1 $\boxed{18}$ $\sum_{n=1}^{\infty} \frac{n!}{(2n)^n} x^n$ KS = ? $\frac{1}{R} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ 2 $= \lim_{n\to\infty} \frac{\frac{(n+1)!}{(2n+2)^{n+1}}}{\frac{n!}{(2n)^n}}$ 2 $= \lim_{n\to\infty} (n+1)\cdot\left(\frac{2n}{2n+2}\right)^n \cdot \frac{1}{2n+2}$

$= \lim_{n\to\infty} \frac{1}{2}\left(1 - \frac{2}{2n+2}\right)^n = \lim_{n\to\infty} \frac{1}{2}\left(1-\frac{1}{n+1}\right)^{n+1} \cdot \frac{1}{1-\frac{1}{n+1}} \to \frac{1}{2e}$ 4 $R = 2e$

2 $\boxed{8}$ $f(x) = \sqrt{x}$ $x_0 = 9$

a, $\sqrt{x} = \sqrt{9+x-9} = 3\sqrt{1+\frac{x-9}{9}} = 3\left(1+\frac{x-9}{9}\right)^{\frac{1}{2}} = \sum_{k=0}^{\infty} 3\binom{\frac{1}{2}}{k}\cdot\frac{1}{9^k}(x-9)^k$ 4

KS: $\left|\frac{x-9}{9}\right| < 1 \Rightarrow |x-9| < 9 \Rightarrow R = 9$ 1 3

b, $\sqrt{10} = 3\cdot\sqrt{1+\frac{1}{9}} \approx 3 + 3\cdot\frac{1}{2}\cdot\frac{1}{9} + 3\cdot\frac{1}{9^2}\cdot\frac{\frac{1}{2}\cdot\left(-\frac{1}{2}\right)}{2}\cdot 1 = 3 + \frac{1}{6} - \frac{1}{8\cdot 27}$

3 $\boxed{9}$ a, $f(x) = \frac{1}{x-7}$ $x_0 = 4$ $f(x) = \frac{1}{-3+x-4} = -\frac{1}{3}\,\frac{1}{1-\frac{x-4}{3}} = -\frac{1}{3}\sum_{k=0}^{\infty}\left(\frac{x-4}{3}\right)^k$

$= \sum_{k=0}^{\infty} -\frac{1}{3^{k+1}}(x-4)^k$ 5 , ha $\left|\frac{x-4}{3}\right| < 1$, azaz $x \in (1,7)$

b, $g(x) = \mathrm{ch}(3x)$ $x_0 = 0$ $\mathrm{ch}\, u = \sum_{k=0}^{\infty}\frac{u^{2k}}{(2k)!} \Rightarrow \mathrm{ch}\,3x = \sum_{k=0}^{\infty}\frac{3^{2k}\cdot x^{2k}}{2k!}$ 4

K.T. = $\mathbb{R}$

4 $\boxed{8}$ a, $f(x,y) = (3x^2+4y^2)\,\mathrm{arctg}\,\frac{x}{y}$

$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{r\to 0} \underbrace{\overbrace{(3r^2\cos^2\varphi + 4r^2\sin^2\varphi)}^{\to 0}\,\overbrace{\mathrm{arctg}\frac{\cos\varphi}{\sin\varphi}}^{\text{korlátos}}} = 0$ 4

b, $g(x,y) = \mathrm{arctg}^2\left(\frac{x+1}{y}\right)$

$\lim_{(x,y)\to(0,0)} g(x,y) = \lim_{r\to 0} \mathrm{arctg}^2\left(\frac{r\cos\varphi+1}{r\sin\varphi}\right) = \lim_{r\to 0}\mathrm{arctg}^2\overbrace{\left(\mathrm{ctg}\,\varphi + \frac{1}{r}\cdot\sin\varphi\right)}^{\pm\infty\text{-ben is ugyanode tart arctg}^2} = \frac{\pi^2}{4}$ 4

y = 0-ban nem ért, ott nem baj ha nincs lim

5 $\boxed{17}$ $f(x,y) = \sqrt{x^2+3y^2}$ P(1,2)

a, Folytonos az origóban? $f(0,0) = 0$ ill. $\lim_{(x,y)\to(0,0)}\sqrt{x^2+3y^2} = \lim_{r\to 0} \overset{0}{r}\,\overbrace{\sqrt{1+2\sin^2\varphi}}^{\text{korl}} = 0$

$\Rightarrow$ igen 1 Vagy: igen, mert folytonos fvek kompozíciója.

b, $f'_x = \frac{1}{2}\,\frac{1}{\sqrt{x^2+3y^2}}\cdot 2x$ 2
$f'_y = \frac{1}{2}\,\frac{1}{\sqrt{x^2+3y^2}}\cdot 6y$ 2
$(x,y) \neq (0,0)$

$f'_x(0,0) = \lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0}\frac{\overbrace{\sqrt{h^2}}^{|h|}}{h}$ $\nexists$ 2

$f'_y(0,0) = \lim_{h\to 0}\frac{f(0,h)-f(0,0)}{h} = \lim_{h\to 0}\frac{\sqrt{3h^2}}{h}$ $\nexists$ 2

c, Ahol (egy környezetben) $\exists$ parc deriv & folytonosak, ott van gradiens, így $\mathbb{R}^2\setminus\{(0,0)\}$-n van. Az origóban $\nexists$ parc deriv $\Rightarrow$ $\nexists$ grad sem. 4

d, Érintősík: $f'_x(x_0,y_0)(x-x_0) + f'_y(x_0,y_0)(y-y_0) - (z - f(x_0,y_0)) = 0$

$f'_x(1,2) = \frac{1}{\sqrt{13}}$ $f'_y(1,2) = \frac{6}{\sqrt{13}}$ $f(1,2) = \sqrt{13}$, így

$\frac{1}{\sqrt{13}}(x-1) + \frac{6}{\sqrt{13}}(y-2) - (z-\sqrt{13}) = 0 \Rightarrow (x-1) + 6(y-2) - \sqrt{13}\,z = -13$

4

2015. 04. 27. Inf. an2. PZH2, α