2015. 03. 23. Int. an2. PZH1 MINTAMEGOLDÁS

1, [12] a, [10]

$$y'(x)=\frac{1}{2}(2y+x)^2+\frac{3}{2}\ ; \qquad u(x)=2y(x)+x \quad (3)$$

$$y'=\left(\frac{u-x}{2}\right)'=\frac{1}{2}u'-\frac{1}{2}$$

$$\frac{1}{2}u'-\frac{1}{2}=\frac{1}{2}u^2+\frac{3}{2}\ ; \qquad u'=\frac{du}{dx}=u^2+4 \quad (1)$$

$$\int\frac{du}{u^2+4}=\int dx \quad (1)\ ; \qquad \int dx=x+c \quad (1)$$

$$\int\frac{du}{u^2+4}=\frac{1}{4}\int\frac{du}{1+\left(\frac{u}{2}\right)^2}=\frac{1}{4}\operatorname{arc\,tg}\left(\frac{u}{2}\right)\cdot 2+c=\frac{1}{2}\operatorname{arc\,tg}\left(\frac{u}{2}\right)+c \quad (3)$$

$$\frac{1}{2}\operatorname{arc\,tg}(\underbrace{y+\frac{x}{2}}_{u})=x+c \implies \underline{\underline{y_{ált}(x)=\operatorname{tg}(2(x+c))-\frac{x}{2}}} \quad (1)$$

b, [2]

$$1=\operatorname{tg}(2c)-0 \implies 2c=\frac{\pi}{4}\ ;\ c=\frac{\pi}{8}\ ;\ \underline{\underline{y_{kezd}(x)=\operatorname{tg}\left(2x+\frac{\pi}{4}\right)-\frac{x}{2}}} \quad (2)$$

2, [10]

$$y'+2(x+1)y=e^{-x^2}$$

(H): $y'=-2(x+1)y \qquad \int\frac{dy}{y}=-2\int(x+1)\,dx$

$$\ln|y|=-x^2-2x+c\ ;\qquad \underline{y_{H,ált}(x)=K\,e^{-x^2-2x}\ ;\ K\in\mathbb{R}} \quad (5)$$

$$y_{I,p}(x)=K(x)\,e^{-x^2-2x} \ (1)\ ; \qquad y_{I,p}'=K'(x)\,e^{-x^2-2x}+K(x)(-2x-2)e^{-x^2-2x} \ (1)$$

Beírva:

$$\left(K'(x)\cancel{-2(x+1)K(x)}\right)e^{-x^2-2x}+\cancel{2(x+1)K(x)\,e^{-x^2-2x}}=e^{-x^2}$$

$$K'(x)=e^{2x} \ (1)\ ;\qquad K(x)=\int e^{2x}dx=\frac{1}{2}e^{2x}\ ;\qquad \underline{y_{I,p}(x)=\frac{1}{2}e^{-x^2}} \quad (1)$$

$$y_{I,ált}(X)=y_{H,ált}(X)+y_{I,p}(x)=\underline{\underline{K\,e^{-x^2-2x}+\frac{1}{2}e^{-x^2}}}\ ;\ K\in\mathbb{R}. \quad (1)$$

3, [8]

$f(n+1) = 3f(n) + 10f(n-1)$

$q^{n+1} = 3q^n + 10q^{n-1} \Rightarrow q^2 - 3q - 10 = (q-5)(q+2) = 0$

$q_1 = 5;\ q_2 = -2;$ ② $f_{ált}(n) = A\,5^n + B(-2)^n$ ②

$f(0) = 8 \Rightarrow A + B = 7$
$f(1) = -1 \Rightarrow 5A - 2B = 14$ ② $A = 4$, $B = 3$; $f(n) = 4\cdot 5^n + 3\cdot(-2)^n$ ②

4, [10] $y'' - 2y' + 5y = 3\sin(2x)$

(H) $\lambda^2 - 2\lambda + 5 = 0$; $\lambda_{1,2} = 1 \pm \sqrt{1-5} = 1 \pm 2i$ ②

$y_{H,ált}(x) = C_1 e^x \sin(2x) + C_2 e^x \cos(2x)$ ② $C_1, C_2 \in \mathbb{R}$ Nincs rezonancia!

$y_{I,p}(x) = A\sin(2x) + B\cos(2x)$ ② $/\cdot 5$

$y'_{I,p}(x) = 2A\cos(2x) - 2B\sin(2x)$ $/\cdot(-2)$

$y''_{I,p}(x) = -4A\sin(2x) - 4B\cos(2x)$ $/\cdot 1$

⊕

$3\sin(2x) = \sin(2x)\underbrace{(5A + 4B - 4A)}_{A+4B} + \cos(2x)\underbrace{(5B - 4A - 4B)}_{B-4A}$

$A + 4B = 3$ $/\cdot 4$ ②
$-4A + B = 0$

⊕

$0 + 17B = 12 \Rightarrow B = \frac{12}{17}$; $A = 3 - \frac{4\cdot 12}{17} = \frac{51-48}{17} = \frac{3}{17}$

$y_{I,p}(x) = \frac{3}{17}\sin(2x) + \frac{12}{17}\cos(2x)$ ①

$y_{I,ált}(x) = C_1 e^x \sin(2x) + C_2 e^x \cos(2x) + \frac{3}{17}\sin(2x) + \frac{12}{17}\cos(2x)$ ①

5, a, $\boxed{5}$
$$\sum_{n=1}^{\infty} \frac{3^{n+1} + (-2)^{2n+3}}{5^n} = 3\sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^n - 8\sum_{n=1}^{\infty}\left(\frac{4}{5}\right)^n = \quad (3)$$

Geometrické řady

$$= 3\cdot\frac{3/5}{1-3/5} - 8\,\frac{4/5}{1-4/5} = \frac{9}{2} - 32 = -\frac{55}{2} \quad (2)$$

b, $\boxed{5}$
$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \quad (2)$$

$$S_n = \sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+1}\right) = \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \dots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right) = 1 - \frac{1}{n+1} \quad (2)$$

Teleskopický!

$$S = \lim_{n\to\infty} S_n = \underline{\underline{1}} \quad (1)$$