right)$$

$$D_2 = \frac{\partial P}{\partial q_2} = m_2 g\, l_{c2} C_{12}$$

$$\boxed{\begin{aligned} & D_{11} \ddot{q}_1 + D_{12} \ddot{q}_2 + 2 D_{112} \dot{q}_1 \dot{q}_2 + D_{122} \dot{q}_2^2 + D_1 = \tau_1 \\ & D_{12} \ddot{q}_1 + D_{22} \ddot{q}_2 - D_{112} \dot{q}_1^2 + D_2 = \tau_2 \end{aligned}}$$

$$\bar{\bar{H}} \ddot{\bar{q}} + \bar{h} = \tau , \qquad \ddot{\bar{q}} = \bar{\bar{H}}^{-1} \left(-\bar{h} + \bar{\tau} \right) = -\bar{\bar{H}}^{-1} \bar{h} + \bar{\bar{H}}^{-1} \overbrace{\bar{\tau}}^{\bar{u}}$$

$$\bar{x} = \begin{bmatrix} \bar{q} \\ \dot{\bar{q}} \end{bmatrix} \begin{matrix} \} \, \bar{x}_1 \\ \} \, \bar{x}_2 \end{matrix} , \qquad \dot{\bar{x}} = \bar{f}(\bar{x}) + \bar{g}(x) \bar{u}$$

$$\boxed{\dot{\bar{x}}_1 = \bar{x}_2 , \qquad \dot{\bar{x}}_2 = -\bar{\bar{H}}^{-1}(\bar{x}_1) \bar{h}(\bar{x}_1, \bar{x}_2) + \bar{\bar{H}}^{-1}(x_1) \bar{u}}$$

Lagrange: $\sum_i D_{ij}(\bar{q}) \ddot{q}_j + \sum_j \sum_k D_{ijk}(\bar{q}) \dot{q}_j \dot{q}_k + D_i(\bar{q}) = \tau_i$

Appell: $\bar{\bar{H}}(\bar{q}) \ddot{\bar{q}} + \bar{h}(\bar{q}, \dot{\bar{q}}) = \bar{\tau}$

$\Rightarrow$ 3. alak: $\bar{\bar{C}}(\bar{q}, \dot{\bar{q}}) = [c_{ik}], \quad c_{ik} = \sum_j D_{ijk}(\bar{q}) \dot{q}_j \Rightarrow \bar{\bar{C}}(\bar{q}, \dot{\bar{q}}) \dot{\bar{q}}$

$\bar{D}(\bar{q})$

$\bar{\bar{H}}(\bar{q}) \ddot{\bar{q}} + \bar{\bar{C}}(\bar{q}, \dot{\bar{q}}) \dot{\bar{q}} + \bar{D}(\bar{q}) = \bar{\tau}$

$\dot{\bar{\bar{H}}} - 2 \bar{\bar{C}}$ antiszimmetrikus $\Rightarrow$ bármely változóban nulla a kvadratikus alak

4. alak: $\bar{\alpha}$ független paraméterek

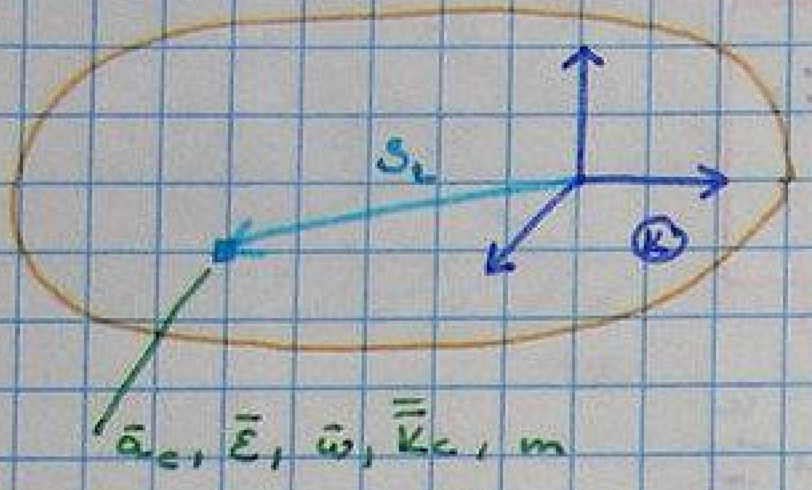
$$\bar{\bar{Y}} (\underbrace{\bar{q}, \dot{\bar{q}}}_{\substack{\text{állapotváltozók} \\ \bar{x}}}, \underbrace{\dot{\bar{q}}}_{\dot{\bar{q}}_r}, \underbrace{\ddot{\bar{q}}}_{\ddot{\bar{q}}_r}) \bar{\alpha} = \bar{\tau} , \qquad \bar{\alpha} = ?$$

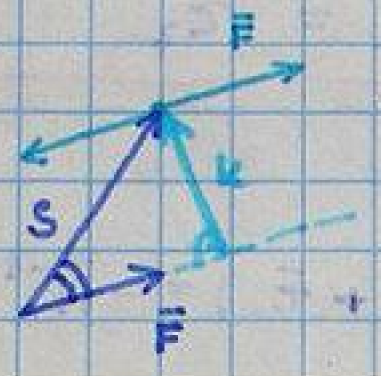
2009.03.18.

Newton - Euler egyenletek

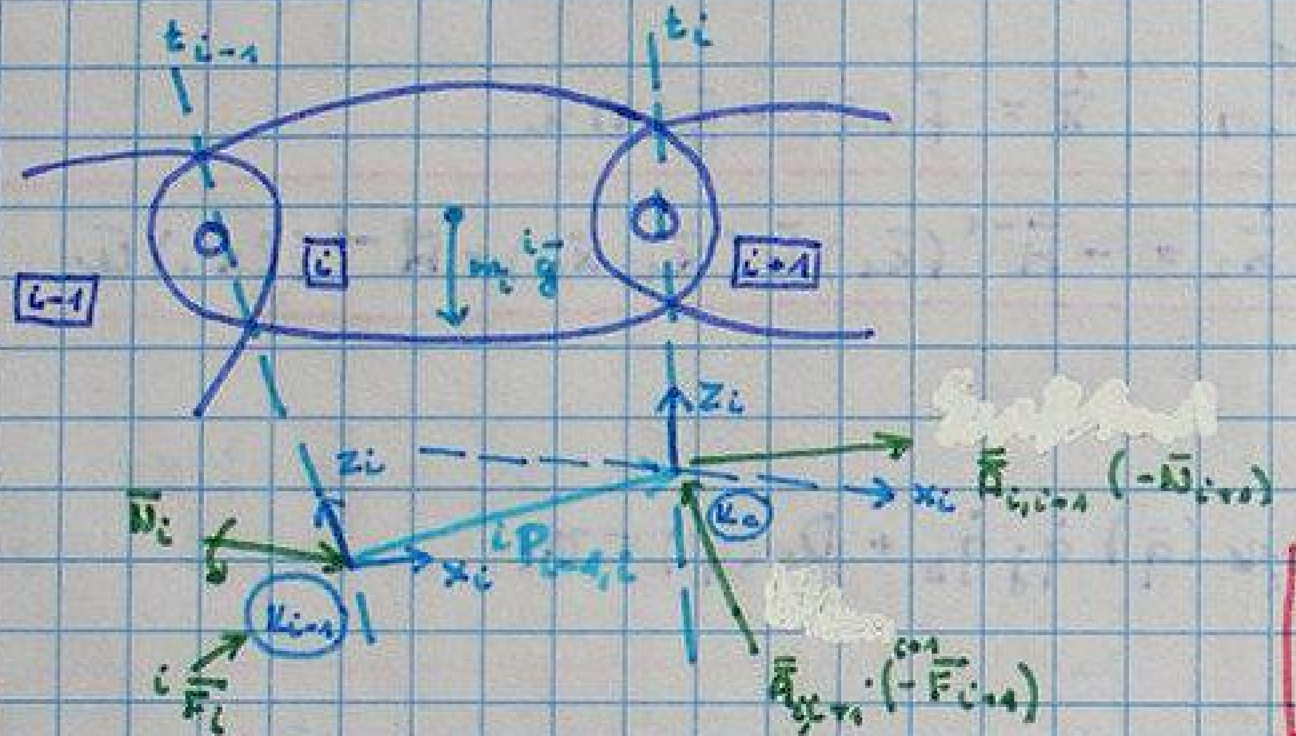
$$(1)\quad m\bar{a}_c = \bar{F}_{ext}$$

$$(2)\quad \bar{\bar{K}}_c \bar{\varepsilon} + \bar{\omega} \times (\bar{\bar{K}}_c \bar{\omega})^{\cdot} = \bar{N}_{ext}$$





$$\bar{N} = \bar{F} \times \bar{S}$$



$$\boxed{m_i \bar{a}_{ci} = m_i\,{}^i\bar{g} + {}^i\bar{F}_i - \bar{\bar{A}}_{i,i+1}\,{}^{i+1}\bar{F}_{i+1}}$$

$$\Rightarrow \boxed{{}^i\bar{F}_i = m_i(\bar{a}_{ci} - {}^i\bar{g}) + \bar{\bar{A}}_{i,i+1}\,{}^{i+1}\bar{F}_{i+1}}$$

$$\bar{\bar{K}}_{ci}\bar{\varepsilon}_i + \bar{\omega}_i \times (\bar{\bar{K}}_{ci}\bar{\omega}_i) = {}^i\bar{N}_i + {}^i\bar{F}_i \times ({}^iP_{i-1,i} + \bar{S}_{ci}) - \bar{\bar{A}}_{i,i+1}\cdot{}^{i+1}\bar{N}_{i+1} - (\bar{\bar{A}}_{i,i+1}\cdot{}^{i+1}\bar{F}_{i+1}) \times \bar{S}_{ci}$$

$$\boxed{{}^i\bar{N}_i = \bar{\bar{K}}_{ci}\bar{\varepsilon}_i + \bar{\omega}_i \times (\bar{\bar{K}}_{ci} w_i) - {}^i\bar{F}_i \times ({}^i\bar{P}_{i-1,i} + \bar{S}_{ci}) + \bar{\bar{A}}_{i,i+1}\cdot{}^{i+1}\bar{N}_{i+1} + (\bar{\bar{A}}_{i,i+1}\cdot{}^{i+1}\bar{F}_{i+1}) \times \bar{S}_{ci}}$$

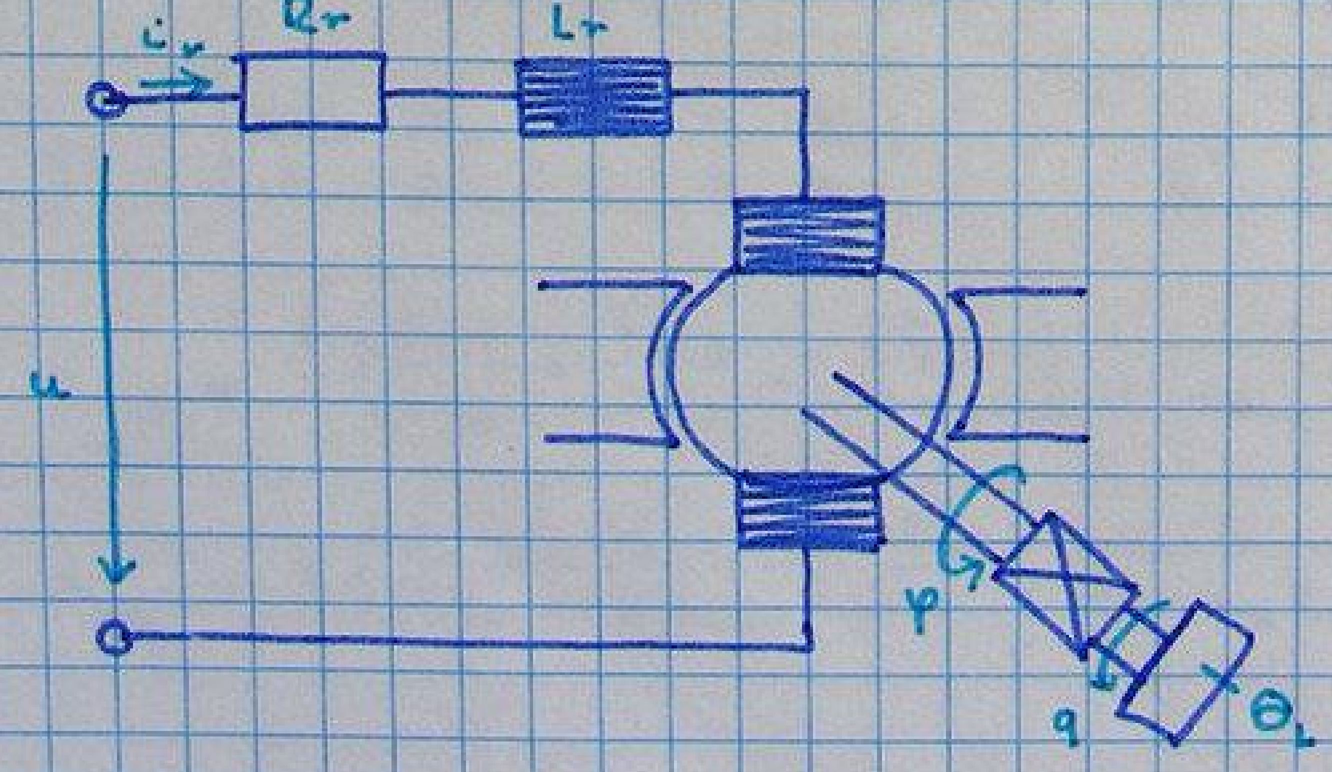
$${}^iP_{i-1,1} = \bar{\bar{A}}^T_{i-1,i}\,\bar{P}_{i-1,i}$$

$${}^i\bar{t}_{i-1} = \bar{\bar{A}}^T_{i-1,i}\begin{pmatrix}0\\0\\1\end{pmatrix} = \bar{n}_{i-1,i}$$

R csukló $\Rightarrow \tau_i = \bar{n}_{i-1,i}\cdot{}^i\bar{N}_i$

T csukló $\Rightarrow \tau_i = \bar{n}_{i-1,i}\cdot{}^i\bar{F}_i$

Aktuátor:



$$\nu = \frac{\dot{\varphi}}{\dot{q}} \quad \text{áttétel}$$

$$R_r i_r + L_r \frac{di_r}{dt} = u - c_1\dot{\varphi}$$

$$\tau_m = c_2 i_r$$

↳ torque / nyomaték

$$N_m = N_s \Leftrightarrow \tau_m\dot{\varphi} = \tau\dot{q} \Rightarrow \tau = \frac{\dot{\varphi}}{\dot{q}}\tau_m \Rightarrow \nu\tau_m$$

m: motor oldal, s: szegmens oldal

Robot + motor + áttétel:

viszkózus súrlódás

$$\sum_j D_{ij}\ddot q_j + \sum_j\sum_k D_{ijk}\,\dot q_j\,\dot q_k + D_i + \Theta_{ri}\cdot\nu_i^2\,\ddot q_i + \nu_i f_{ri}\dot\varphi_i + f_{\text{áll},i}\,\dot q_i = \tau_i = \nu_i c_{2i}\, i_{ri}$$

$$\tau_i = \nu_i c_{2i}\, i_{ri}$$

$$R_{ri}\, i_{ri} + L_{ri}\frac{di_{ri}}{dt} = u_i - c_{1i}\dot\varphi_i$$

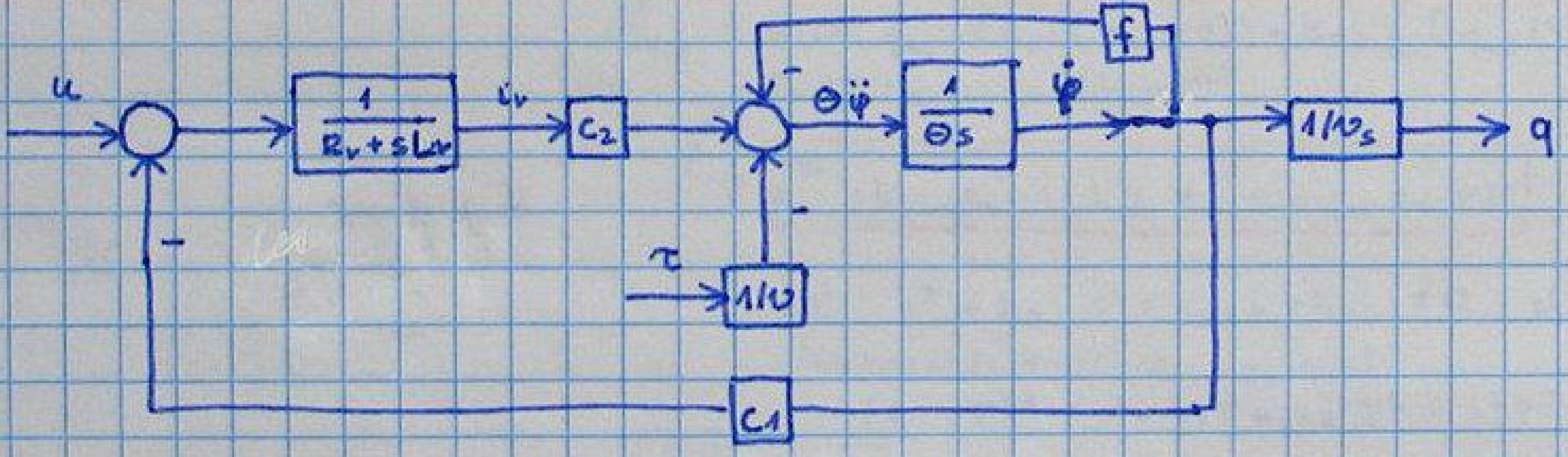
Pálya: $\quad q_a(t) \rightarrow \overline{D_{ii}\,(q_a(t))} = \bar D_{ii} \rightarrow \bar D_{ii}\,\ddot q_i$

$$(\bar D_{ii} + \nu_i^2\Theta_{ri})\,\ddot q_i + (f_{\text{áll},i} + \nu_i^2 f_{ri})\,\dot q_i + \underbrace{\sum_j D_{ij}\ddot q_j + \sum_j\sum_k D_{ijk}\cdot\dot q_j\,\dot q_k + D_i - \bar D_{ii}\,\ddot q_i}_{\tau_i^* \quad \text{(zavarás)}} =$$

$$= \nu_i c_{2i}\, i_{ri}$$

$$\Big(\underbrace{\Theta_{ri} + \frac{D_{ii}}{\nu_i^2}}_{\Theta_i}\Big)\ddot\varphi_i + \Big(\underbrace{f_{ri} + \frac{f_{\text{áll},i}}{\nu_i^2}}_{f_i}\Big)\dot\varphi_i = c_{2i}\, i_{ri} - \frac{\tau_i^*}{\nu_i}$$

$$\boxed{\begin{aligned} \Theta_i\ddot\varphi_i + f_i\dot\varphi_i &= c_{2i}\, i_{ri} - \frac{\tau_i^*}{\nu_i} \\ R_{ri}\, i_{ri} + L_{ri}\frac{di_{ri}}{dt} &= u_i - c_{1i}\dot\varphi_i \end{aligned}}$$



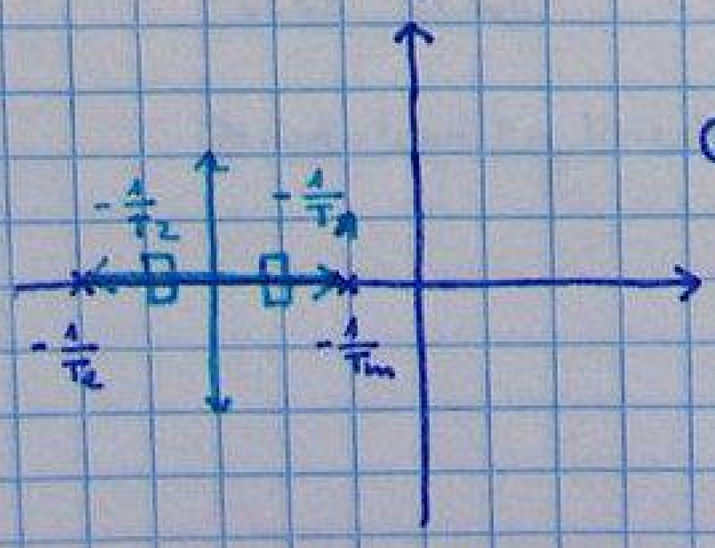
τ_i^* kiszabályozandó

$$\frac{\frac{1}{\Theta s}}{1+\frac{1}{\Theta s}f} = \frac{1}{f+\Theta s} = \frac{1/f}{1+s\underbrace{\frac{\Theta}{f}}_{T_m}}, \qquad \frac{1}{R_r+sL_r} = \frac{1/R_r}{1+s\underbrace{\frac{L_r}{R_r}}_{T_e}}$$

$$W_{\varphi u} = \frac{\frac{1/R_r}{1+sT_e}\cdot c_2\cdot\frac{1/f}{1+sT_m}}{1+\frac{1/R_r}{1+sT_e}\cdot c_2\cdot\frac{1/f}{1+sT_m}\cdot c_1} = \frac{\frac{c_2}{R_r f}}{(1+sT_m)(1+sT_e) + \frac{c_1 c_2}{R_r f}}$$

$$s^2 T_m T_e + s(T_m T_e) + 1 + \frac{c_1 c_2}{R_r f} = 0$$

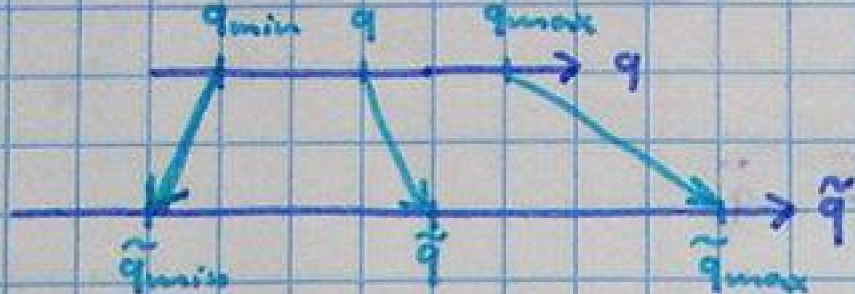
$$W_{\varphi u} = \frac{A}{1+2\xi T s + T^2 s^2} = \begin{cases} \xi > 1 \;\Rightarrow\; \dfrac{A}{(1+sT_1)(1+sT_2)} & T_1, T_2 \\ \xi < 1 \;\Rightarrow\; \dfrac{A}{1+2\xi T s + T^2 s^2} & \text{lengőtag} \end{cases}$$



S

Decentralizált (tengelyenkénti) 3-hurkos kaszkád szabályozás

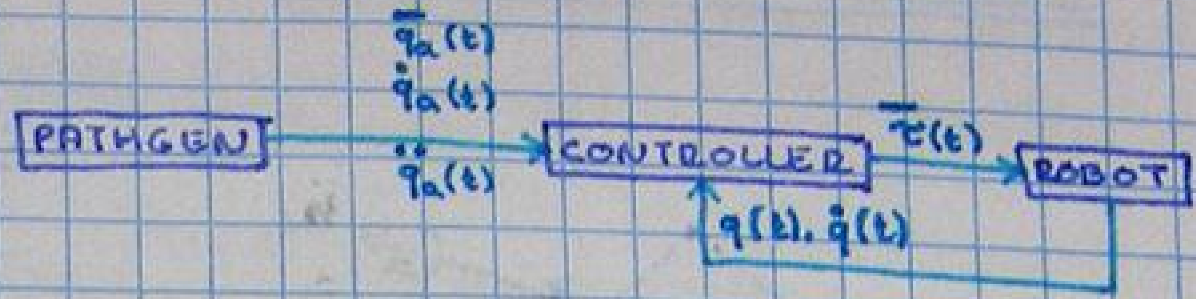
belső áramszabályozás: analóg PI
középső fordulatszámszabályozás: analóg PI
külső pozíciószabályozás: mintavételes PID
érzékelők: $\tilde{q}$ "számláló" (3-hurkos, forgásirányfüggő)



$\tilde{\tau}$ feszültség: $\tilde{\tau} = \gamma\, i_r$
$\tilde{n}$ feszültség: $\tilde{n} = \delta\, \dot{\varphi}$

2008. 03. 25.

Pálya generálás - PATHGEN
Kontroller - CONTROLLER



$\dot{q}_a(t) := \dot{\bar{q}}_a(t), \dots$

S-függvény (flad, idő, állapotváltozók)

m — matlab
C — C

Embedded-function / Beágyazott függvény (csak meghatározott függvények hívására)

$$\boxed{\bar{\bar{H}}(\bar{q})\ddot{\bar{q}} + \bar{h}(\bar{q}, \dot{\bar{q}}) = \bar{\tau}}$$

1.) Kiszámított nyomatékok módszere / Computed Torque Technic (CTT)

Robot: $\bar{\bar{H}}\ddot{\bar{q}} + \bar{h} = \bar{\tau}$

CTT szabályozás: $\bar{\tau} := \bar{\bar{H}}\bar{u} + \bar{h}$

ZR: $\bar{\bar{H}}\ddot{\bar{q}} + \cancel{\bar{h}} = \bar{\bar{H}}\bar{u} + \cancel{\bar{h}}, \quad \exists \bar{\bar{H}}^{-1}$

$\ddot{\bar{q}} = \bar{u} \iff \ddot{q}_i = u_i$ szétcsatolt kettős integrál

Decentralizált szabályozók: $u_i := \ddot{q}_{ai} + PID$

$\ddot{q}_{ai} = u_i := \ddot{q}_{ai} + k_{Pi}(q_{ai} - q_i) + k_{Ii}\int_0^t [q_{ai}(\tau) - q_i(\tau)]d\tau + k_{Di}(\dot{q}_{ai} - \dot{q}_i)$

$e_i := q_{ai} - q_i$ (hiba)

$$\boxed{e_i''' + k_{Di} e_i'' + k_{Pi} e_i' + k_{Ii} e_i = 0}$$

KE: $s^3 + k_{Di}s^2 + k_{Pi}s + k_{Ii} = 0$ stabil és gyors

karakterisztikus egyenlet

$(1 + sT)^3 = 1 + 3sT + 3s^2T^2 + s^3T^3 = 0$

$\Rightarrow / : T^3 \Rightarrow$

$$s^2 + \underbrace{\frac{3}{T}}_{k_{Di}} s^2 + \underbrace{\frac{3}{T^2}}_{k_{Pi}} s + \underbrace{\frac{1}{T^3}}_{k_{Ii}} = 0$$

pl.: $T = 50ms$, $i = 1, 2, 3$
$T = 25ms$, $i = 4, 5, 6$

Valóságban: → névleges modell $\hat{\bar{\bar{H}}}, \hat{\bar{h}}$
→ valódi modell $\bar{\bar{H}}, \bar{h}$

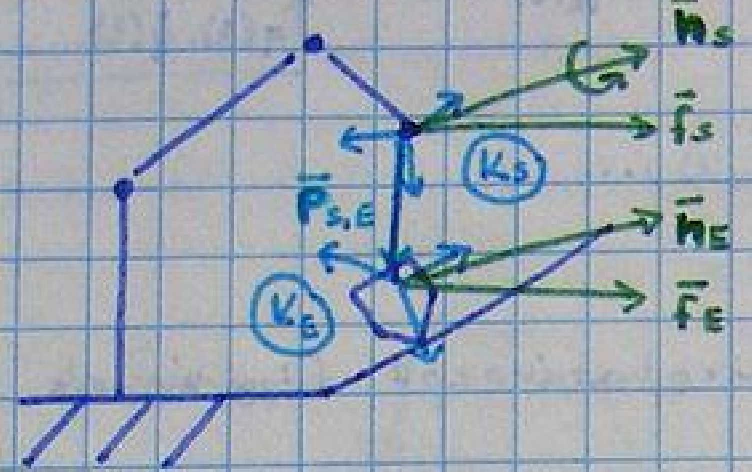
$\bar{\bar{H}}\ddot{\bar{q}} + \bar{h} = \hat{\bar{\bar{H}}}\bar{u} + \hat{\bar{h}}$

$$\boxed{\ddot{\bar{q}} = \bar{\bar{H}}^{-1}(\hat{\bar{h}} - \bar{h}) + \bar{\bar{H}}^{-1}\hat{\bar{\bar{H}}}\bar{u}}$$ csak közelítőleg szétcsatolt!

Felső szint: $\bar{\tau} := \hat{\bar{\bar{H}}}(\bar{q})\bar{u} + \hat{\bar{h}}(\bar{q}, \dot{\bar{q}})$

Decentralizált lineáris: $u_i := \ddot{q}_{ai} + PID$

Erő és nyomaték áthelyezés:



Bázisfüggetlen alak:

$$\bar{f}_S = \bar{f}_E$$
$$\bar{n}_E = \bar{n}_S + \bar{f}_S \times \bar{p}_{S,E}$$

Bázisfüggő alak:

$$K_S \xrightarrow{\bar{\bar{T}}_{S,E}} K_E$$
$\bar{f}_S$ $\qquad$ $\bar{f}_E = ?$

$\bar{n}_S$ $\qquad$ $\bar{n}_E = ?$

$$\bar{f}_E = \bar{\bar{A}}_{SE}^T \bar{f}_S$$

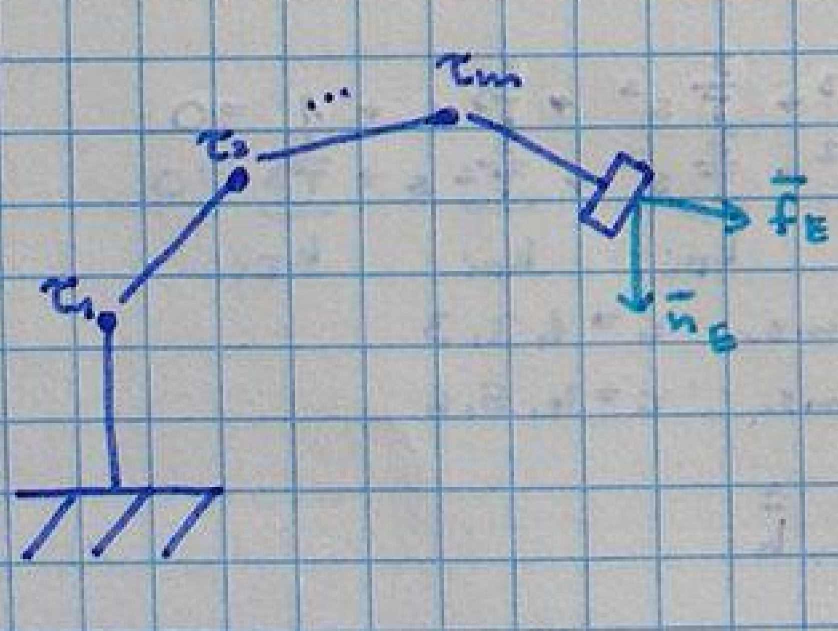
$$\bar{n}_E = \bar{\bar{A}}_{SE}^T \{ \bar{n}_S + \underbrace{\bar{f}_S \times \bar{p}_{SE}}_{-\bar{p}_{SE} \times \bar{f}_S} \} = \bar{\bar{A}}_{SE}^T \bar{n}_S + \underbrace{\bar{\bar{A}}_{SE}^T [\bar{p}_{SE} \times]^T}_{([\bar{p}_{SE} \times] \bar{\bar{A}}_{SE})^T} \bar{f}_S = ([\bar{p}_{SE} \times] \bar{\bar{A}}_{SE})^T \bar{f}_S + \bar{\bar{A}}_{SE}^T \bar{n}_S$$

$$\begin{pmatrix} \bar{f}_E \\ \bar{n}_E \end{pmatrix} = \begin{bmatrix} \bar{\bar{A}}_{SE}^T & \bar{\bar{O}} \\ ([\bar{p}_{SE} \times] \bar{\bar{A}}_{SE})^T & \bar{\bar{A}}_{SE}^T \end{bmatrix} \begin{pmatrix} \bar{f}_S \\ \bar{n}_S \end{pmatrix}$$

$$K_1 \xrightarrow{\bar{\bar{T}}_{1,2}} K_2$$

$$\begin{pmatrix} \bar{f}_1 \\ \bar{n}_1 \end{pmatrix} \longrightarrow \begin{pmatrix} \bar{f}_2 \\ \bar{n}_2 \end{pmatrix} = \begin{bmatrix} \bar{\bar{A}}_{12}^T & \bar{\bar{O}} \\ ([p_{12} \times] \bar{\bar{A}}_{12})^T & \bar{\bar{A}}_{12}^T \end{bmatrix} \begin{pmatrix} \bar{f}_1 \\ \bar{n}_1 \end{pmatrix}$$

Ekvivalens csuklónyomatékok:



$$\bar{\bar{F}}_E = \begin{pmatrix} \bar{f}_E \\ \bar{n}_E \end{pmatrix}, \quad \bar{\tau} = \begin{pmatrix} \tau_1 \\ \vdots \\ \tau_m \end{pmatrix}, \quad \begin{pmatrix} \bar{v} \\ \bar{\omega} \end{pmatrix} = \bar{\bar{J}} \dot{\bar{q}}$$

$$\downarrow$$

$$\delta \bar{x} = \bar{\bar{J}} \delta \bar{q} \qquad \bar{x} = \begin{pmatrix} \bar{p} \\ t_\varphi \end{pmatrix}$$

$$\langle \bar{F}, \underbrace{\delta x}_{\bar{\bar{J}} \delta \bar{q}} \rangle = \langle \bar{\tau}, \delta \bar{q} \rangle$$

T

$$\langle \bar{\bar{J}}^T \bar{F}, \delta \bar{q} \rangle = \langle \bar{\tau}, \delta \bar{q} \rangle, \quad \forall \delta \bar{q} \Rightarrow \boxed{\bar{\tau} = \bar{\bar{J}}^T \bar{F}}$$

2009. 03. 26.

Robot dinamikus modellje: Descartes-koordinátákban:

$$\bar{\bar{H}}\ddot{\bar{q}} + \bar{h} = \bar{\tau}$$

$\bar{F}$ ált. erő a TCP-ben

$\bar{\bar{J}}$ a Jacobi-mátrix TCP-ig

$$\dot{\bar{x}} = \bar{\bar{J}}\cdot\dot{\bar{q}}\ ; \quad \ddot{\bar{x}} = \bar{\bar{J}}\cdot\ddot{\bar{q}} + \underbrace{\frac{d\bar{\bar{J}}}{dt}\cdot\dot{\bar{q}}}_{\bar{\alpha}} = \overbrace{\bar{\bar{J}}}^{\begin{pmatrix}\bar{\bar{\Omega}}\\ \bar{\bar{\Gamma}}\end{pmatrix}}\cdot\ddot{\bar{q}} + \underbrace{\begin{pmatrix}\bar{\Theta}\\ \bar{\Phi}\end{pmatrix}}_{\bar{\alpha}} \Rightarrow \ddot{\bar{q}} = \bar{\bar{J}}^{-1}(\ddot{\bar{x}} - \bar{\alpha})$$

$$\bar{\bar{H}}\bar{\bar{J}}^{-1}(\ddot{\bar{x}} - \bar{\alpha}) + \bar{h} = \bar{\bar{J}}^{T}\bar{F}$$

$$\bar{\bar{H}}^{*} := \underbrace{(\bar{\bar{J}}^{T})^{-1}}_{\bar{\bar{J}}^{-T}}\bar{\bar{H}}\bar{\bar{J}}^{-1} = \bar{\bar{J}}^{-T}\bar{\bar{H}}\bar{\bar{J}}^{-1} \qquad \bar{h}^{*} := \bar{\bar{J}}^{-T}\bar{h} - \bar{\bar{H}}^{*}\bar{\alpha}$$

$$\Rightarrow \boxed{\bar{\bar{H}}^{*}\ddot{\bar{x}} + \bar{h}^{*} = \bar{F}}\ ; \quad \boxed{\bar{\bar{H}}^{*} = \bar{\bar{J}}^{-T}\bar{\bar{H}}\bar{\bar{J}}^{-1}\ ; \quad \bar{h}^{*} = \bar{\bar{J}}^{-T}\bar{h} - \bar{\bar{H}}^{*}\bar{\alpha}}$$

$$\bar{\bar{H}}^{*}\ddot{\bar{x}} + \bar{h}^{*} = \bar{F} \quad \text{(robot)}$$

Szabad mozgás nemlineáris szétcsatolása

Irányítás: $\boxed{\bar{F} := \bar{\bar{H}}^{*}\bar{u}^{*} + \bar{h}^{*}}$ (centralizált szabályozó)

Zárt rendszer (ZR): $\bar{\bar{H}}^{*}\ddot{\bar{x}} + \bar{h}^{*} = \bar{\bar{H}}^{*}\bar{u}^{*} + \bar{h}^{*}, \quad \exists\, \bar{\bar{H}}^{*-1}$

$$\ddot{\bar{x}} = \bar{u}^{*} \Longleftrightarrow \ddot{x}_i = u_i^{*}, \quad i = 1, \dots, 6 \qquad \text{(szétcsatolt kettősintegrátorok)}$$

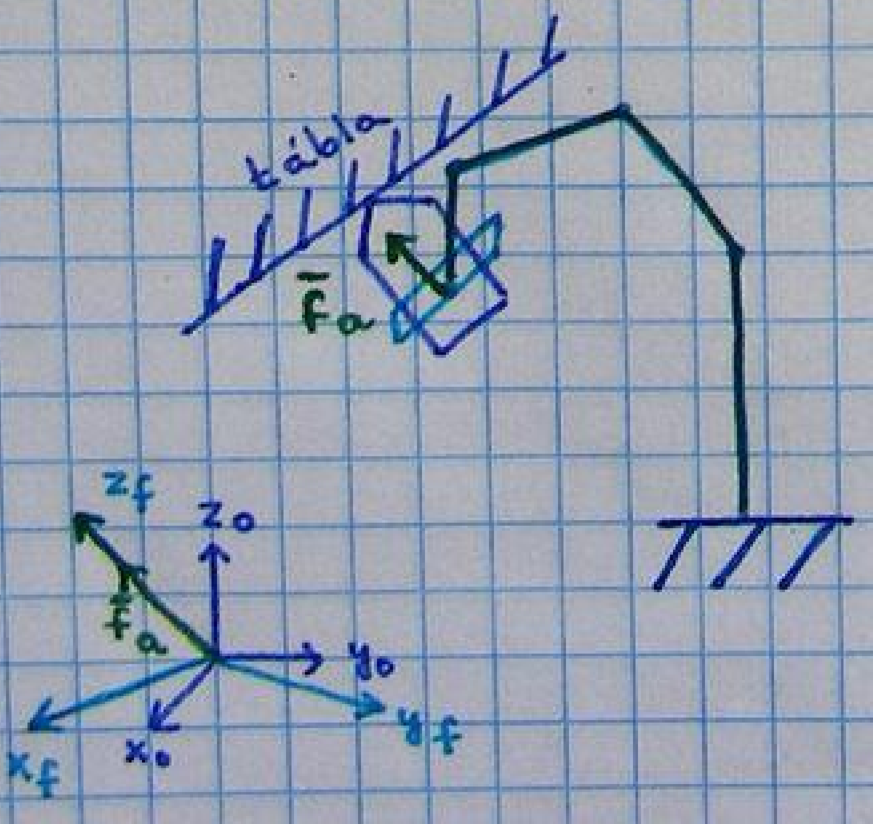
Decentralizált: $u_i^{*} := \ddot{x}_{ai} + PID = \ddot{x}_{ai} + k_{pi}(x_{ai} - x_i) + k_{Ii}\cdot\int_0^T (x_{ai} - x_i)\,dz + k_{Di}(\dot{x}_{ai} - \dot{x}_i)$

$$\boxed{u_i^{*} = \ddot{x}_{ai} + k_{pi}(x_{ai} - x_i) + k_{Ii}\int_0^T (x_{ai} - x_i)\,dz + k_{Di}(\dot{x}_{ai} - \dot{x}_i)}$$

$$\boxed{\bar{\tau} := \bar{\bar{J}}^{T}\bar{F} = \bar{\bar{H}}\bar{\bar{J}}^{-1}(\bar{u}^{*} - \bar{\alpha}) + \bar{h}} \quad \text{(implementálás}$$

pl.: táblára krétával író robotkar

$\bar{F}$ erő: ne legyen túl gyenge (levegőbe írás) sem erős (törik a kréta)



Operációs tármódszer (Khatib)

Pozíció specifikációs mátrix:

$$\bar{\bar{\Sigma}}_f = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}, \quad \sigma_x = 1, \text{ ha } x_f \text{ irányban szabad mozgás lehet}$$

stb.

Erő specifikációs mátrix: $\tilde{\bar{\bar{\Sigma}}}_f = \bar{\bar{I}} - \bar{\bar{\Sigma}}_f$

$$K_f \xrightarrow{\bar{\bar{A}}_f} K_\sigma \qquad \bar{v}_f = \bar{\bar{A}}_f \bar{S}_\sigma$$

Hasonlóan: τ_a spec. nyomaték, $\bar{\bar{\Sigma}}_\tau$, $\tilde{\bar{\bar{\Sigma}}}_\tau$, K_τ, $\bar{\bar{A}}_\tau$

$$K_\tau \xrightarrow{\bar{\bar{A}}_\tau} K_\sigma \qquad \bar{v}_\tau = \bar{\bar{A}}_\tau \bar{S}_\sigma$$

- $\bar{x}_a - \bar{x}$ poz./orient. hiba és $\bar{F}_a - \bar{F}$ erő/nyomaték hiba számítása K_σ-ban
- hibák áttranszformálása K_f, K_τ-ba
- nemirányítható hibák elnyomása (nullázása) K_f, K_τ-ban a maszk mátrixok figyelembevételével
- megmaradt hibák vissza transzformálása K_σ-ba

Általánosított feladatspec. mátrixok:

$$\bar{\bar{S}} = \begin{bmatrix} \bar{\bar{A}}_f^T \bar{\bar{\Sigma}}_f \bar{\bar{A}}_f & \bar{\bar{0}} \\ \bar{\bar{0}} & \bar{\bar{A}}_\tau^T \bar{\bar{\Sigma}}_\tau \bar{\bar{A}}_\tau \end{bmatrix}, \qquad \tilde{\bar{\bar{S}}} = \begin{bmatrix} \bar{\bar{A}}_f^T \tilde{\bar{\bar{\Sigma}}}_f \bar{\bar{A}}_f & \bar{\bar{0}} \\ \bar{\bar{0}} & \bar{\bar{A}}_\tau^T \tilde{\bar{\bar{\Sigma}}}_\tau \bar{\bar{A}}_\tau \end{bmatrix}$$

$$\bar{F} := \bar{F}_{mozgás} + \bar{F}_{ccgf} + \bar{F}_{aktív}$$

$$\bar{F}_{mozgás} := \hat{\bar{\bar{H}}}^* \bar{\bar{S}} \bar{u}^*_{mozgás}$$

$$\bar{F}_{ccgf} := \hat{\bar{h}}^*$$

$$\bar{F}_{aktív} := \tilde{\bar{\bar{S}}} \bar{u}^*_{aktív} + \hat{\bar{\bar{H}}}^* \hat{\bar{\bar{S}}} \bar{u}^*_{csillapítás}$$

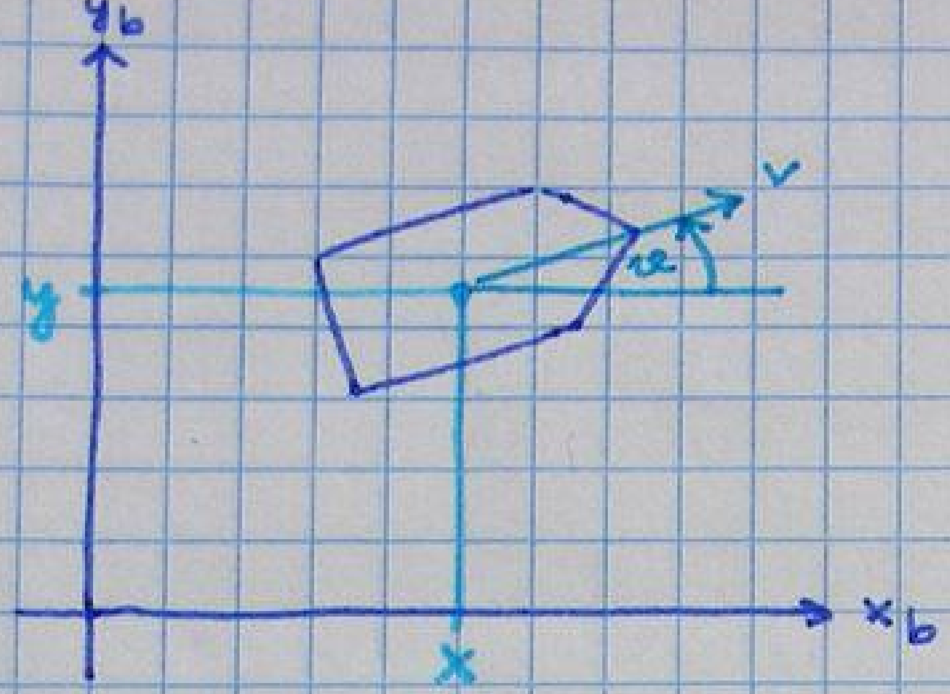
$$\bar{u}^*_{mozgás} := \ddot{\bar{x}}_a + PID = \ddot{\bar{x}}_a + k_P(\bar{x}_a - \bar{x}) + k_I \int_0^t (\bar{x}_a - \bar{x})\,dt + k_D(\dot{\bar{x}}_a - \dot{\bar{x}})$$

$$\bar{u}^*_{aktív} := \bar{F}_a + PI = \bar{F}_a + k_{PF}(\bar{F}_a - \bar{F}) + k_{IF} \int_0^t (\bar{F}_a - \bar{F})\,d\tau$$

$$\bar{u}^*_{csillapítás} := -K_{VF}\,\dot{\bar{x}}$$

Implementálás: $\boxed{\bar{\tau} := \bar{\bar{J}}^T \bar{F} = \hat{\bar{\bar{H}}} \bar{\bar{J}}^{-1} \left\{ \bar{\bar{S}} u^*_{mozgás} + \hat{\bar{\bar{S}}} u^*_{csillapítás} - \bar{\alpha} \right\} + \bar{\bar{J}}^T \tilde{\bar{\bar{S}}} u^*_{aktív} + \bar{h}_{ccgf}}$

b: bázis koordinátarendszer



$\dot{x} = v\,C\varphi;\quad \dot{y} = v\,S\varphi;\quad \dot{\varphi} = \omega$

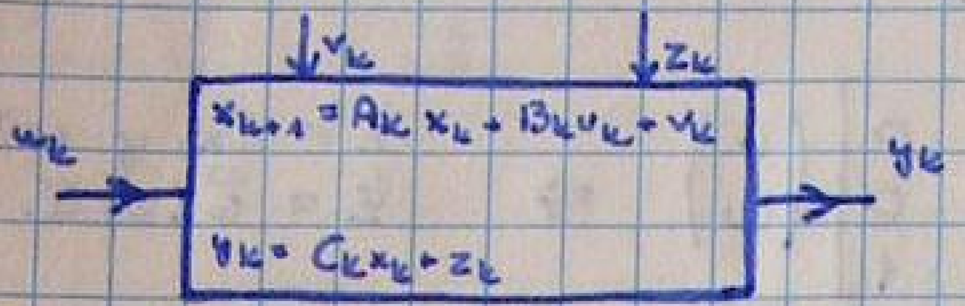
$\Downarrow$

$$\underbrace{\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{pmatrix}}_{\dot{\bar{z}}} = \underbrace{\begin{bmatrix} C\varphi & 0 \\ S\varphi & 0 \\ 0 & 1 \end{bmatrix}}_{\bar{\bar{G}}(\bar{z})} \underbrace{\begin{pmatrix} v \\ \omega \end{pmatrix}}_{\bar{u}} \Rightarrow \begin{array}{l} \bar{z} = (x, y, \varphi)^T \\ \dot{\bar{z}} = \bar{\bar{G}}(z)\,\bar{u} \end{array}$$

Kétféle mozgás:
- helyzetszabályozás
  (lineáris állapotvisszacsatolás $\underline{K}$)
- pályakövetés

2009.04.08.

Kalman-szűrő



$x(0) \to E\,x(0) = x_0,$

$\Sigma_0 = E[(x(0) - x_0)(x(0) - x_0)^T]$

$E v_k = 0$

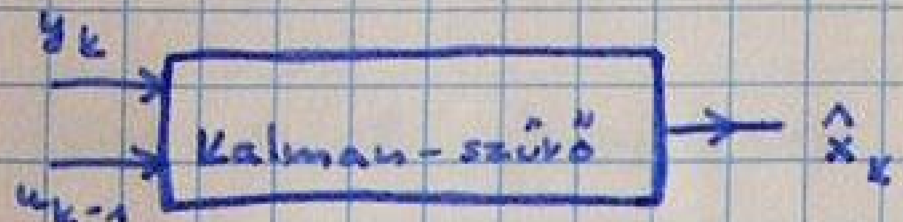
$E[v_k v_l^T] = R_{v,k}\,\delta_{k,l}$

$E z_k = 0$

$E[z_k z_l^T] = R_{z,k}\,\delta_{k,l}$

$E v_k z_k^T = 0$

$x(0)$ nem korrelál z_k, v_k



$\hat{x}_k = F_k \hat{x}_{k-1} + G_k y_k + H_k u_{k-1}$

$\hat{x}_0 = E x(0) = x_0$

(1) $E(x_k - \hat{x}_k) = 0, \ \forall k$

(2) $\Sigma_k = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \to \inf$

$x_{k+1} = A_k x_k + B_k u_k + v_k$

$\hat{x}_{k+1} = F_k \hat{x}_k + G_{k+1}[(A_k x_k + B_k u_k + v_k) + z_{k+1}] + H_k u_k$

$\boxed{H_k = B_k - G_{k+1} C_{k+1} B_k}$

$\boxed{F_k = (I - G_{k+1} C_{k+1}) A_k}$

$x_{k+1} - \hat{x}_{k+1} = (I - G_{k+1} C_{k+1}) A_k x_k + (B_k - G_{k+1} C_{k+1} B_k - H_k) u_k +$
$+ (I - G_{k+1} C_{k+1}) v_k - G_{k+1} z_{k+1} - F_k \hat{x}_k$

$x_{k+1} - \hat{x}_{k+1} = (I - G_{k+1} C_{k+1})[A_k x_k + B_k u_k + v_k - A_k \hat{x}_k - B_k u_k] - G_{k+1} z_{k+1}$

$\boxed{\bar{x}_{k+1} = A_k \hat{x}_k + B_k u_k}$

$x_{k+1} - \hat{x}_{k+1} = (I - G_{k+1} C_{k+1})(x_{k+1} - \bar{x}_{k+1}) - G_{k+1} z_{k+1}$

$E[(x_{k+1} - \bar{x}_{k+1})(x_{k+1} - \bar{x}_{k+1})^T] = \Pi_{k+1}$

$\Sigma_{k+1} = (I - G_{k+1} C_{k+1}) \Pi_{k+1} (I - G_{k+1} C_{k+1})^T + G_{k+1} R_{z,k+1} G_{k+1}^T \to \inf$

$-(I - G_{k+1} C_{k+1}) \Pi_{k+1} C_{k+1}^T + G_{k+1} R_{z,k+1} = 0$

$\hookrightarrow \boxed{G_{k+1} = \Pi_{k+1} C_{k+1}^T (C_{k+1} \Pi_{k+1} C_{k+1}^T + R_{z,k+1})^{-1}}$

Aktuális Kalman-szűrő algoritmus

① Inic.: $\hat{x}_0 = Ex(0) = x_0, \ \Sigma_0$

② Előretartó rekurzió

$(k-1)T$ → ① $\quad T$ → ②

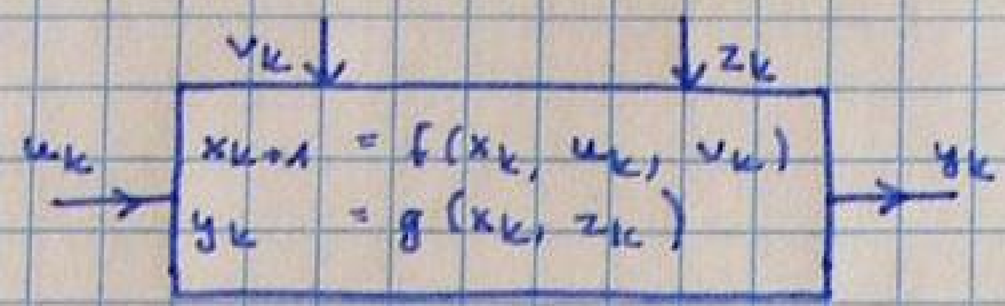
$\hat{x}_{k-1}$ $\quad y_k$

u_{k-1} $\quad \hat{x}_k = ?$

$$\bar{x}_k = A_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1}$$
$$\Pi_k = A_{k-1}\Sigma_{k-1}A_{k-1}^T + R_{v,k-1}$$
$$\Sigma_k = \Pi_k - \Pi_k C_k^T (C_k\Pi_k C_k^T + R_{z,k}) C_k \Pi_k$$
$$G_k = \Pi_k C_k^T (C_k \Pi_k C_k^T + R_{z,k})^{-1}$$
$$\hat{x}_k = \bar{x}_k + G_k(y_k - C_k\bar{x}_k)$$

① time update

② measurement update



$$\hat{x}_{k-1}, v_{k-1} = 0 \longrightarrow f(\hat{x}_{k-1}, u_k, 0) + \frac{\partial f(\hat{x}_{k-1}, u_{k-1}, 0)}{\partial x}(x_k - \hat{x}_{k-1}) + \frac{\partial f(\hat{x}_{k-1}, u_{k-1}, 0)}{\partial v} v_k,$$

$$A_{k-1} = \frac{\partial f(\hat{x}_{k-1}, u_{k-1}, u)}{\partial x}, \qquad B_{v,k-1} = \frac{\partial f(\hat{x}_{k-1}, u_{k-1}, 0)}{\partial v}$$

$$\bar{x}_k, z_k = 0 \longrightarrow g(\bar{x}_k, 0) + \frac{\partial g(\bar{x}_k, 0)}{\partial x}(x_k - \bar{x}_k) + \frac{\partial g(\bar{x}_k, 0)}{\partial z} z_k,$$

$$C_k = \frac{\partial g(\bar{x}_k, 0)}{\partial x}, \qquad C_{z,k} = \frac{\partial g(\bar{x}_k, 0)}{\partial z}$$

$$E[B_{v,k-1} v_{k-1} v_{k-1}^T B_{v,k-1}^T] = B_{v,k-1} R_{v,k-1} B_{v,k-1}^T =: R_{v,k-1}$$

$$E[C_{z,k}\, z_k z_k^T C_{z,k}^T] = C_{z,k} R_{z,k} C_{z,k}^T =: R_{z,k}$$

EKF-szűrő:

Inic.: $\hat{x}_0 = Ex(0) = x_0, \ \Sigma_0$

Előretartó rekurzió:

$\bar{x}_k = f(\hat{x}_{k-1}, u_{k-1}, 0)$

Nemlineáris rendszer linearizálása: A_{k-1}; B_{k-1}; C_k; $C_{z,k}$; korr. $R_{v,k-1}$; $R_{z,k}$

①

② $\hat{x}_k = \bar{x}_k + G_k(y_k - g(\bar{x}_k, 0))$

$(k-1)T$ → ① $\quad kT$ →

$\hat{x}_{k-1}$ $\quad y_k$

u_{k-1} $\quad \hat{x}_k = ?$

$\bar{x}_k$