1 EA

Maxwell-egyenletek

$$\text{rot}\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\text{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\text{div}\,\bar{B} = \emptyset$$

$$\text{div}\,\bar{D} = \rho$$

Descartes-ben

$$\text{rot}\,\bar{v} = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

$$\text{div}\,\bar{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\text{grad}\,\varphi = \frac{\partial \varphi}{\partial x}\bar{e}_x + \frac{\partial \varphi}{\partial y}\bar{e}_y + \frac{\partial \varphi}{\partial z}\bar{e}_z$$

$\bar{F} = Q(\bar{E} + \bar{v} \times \bar{B})$ Lorentz-tv.

a kapcs. a gerj. mennyiségek és a teljes. között

rotáció

$$\lim_{\Delta A \to 0} \frac{\oint \bar{v}\, d\bar{l}}{\Delta A} = \text{rot}\,\bar{v}$$

· Stokes-t.

$$\int_A \text{rot}\,\bar{v}\, d\bar{A} = \oint_l \bar{v}\, d\bar{l}$$

divergencia

$$\lim_{\Delta V \to 0} \frac{\oint_A \bar{v}\, d\bar{A}}{\Delta V} = \text{div}\,\bar{v}$$

Gauss-t.

$$\int_V \text{div}\,\bar{v}\, dV = \oint_A \bar{v}\, d\bar{A}$$

Anyagjellemzők

$\bar{B} = \mu \bar{H}$, $\bar{D} = \varepsilon \bar{E}$

$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$

$w = \frac{1}{2} \bar{E}\bar{D} + \frac{1}{2} \bar{H}\bar{B}$

Anyagjellemzők értelmezése

$\bar{B} = \mu_0 \bar{H} + \mu_0 \bar{M}$

$\bar{M}(\bar{H}) = (\mu_r - 1)\bar{H} \Rightarrow \bar{B} = \mu_0 \mu_r \bar{H}$

$\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$

Mikroszkópikus M. E

$\text{rot}\, \bar{B} = \mu_0 \bar{J} + \mu_0 \varepsilon_0 \frac{\partial \bar{E}}{\partial t}$ H, D helyett B, E

$\text{rot}\, \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$\text{div}\, \bar{B} = 0$

$\text{div}\, \bar{E} = \frac{\rho}{\varepsilon_0}$

$D = \mu_0 \bar{E} + \bar{P}$

$B = \mu_0 \bar{H} + \mu_0 \bar{M}$

$$\text{rot}\,\bar{H} = \bar{J} + \varepsilon_0 \frac{\partial \bar{E}}{\partial t} + \frac{\partial \bar{P}}{\partial t}$$

$$\text{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\text{div}\,\bar{B} = 0$$

$$\text{div}\,(\varepsilon_0 \bar{E} + \bar{P}) = \varrho$$

$$\bar{H} = \frac{\bar{B}}{\mu_0} - \bar{M}$$

$$\text{rot}\,\frac{\bar{B}}{\mu_0} - \text{rot}\,\bar{M} = \bar{J} + \varepsilon_0 \frac{\partial \bar{E}}{\partial t} + \frac{\partial \bar{P}}{\partial t}$$

$$\text{rot}\,\bar{B} = \mu_0 \bar{J} + \boxed{\mu_0\, \text{rot}\,\bar{M} + \mu_0 \frac{\partial \bar{P}}{\partial t}} + \mu_0 \varepsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$\text{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\text{div}\,\bar{B} = \emptyset$$

$$\text{div}\,\bar{E} = \frac{\varrho}{\varepsilon_0} + \frac{\text{div}\,\bar{P}}{\varepsilon_0}$$

2. EA | Anyagjellemzők

$$\bar{B} = \mu_0 \bar{H} + \mu_0 \bar{M}$$

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$$

Anyagok jellemzése:

1) • Izotrop

$\bar{D} = \varepsilon \bar{E}$; $\bar{B} = \mu \bar{H}$

• Anizotrop

$\bar{D} = \underline{\underline{\varepsilon}} \bar{E}$; $\bar{B} = \underline{\underline{\mu}} \bar{H}$ $\underline{\underline{\varepsilon}}, \underline{\underline{\mu}}$ mátrixok.

• Bian izotrop

$\bar{D} = \underline{\underline{\varepsilon}} \bar{E} + \underline{\underline{\xi}} \bar{H}$ elektromos eltolás nem csak a vill. térerősségtől függenek.

$\bar{B} = \underline{\underline{\mu}} \bar{H} + \underline{\underline{\zeta}} \bar{E}$

2) • Lineáris

$\bar{D}_1 = F_D(\bar{E}_1)$

$\bar{D}_2 = F_D(\bar{E}_2)$ $\Rightarrow F_D(c_1 \bar{E}_1 + c_2 \bar{E}_2) = c_1 \bar{D}_1 + c_2 \bar{D}_2$

• Nem lineáris

Ami nem lineáris.

2, Időbeli diszperzió

$$\bar P(\bar r,t)=\varepsilon_0\int_{-\infty}^{t}\underline{\underline{R}}(\bar r,t-\tau)\,\bar E(\bar r,\tau)\,d\tau \Rightarrow \hat{P}(\bar r,\omega)=\underbrace{\varepsilon_0\,\tilde{\underline{\underline{R}}}(\bar r,\omega)}_{\underline{\underline{\varepsilon}}(\bar r,\omega)}\,\hat{\bar E}(\bar r,\omega)$$

Lin. anyag esetén

$$\mathcal{F}\{P(\bar r,t)\}=\hat{P}(\bar r,\omega)$$

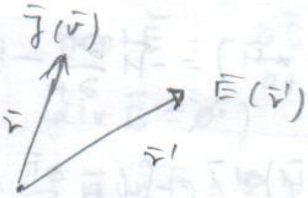
$$\mathcal{F}\{\underline{\underline{R}}(\bar r,t)\}=\tilde{\underline{\underline{R}}}(\bar r,\omega)$$

$$\mathcal{F}\{\bar E(\bar r,t)\}=\hat{\bar E}(\bar r,\omega)$$

tehát $\underline{\underline{\varepsilon}}$ frekvencia függő → minden esetben

3, Térbeli diszperzió

$$\bar J(\bar r)=\int_V \underline{\underline{K}}(\bar r,\bar r')\cdot\bar E(\bar r)\,dV'$$

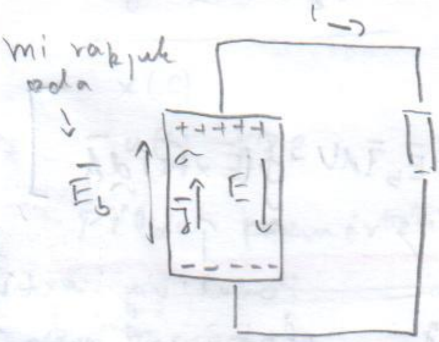


Differenciális Ohm-tv.

$$\bar J=\sigma\left(\bar E+\bar E_b\right)+\bar J_b$$

E_b értelmezése

$$\bar J=\sigma(\bar E+\bar E_b)$$



Energia

$$\mathrm{rot}\,\bar H = \bar J + \frac{\partial \bar D}{\partial t} \;\Big|\; \bar E \qquad \bar H\,\mathrm{rot}\,\bar E - \bar E\,\mathrm{rot}\,\bar H = -\bar H\frac{\partial \bar B}{\partial t} - \bar E\frac{\partial \bar D}{\partial t} - \bar E\bar J$$

$$\mathrm{rot}\,\bar E = -\frac{\partial \bar B}{\partial t} \;\Big|\; \bar H \qquad \left[\mathrm{div}(\bar E\times\bar H) = \bar H\,\mathrm{rot}\,\bar E - \bar E\,\mathrm{rot}\,\bar H\right]$$

$$\mathrm{div}\,\bar B = 0$$

$$\mathrm{div}\,\bar D = \rho$$

$$\bar D = \varepsilon\bar E,\quad \bar B = \mu\bar H \qquad \Rightarrow \bar E = \frac{\bar J}{\sigma} - \frac{\bar J_b}{\sigma} - \bar E_b$$

$$\bar J = \sigma(\bar E + \bar E_b) + \bar J_b$$

$$\mathrm{div}(\bar E\times\bar H) = -\bar H\frac{\partial \bar B}{\partial t} - \bar E\frac{\partial \bar D}{\partial t} - \frac{\bar J^2}{\sigma} + \frac{\bar J_b\bar J}{\sigma} + \bar E_b\bar J \;\Big|\; \int_V$$

V-t körül-vevő

$$\oint_A(\bar E\times\bar H)\,d\bar A = -\int_V\left(\bar H\frac{\partial \bar B}{\partial t} + \bar E\frac{\partial \bar D}{\partial t}\right)dV - \int_V\frac{\bar J^2}{\sigma}dV + \int_V\frac{\bar J_b\bar J}{\sigma}dV + \int_V\bar E_b\bar J\,dV$$

$$\underbrace{-\int_V\left(\bar H\frac{\partial \bar B}{\partial t} + \bar E\frac{\partial \bar D}{\partial t}\right)dV}_{\text{telj sűrűség}} = \int\frac{\bar J^2}{\sigma}dV - \int_V\frac{\bar J_b\bar J}{\sigma}dV - \int_V\bar E_b\bar J\,dV + \oint_A\bar E\times\bar H\,dA$$

$$\bar H\frac{\partial \bar B}{\partial t} = \mu\bar H\frac{\partial \bar H}{\partial t} = \frac{\partial}{\partial t}\frac{1}{2}\mu H^2 = \frac{\partial}{\partial t}\overbrace{\frac{1}{2}\bar H\bar B}^{w_m} \qquad w_e = \frac{1}{2}\bar E\bar D$$

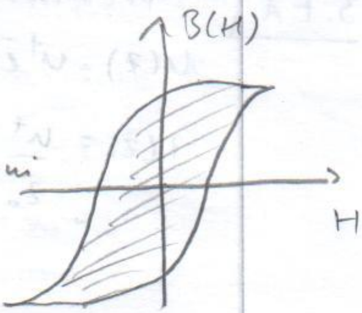
$$\left[-\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\bar E\bar D + \frac{1}{2}\bar H\bar B\right)dV = \int_V\frac{\bar J^2}{\sigma}dV - \int_V\frac{\bar J_b\bar J}{\sigma}dV - \int_V\bar E_b\bar J\,dV + \oint_A(\bar E\times\bar H)\,d\bar A\right]$$

csak pozitív Joule-hő

beiktatott áramsűrűség teljesítménye

Pointing-vector elsugárzott energ.

$$\bar{H}\frac{\partial \bar{B}}{\partial t} = \frac{\partial}{\partial t}\int_0^B \bar{H}\,d\bar{B} = \frac{\partial w_m}{\partial t} \Rightarrow \Delta w_m = \int_{B_1}^{B_2} \bar{H}\,d\bar{B}$$

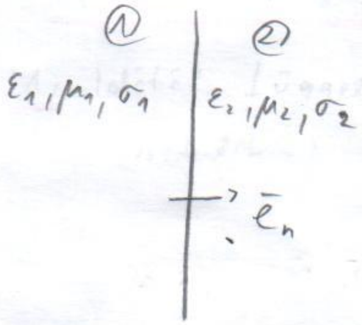


$$\bar{E}\frac{\partial \bar{D}}{\partial t} = \frac{\partial}{\partial t}\int_0^D \bar{E}\,d\bar{D} = \frac{\partial w_e}{\partial t}$$

Megtudjuk határozni a telj sűrűség változást

$$w = \frac{1}{2}\bar{E}\bar{D} + \frac{1}{2}\bar{H}\bar{B}$$

Térjellemzők viselkedése közegek határán



$$\bar{e}_n \times (\bar{H}_2 - \bar{H}_1) = \bar{K} \qquad \left(\text{rot}\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}\right)$$

$$\bar{e}_n \times (\bar{E}_2 - \bar{E}_1) = \emptyset \qquad \left(\text{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}\right)$$

$$\bar{e}_n \cdot (\bar{B}_2 - \bar{B}_1) = \emptyset \qquad (\text{div}\,\bar{B} = \emptyset)$$

$$\bar{e}_n (\bar{D}_2 - \bar{D}_1) = \sigma \qquad (\text{div}\,\bar{D} = \rho)$$

Példa kezdetiérték-problémára

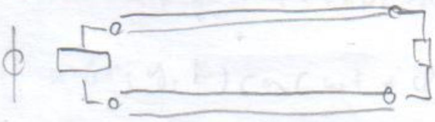
$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{B}u$$

$$y = \underline{C}^T \underline{x} + Du$$

$x(0)$

$u(t) \quad t > 0$

Példa peremérték problémára

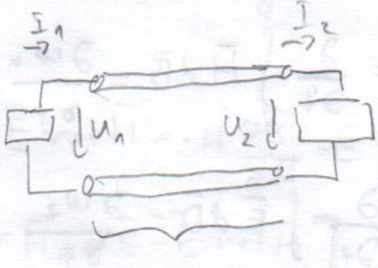


$$\frac{\partial^2 u}{\partial z^2} + \gamma^2 u = 0$$

3. EA

$$u(z) = u^+ e^{-\gamma z} + u^- e^{\gamma z}$$

$$I(z) = \frac{u^+}{Z_0} e^{-\gamma z} - \frac{u^-}{Z_0} e^{\gamma z}$$



• tart. belüli diff: $\frac{d^2 u}{dz^2} - \gamma^2 u = 0$

$$-\frac{du}{dz} = (R' + j\omega L') I$$

Peremérték feladat:

A tartományon belül érvényes diff. egyenletek és források + A tart. határon kívüli összessége

Példa kezdeti ért. feladatra:

$$\underline{x}' = \underline{\underline{A}}\,\underline{x} + \underline{B} u$$

$$y = \underline{C}^T x + D u$$

$x(t_0)$ -	$\underline{x}(-0)$
$u(t)$ $t>0$	$u(t)$ $t \geq 0$

Időbeli változás kezelése: speciális időfüggések

$$\operatorname{rot} \bar{H}(\bar{r}, t) = \bar{J}(\bar{r}, t) + \frac{\partial \bar{D}(\bar{r}, t)}{\partial t}$$

$$\operatorname{rot} \bar{E}(\bar{r}, t) = -\frac{\partial \bar{B}(\bar{r}, t)}{\partial t}$$

$$\operatorname{div} \bar{B}(\bar{r}, t) = 0$$

$$\operatorname{div} \bar{D}(\bar{r}, t) = \rho(\bar{r}, t)$$

$$\bar{B}(\bar{r}, t) = \mu(\bar{r}) \bar{H}(\bar{r}, t)$$

$$\bar{D}(\bar{r}, t) = \varepsilon(\bar{r}) \bar{E}(\bar{r}, t)$$

$$\bar{J}(\bar{r}, t) = \sigma(\bar{r}) \left(\bar{E}(\bar{r}, t) + \bar{E}_b(\bar{r}, t)\right) + \bar{J}_b(\bar{r}, t)$$

Lineáris esetben elképzelhető, hogy idő és tér koordináták

szétválaszthatóak:

$$E(x,y,z,t) = E_x(x,y,z) f_x(t) \bar{e}_x +$$
$$E_y(x,y,z) f_y(t) \bar{e}_y +$$
$$E_z(x,y,z) f_z(t) \bar{e}_z$$

$\bar{E}(\bar{r},t) = \bar{E}(\bar{r}) f(t)$ kiterjeszthető

Speciális időfüggések:

1, Időtől független: $\frac{\partial}{\partial t} = \emptyset$

sztatikus: $\mathrm{rot}\, \bar{H} = \bar{J}$

$\mathrm{rot}\, \bar{E} = \emptyset$ Ezt így nem

$\mathrm{div}\, \bar{B} = \emptyset$ használjuk

$\mathrm{div}\, \bar{D} = \varrho$

2, Szinuszos állandósult állapot.

stabilitás – anyag
– külső források

$$\bar{E}(x,y,z,t) = E_x(x,y,z) \cos(\omega t + \varphi_x) \bar{e}_x +$$
$$E_y(x,y,z) \cos(\omega t + \varphi_y) \bar{e}_y +$$
$$E_z(x,y,z) \cos(\omega t + \varphi_z) \bar{e}_z$$

$\bar{H}, \bar{B}, \bar{D}, \varrho, \bar{J}$ $\quad \omega = \text{állandó}$

Komplex térerősség

$$E_x(x,y,z) \cos(\omega t + \varphi_x) = \mathrm{Re}\{\underbrace{\tilde{E}_x(x,y,z)}_{E_x(x,y,z)e^{j\varphi_x}} e^{j\omega t}\}$$ $\boxed{\text{Ezt az alakot használjuk.}}$

csak hely függő

$$\tilde{\bar{E}}(x,y,z) = \tilde{E}_x(x,y,z)\bar{e}_x + \tilde{E}_y(x,y,z)\bar{e}_y + \tilde{E}_z(x,y,z)\bar{e}_z$$

$t = t_0$

$$\bar{E}(x,y,z,t_0) = \mathrm{Re}\{\tilde{\bar{E}}(x,y,z) e^{j\omega t_0}\}$$

A térerősség vektorok végpontja időben egy ellipszisen mozog.

$\boxed{\hat{\bar{E}} \text{ helyett } \bar{E}}$

$\text{rot}\, \bar{H}(\bar{r}) = \bar{J}(\bar{r}) + j\omega \bar{D}(\bar{r})$ időbeli deriválás szinuszos esetben

$\text{rot}\, \bar{E}(\bar{r}) = -j\omega \bar{B}(\bar{r})$

$\text{div}\, \bar{B}(\bar{r}) = 0$

$\text{div}\, \bar{D}(\bar{r}) = \rho(\bar{r})$

$p(t) = u(t)\, i(t) \quad u(t) \leftrightarrow \hat{U}$

$P = \frac{1}{T}\int_0^T p(t)\, dt \quad i(t) \leftrightarrow \hat{I}$

$P = \text{Re}\{\frac{1}{2}\hat{U}\hat{I}^*\}$

$\frac{1}{T}\int_0^T \hat{U} \cos(\omega t + \varphi_u)\, \hat{I} \cos(\omega t + \varphi_i)\, dt = \frac{1}{2}\hat{U}\hat{I}\cos(\varphi_u - \varphi_i)$

Maxwell egyenletek szinuszos gerj esetén

$$\text{rot}\,\bar{H}(\bar{r}) = \bar{J}(\bar{r}) + j\omega \bar{D}(\bar{r})$$

$$\text{rot}\,\bar{E}(\bar{r}) = -j\omega \bar{B}(r)$$

$$\text{div}\,\bar{B}(\bar{r}) = 0$$

$$\text{div}\,\bar{D}(\bar{r}) = \rho(\bar{r})$$

$$\bar{B} = \mu(\bar{r})\,\bar{H}(\bar{r})$$

$$\bar{D} = \varepsilon(\bar{r})\,\bar{E}(r)$$

$$\bar{J} = \sigma\left(\bar{E}(\bar{r}) + \bar{E}_b(\bar{r})\right) + \bar{J}_b(\bar{r})$$

$$\bar{E}(\bar{r},t) = \text{Re}\left\{ \bar{E}(r)e^{j\omega t} \right\}$$

időfv. komplex csúcsérték

$\bar{H}$

Állandósult állapot szinuszos gerj esetén:
- A kimenet is periodikus

rossz vezetőkben ($\sigma \ll 1\,\frac{S}{m}$)

$$\text{rot}\,\bar{H} = \sigma\bar{E} + j\omega\varepsilon\bar{E} \qquad (\bar{J}_b = 0,\ \bar{E}_b = 0)$$ nem nagyon vannak áramok

$$\text{rot}\,\bar{H} = (\sigma + j\omega\varepsilon)\bar{E} = j\omega\underbrace{\left(\varepsilon - j\frac{\sigma}{\omega}\right)}_{\hat{\varepsilon}}\bar{E}$$

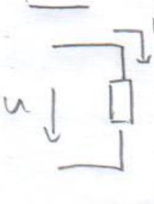
$\hat{\varepsilon}$ - komplex permittivitás

$\hat{\varepsilon} = \varepsilon' - j\varepsilon''$ $\quad \text{tg}\,\delta = \frac{\varepsilon''}{\varepsilon'}$ anyag veszteség

- ε, μ, σ frekvencia függése

Teljesítmények szinuszos gerj. esetén

Bev. $u(t) = U\cos(\omega t + \varphi)$ $\quad P + jQ = \frac{1}{2}UI^* = \frac{1}{2}Ue^{j\varphi}\frac{1}{2}e^{-j\rho} = \frac{1}{2}UIe^{j(\varphi-\rho)}$



$i(t) = I\cos(\omega t + \rho)$

$P(t) = u(t)\cdot i(t)$

$$P = \frac{1}{T}\int_0^T u(t)\,i(t)\,dt = \frac{1}{2}U\cdot I\cos(\varphi - \rho)$$

$$\frac{1}{T}\int_0^T \frac{1}{2}\bar{E}(\bar{r},t)\dot{D}(\bar{r},t)\,dt = \mathrm{Re}\left\{\frac{1}{2}\cdot\frac{1}{2}\bar{E}(\bar{r})\bar{D}^*(\bar{r})\right\}$$

Szinuszos EM terek energia vizsgálata

$$① \quad \bar{E}\,\mathrm{rot}\,\bar{H}^* = \bar{E}\bar{J}^* - j\omega\bar{E}\bar{D}^* \qquad \mathrm{rot}\,H^* = \bar{J}^* - j\omega\bar{D}^* \;\Big|\; \bar{E}\cdot$$

$$② \quad \bar{H}^*\,\mathrm{rot}\,\bar{E} = -j\omega\bar{H}^*\cdot\bar{B}$$

② − ①

$$\bar{H}^*\,\mathrm{rot}\,\bar{E} - \bar{E}\,\mathrm{rot}\,\bar{H}^* = j\omega\bar{H}^*\bar{B} - \bar{E}\bar{J}^* + j\omega\bar{E}\bar{D}^* \quad \Big|\quad \bar{J} = \sigma(\bar{E}+E_b) + \bar{J}_b$$

$$\mathrm{div}(\bar{E}\times\bar{H}^*) = -j\omega H^*\bar{B} - \frac{|\bar{J}|^2}{\sigma} + \frac{\bar{J}_b\bar{J}^*}{\sigma} + \bar{E}_b\bar{J}^* + j\omega\bar{E}\bar{D}^* \quad \Big|\quad \int_V dV$$

$$\boxed{j\omega\frac{1}{2}\int_V H^*\bar{B} - \bar{E}\bar{D}^*\,dV = \int_V \frac{1}{2}\frac{J_b\bar{J}^*}{\sigma}dV + \int_V \frac{1}{2}\bar{E}_b\bar{J}^*dV - \int \frac{1}{2}\frac{|\bar{J}|^2}{\sigma}dV + \oint_A \frac{1}{2}(\bar{E}\times\bar{H}^*)\,dA}$$

lin. anyag esetén: $\mu|H|^2 \quad \varepsilon|\bar{E}|^2$

$$\emptyset = \int_V \mathrm{Re}\left\{\frac{1}{2}\frac{\bar{J}_b\bar{J}^*}{\sigma}\right\}dV + \int_V \mathrm{Re}\left\{\frac{1}{2}\bar{E}_b\bar{J}^*\right\}dV - \int_V \frac{1}{2}\frac{|\bar{J}|^2}{\sigma}dV + \oint_A \mathrm{Re}\left\{\frac{1}{2}(\bar{E}\times\bar{H}^*)\right\}dA$$

kívülről beiktatott terekből származó hatásos telj.

eldisz. telj.

elsugárzott telj.

→ Ha ez nem lenne igaz, nem lenne állandósult állapot

3, Periódikus gerjesztés, lineáris stabilis közegben

- T periódusidő, $\omega_0 = \frac{2\pi}{T}$ => Fourier sor -> szuperpozíció
- Minden frekvenciára ($\omega_0, 2\omega_0, 3\omega_0 ..$) külön-külön szinusz állandósult állapotbeli analízis (komplex csúcsért. felhasználásával)
- Eredmény: az egyes frekvenciákra számolt terek összege

4, Nem periodikus, lineáris és stabilis anyagjellemzők. (tipikusan valamilyen hosszúságú impulzusok) (nem stabilis: $\bar{J}_b$ függ $\bar{E}$-től)

$$\int_{-\infty}^{\infty} |\bar{E}(\bar{r},t)|^2 dt < \infty \qquad \bar{E}(\bar{r}, j\omega) = \int_{-\infty}^{\infty} \bar{E}(\bar{r},t)\, e^{-j\omega t} dt = \mathcal{F}\{\bar{E}(\bar{r},t)\}$$

$$\mathcal{F}\left\{\frac{\partial \bar{E}(\bar{r},t)}{\partial t}\right\} = j\omega\, \mathcal{F}\{\bar{E}(\bar{r},t)\} = j\omega \bar{E}(\bar{r}, j\omega)$$

$$\mathrm{rot}\, \bar{H}(\bar{r}, j\omega) = \bar{J}(\bar{r}, j\omega) + j\omega \bar{D}(\bar{r}, j\omega)$$

$$\mathrm{rot}\, \bar{E}(\bar{r}, j\omega) = -j\omega \bar{B}(\bar{r}, j\omega)$$

$$\mathrm{div}\, \bar{B}(\bar{r}, j\omega) = 0$$

$$\mathrm{div}\, \bar{D}(\bar{r}, j\omega) = \rho(\bar{r}, j\omega)$$

$$\bar{E}(\bar{r},t) = \mathcal{F}^{-1}\{\bar{E}(r, j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{E}(\bar{r}, j\omega)\, e^{j\omega t} d\omega$$

$\bar{D} = \varepsilon \bar{E}$ szinuszos állandósult áll. $D(\bar{r}) = \varepsilon(\bar{r}) E(\bar{r})$ ω adott

$$\bar{D}(\bar{r}, j\omega) = \varepsilon(\bar{r}, j\omega)\, E(\bar{r}, j\omega)$$

Időbeli diszperzió:
Az anyag kül. sebességű vált. másképp reagál.

$$\bar{D}(\bar{r}, t) = \varepsilon(\bar{r}, t) * \bar{E}(\bar{r}, t)$$

$$\int_{-\infty}^{\infty} \varepsilon(\bar{r}, t-\tau)\, E(\bar{r}, \tau)\, d\tau$$

5, Belépő gerjesztés, nem feltétlenül "stabilis" anyag, ellemzők

$$\bar{E}(\bar{r},s) = \mathcal{L}\{\bar{E}(\bar{r},t)\} = \int_{-\infty}^{\infty} \bar{E}(\bar{r},t)\, e^{-st}\, dt$$

$$\operatorname{rot} \bar{H}(\bar{r},s) = \bar{J}(\bar{r},s) + s\,\bar{D}(\bar{r},s)$$

$$\operatorname{rot} \bar{E}(\bar{r},s) = -s\,\bar{B}(\bar{r},s)$$

$$\operatorname{div} \bar{B}(\bar{r},s) = \emptyset$$

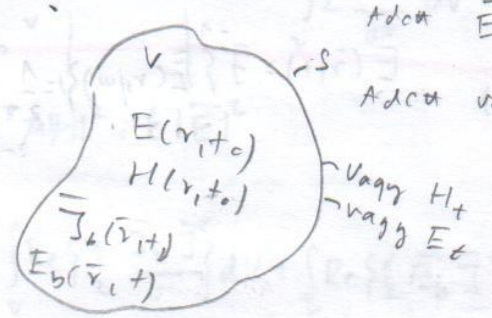
$$\operatorname{div} \bar{D}(\bar{r},s) = \rho(\bar{r},s)$$

Maxwell egyenletek egyértelmű megoldhatósága (lin anyagok esetén)

Adott $\bar{E}(\bar{r},t_0)$, $\bar{H}(\bar{r},t_0)$ $\quad \bar{r} \in V$

Adott $\bar{E}_b(\bar{r},t)$, $\bar{J}_b(\bar{r},t)$ $\quad \bar{r} \in V \quad t \geq t_0$

Adott vagy $\bar{E}_t(\bar{r},t)$ vagy $\bar{H}_t(\bar{r},t)$ $\quad \bar{r} \in S \quad t \geq t_0$



5.EA

Biz. indirekt bizonyítás

[ ] : adott
[ ]' : egyik megoldás
[ ]'' : másik mo.

$$\mathrm{rot}\,\bar{H}' = \bar{J}' + \frac{\partial \bar{D}'}{\partial t}$$
$$\mathrm{rot}\,\bar{E}' = -\frac{\partial \bar{B}'}{\partial t}$$
$$\mathrm{div}\,\bar{B}' = \emptyset$$
$$\mathrm{div}\,\bar{D}' = \rho'$$
$$\bar{D}' = \varepsilon \bar{E}'$$
$$\bar{B}' = \mu \bar{H}'$$
$$\bar{J}' = \sigma(\bar{E}' + \bar{E}_b) + \bar{J}_b$$

$$\mathrm{rot}\,\bar{H}'' = \bar{J}'' + \frac{\partial \bar{D}''}{\partial t}$$
$$\mathrm{rot}\,\bar{E}'' = -\frac{\partial \bar{B}''}{\partial t}$$
$$\mathrm{div}\,\bar{B}'' = \emptyset$$
$$\mathrm{div}\,\bar{D}'' = \rho''$$
$$\bar{D}'' = \varepsilon \bar{E}''$$
$$\bar{B}'' = \mu \bar{H}''$$
$$\bar{J}'' = \sigma(\bar{E}'' + \bar{E}_b) + \bar{J}_b$$

$$\bar{H}_0 = \bar{H}'' - \bar{H}'$$
$$\bar{E}_0 = \bar{E}'' - \bar{E}' \longrightarrow \mathrm{rot}\,\bar{H}_0 = \bar{J}_0 + \frac{\partial \bar{D}_0}{\partial t}$$
. . . . .

$$\mathrm{rot}\,\bar{E}_0 = -\frac{\partial B_0}{\partial t}$$
$$\mathrm{div}\,\bar{B}_0 = \emptyset$$
$$\mathrm{div}\,\bar{D}_0 = \rho_0$$
$$\bar{D}_0 = \varepsilon \bar{E}_0$$
$$\bar{B}_0 = \mu \bar{H}_0$$
$$\bar{J}_0 = \sigma \bar{E}_0$$

$\bar{E}_0(\bar{r}, t_0) = \emptyset$ $\quad \bar{r} \in V$

$\bar{H}_0(\bar{r}, t) = 0$

vagy $\bar{H}_{t_0}(\bar{r}, t) = \emptyset$

vagy $\bar{E}_{t_0}(\bar{r}, t) = 0$ $\quad \bar{r} \in S$

$$\bar{E}_0\,\mathrm{rot}\,\bar{H}_0 = \bar{E}_0 \bar{J}_0 + \varepsilon \bar{E}_0 \frac{\partial \bar{E}_0}{\partial t}$$
$$\bar{H}_0\,\mathrm{rot}\,\bar{E}_0 = -\mu \bar{H}_0 \frac{\partial \bar{H}_0}{\partial t}$$

$$\underbrace{\bar{H}_0\,\mathrm{rot}\,\bar{E}_0 - \bar{E}_0\,\mathrm{rot}\,\bar{H}_0}_{\mathrm{div}(\bar{E}_0 \times \bar{H}_0)} = -\mu \bar{H}_0 \frac{\partial \bar{H}_0}{\partial t} - \frac{|\bar{J}_0|^2}{\sigma} - \varepsilon \bar{E}_0 \frac{\partial \bar{E}}{\partial t}$$

$$\frac{1}{2}\mu\frac{\partial|H_0|^2}{\partial t}+\frac{1}{2}\varepsilon\frac{\partial|E_0|^2}{\partial t}=-\frac{|\bar{J}_0|^2}{\sigma}-\operatorname{div}(\bar{E}_0\times\bar{H}_0)\quad\Big|\int_V$$

$$\frac{\partial}{\partial t}\underbrace{\int_V\frac{1}{2}\mu|\bar{H}_0|^2+\frac{1}{2}\varepsilon|\bar{E}_0|^2\,dV}_{t=t_0\text{-ban}=\emptyset}=-\underbrace{\int_V\frac{|\bar{J}_0|^2}{\sigma}dV}_{\geq\emptyset}-\underbrace{\oint_S(\bar{E}_0\times\bar{H}_0)\,dS}_{=\emptyset\text{ hanem vagy: }\bar{E}_{0,t}=\emptyset\text{ v. }\bar{H}_{0,t}=\emptyset}$$

$\bar{H}_0(\bar{r},t)=\emptyset \quad t\geq 0$

$\bar{E}(\bar{r},t)=\emptyset \quad t\geq 0$

Mi van ha $V\to\infty$ akkor $\oint_S(E_0\times H_0)\,dS$ bajba kerül

Források a végtelenben vannak

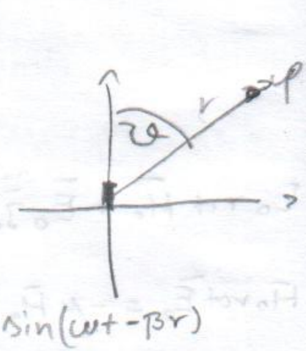
Statikus terek $\sim\frac{1}{r^2}$ csökken a térerősség $\Rightarrow$

Kvázistacionárius tér: vezetőn kívül térben $\sim\frac{1}{r}$ tér erősség

Hullámok: $\sim\frac{1}{r}\Rightarrow \bar{E}_0\times\bar{H}_0\sim\frac{1}{r^2}$

Sommerfeld-féle sugárzási feltétel

$\lim_{r\to\infty} r|E|\leq K \; : \; \lim_{r\to\infty} r|H|\leq K$



$$\lim_{r\to\infty} r\left[\sqrt{\frac{\varepsilon}{\mu}}\bar{E}+\bar{e}_n\times\bar{H}\right]=\emptyset$$

$$\lim_{r\to\infty} r\left[\bar{e}_n\times\sqrt{\frac{\varepsilon}{\mu}}\bar{E}-\bar{H}\right]=\emptyset$$

$E_\vartheta(\bar{r},t)=E_0\frac{1}{r}\sin\vartheta\sin(\omega t-\beta r)$

$H_\varphi(\bar{r},t)=\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0\frac{1}{r}\sin\vartheta\sin(\omega t-\beta r)$

vége

Az elektrodinamika különböző területeire különböző peremérték feladatokat fogalmazunk meg.

Az elektrod. ugyanazon területére is megfogalmazhatunk kül. peremérték feladatokat

Elektrodinamika egyes témaköreire vonatkozó peremérték feladatok

- Maxwell-egyenletek redukált alakja
- Potenciálokat vezetek be.
- Potenciálokra vonatkozó közös. diff. egyenlet
- Peremfeltételek
- végtelen térrészre vonatkozó megoldás megadása

$$\mathrm{rot}\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\mathrm{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\mathrm{div}\,\bar{B} = 0$$

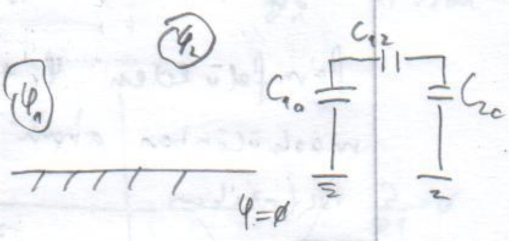
$$\mathrm{div}\,\bar{D} = \rho$$

$$\bar{B} = \mu \bar{H} \quad (\bar{B} = \mu_0 \bar{H} + \mu \bar{M})$$

$$\bar{D} = \varepsilon \bar{E} \quad (\bar{D} = \varepsilon_0 \bar{E} + \bar{P})$$

$$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$$

1, Elektrosztatika $\frac{\partial}{\partial t} = 0$ ρ forrás, E_b



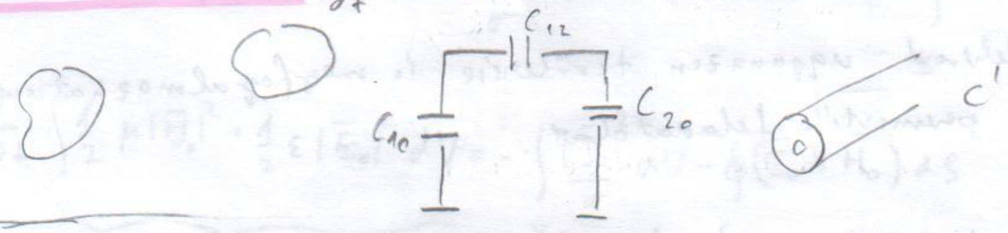
$$\mathrm{rot}\,\bar{E} = 0 \qquad \bar{E} = -\mathrm{grad}\,\varphi$$

$$\mathrm{div}\,\bar{D} = 0$$

$$\bar{D} = \varepsilon \bar{E} \quad (\bar{D} = \varepsilon_0 \bar{E} + \bar{P})$$

6.EA | Az elektrodinamika peremérték-feladatai (PEF)

1, Elektrosztatika $\frac{\partial}{\partial t} = \emptyset$ ϱ = forrás



$\mathrm{rot}\,\bar{E} = \emptyset$ $\bar{E} = -\mathrm{grad}\,\varphi$

$\mathrm{div}\,\bar{D} = \varrho$ $\boxed{-\mathrm{div}(\varepsilon\,\mathrm{grad}\,\varphi) = \varrho}$

$\bar{D} = \varepsilon\bar{E}$

$\mathrm{rot}\,\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t}$

$\mathrm{rot}\,\bar{E} = -\frac{\partial\bar{B}}{\partial t}$

$\mathrm{div}\,\bar{B} = \emptyset$

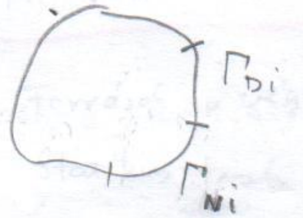
$\mathrm{div}\,\bar{D} = \varrho$

$\bar{B} = \mu\bar{H}$ $(\bar{B} = \mu_0\bar{H} + \mu_0\bar{M})$

$\bar{D} = \varepsilon\bar{E}$ $(\bar{D} = \varepsilon_0\bar{E} + \bar{P})$

$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$

$w = \frac{1}{2}\bar{E}\bar{D} + \frac{1}{2}\bar{H}\bar{B}$



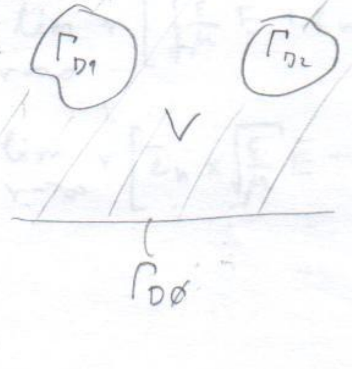
D: Dirichlet
N: Neumann

$\Gamma = \bigcup_i \Gamma_{Di} + \bigcup_i \Gamma_{Ni}$

$\varphi = \varphi_i$ adott Γ_{Di}

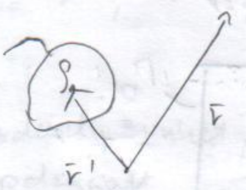
$\varepsilon\frac{\partial\varphi}{\partial n} = \sigma$ adott Γ_{Ni}

fémfelületen φ tangenciális komponense $\emptyset \rightarrow \varphi_{norm.\,komp} =$
máskülönben áram folyna $\rightarrow$ nem lenne sztatikus

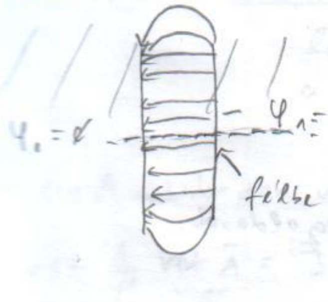


Szabad térben

$$\varphi(\bar r) = \frac{1}{4\pi\varepsilon}\int_V \frac{\rho(\bar r')}{|\bar r - \bar r'|}\,dV'$$



pl. Γ_N szimetria figyelembevételére



$$\Gamma_N = \varepsilon \frac{\partial \varphi}{\partial n} = \emptyset$$

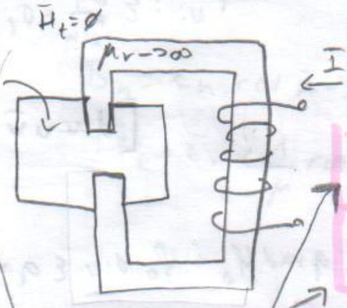
2, Sztatikus mágneses tér $\frac{\partial}{\partial t} = \emptyset$; $\bar J = \emptyset$

$\mathrm{rot}\,\bar H = \emptyset$

$\mathrm{div}\,B = \emptyset$

$B = \mu_0 \bar H + \mu_0 \bar M$

$\bar B = \mu \bar H$



$\bar H = -\mathrm{grad}\,\varphi_m$

$\mathrm{div}(-\mu_0\, \mathrm{grad}\,\varphi_m + \mu_0 \bar M) = \emptyset$

$$\boxed{-\mathrm{div}(\mu_0\,\mathrm{grad}\,\varphi_m) = -\mathrm{div}\,\mu_0 \bar M}$$

$$\boxed{-\mathrm{div}(\mu\,\mathrm{grad}\,\varphi_m) = 0}$$

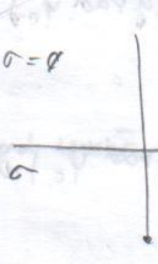
Γ_{Di} $\varphi_M = \varphi_{mi}$ adott Γ_{Ni} : $\mu \frac{\partial \varphi_m}{\partial n} = \sigma_{mi}$ adott

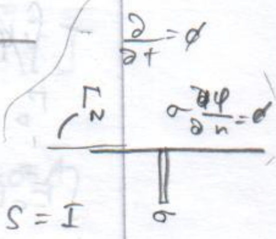
3, Stacionárius áramlási tér $\frac{\partial}{\partial t} = \emptyset$ $\quad \sigma = \emptyset$

$\mathrm{div}\,\bar J = \emptyset$ $\qquad \bar E = -\mathrm{grad}\,\varphi$

$\mathrm{rot}\,\bar E = \emptyset$ $\qquad \mathrm{div}(\sigma(-\mathrm{grad}\,\varphi + \bar E_b) + \bar J_b) = \emptyset$

$\bar J = \sigma(\bar E + \bar E_b) + \bar J_b$





$$\boxed{-\mathrm{div}\,\sigma\,\mathrm{grad}\,\varphi = -\mathrm{div}\,\sigma \bar E_b - \mathrm{div}\,\bar J_b}$$

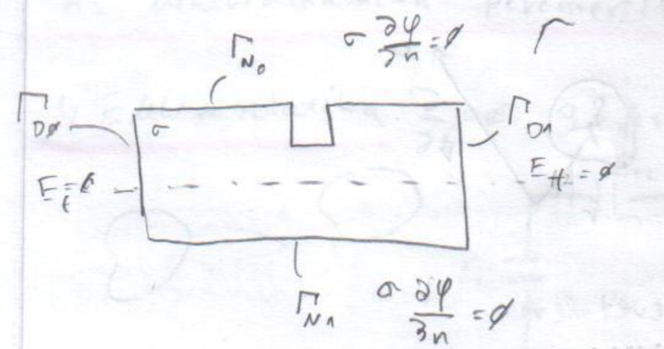
$$\int_S \sigma \frac{\partial \varphi}{\partial n}\,dS = I$$

Γ_{Di} $\varphi = \varphi_i$ adott $\Rightarrow E_t$ adott

$$\sigma \frac{\partial \varphi}{\partial n}\Big|_S = \frac{I}{4\pi r^2}$$

Γ_{ui} $\sigma \frac{\partial \varphi}{\partial n} = f$ (adott) $\Rightarrow J_n$ adott

ρ



Skaláris Poisson egyenletre vezető PEF

pl: $-\mathrm{div}(\varepsilon \,\mathrm{grad}\, \varphi) = \rho$

+fh. φ' és φ'' megoldások

$\varphi_0 = \varphi' - \varphi''$

Γ V $-\mathrm{div}(\varepsilon \,\mathrm{grad}\, \varphi_0) = 0$

$\Gamma_{Di}: \varphi = \varphi_i$ adott $\Rightarrow \Gamma_{Di}\ \varphi_0 = 0$

$\Gamma_{Ni}: \varepsilon \frac{\partial \varphi}{\partial n} = \sigma_i$ adott $\Rightarrow \Gamma_{Ni}\ \varepsilon \frac{\partial \varphi_0}{\partial n} = 0$

$[\mathrm{div}\, u\bar{v} = \bar{v}\,\mathrm{grad}\, u + u\,\mathrm{div}\,\bar{v}]$

$\mathrm{div}\, \varphi_0 \varepsilon \,\mathrm{grad}\, \varphi_0 = \varepsilon \,\mathrm{grad}\, \varphi_0 \,\mathrm{grad}\, \varphi_0 + \varphi_0 \,\mathrm{div}\, \varepsilon \,\mathrm{grad}\, \varphi_0$

$\int_V \bar{E}_0 \bar{D}_0 dV = \int_V \mathrm{grad}\, \varphi_0 (\varepsilon \,\mathrm{grad}\, \varphi_0) dV = \int_V \mathrm{div}(\varphi_0 \varepsilon \,\mathrm{grad}\, \varphi_0) dV -$

$\int_V \varphi_0 \underbrace{\mathrm{div}(\varepsilon \,\mathrm{grad}\, \varphi_0)}_{=0} dV = -\int_{\Gamma_D} \varphi_0 (\varepsilon \,\mathrm{grad}\, \varphi_0) d\bar{\Gamma} - \int_{\Gamma_N} \varphi_0 (\varepsilon \,\mathrm{grad}\, \varphi_0) d\bar{\Gamma} =$

$-\int_{\Gamma_D} \cancel{\varphi_0}^{\,0} (\varepsilon \,\mathrm{grad}\, \varphi_0) \cdot d\bar{\Gamma} - \int_{\Gamma_N} \varphi_0 \overbrace{\varepsilon \frac{\partial \varphi}{\partial n}}^{=0} d\Gamma = 0$

$\int_V \bar{E}_0 \bar{D}_0 = \int \varepsilon |E_0|^2 dV = 0 \Rightarrow \bar{E}_0 = 0$

$\mathrm{grad}(\varphi' - \varphi'') = 0 \Rightarrow \varphi' = \varphi'' + \varphi_0$

$\int_{\Gamma_D} \varphi_0 \frac{\partial \varphi_0}{\partial n} d\Gamma = \varphi_0 \int \varepsilon \frac{\partial \varphi_0}{\partial n} d\Gamma$ — Ekvipotenciális felületen megadjuk az össztöltést akkor is egyértelmű a megoldás

áll.

7.EA

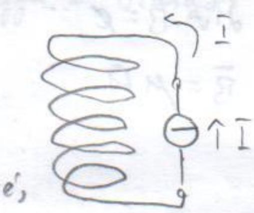
Stacionárius áramok terén $\frac{\partial}{\partial t} = \emptyset$; $\{\bar{J}$

$\mathrm{rot}\,\bar{H} = \bar{J}$ A mágneses tér örvényei mindenhol.

$\mathrm{div}\,\bar{B} = \emptyset \Rightarrow \bar{B} = \mathrm{rot}\,\bar{A}$ vektorpotenciált elő tudjuk írni.

$\bar{B} = \mu\bar{H}$

$\mu_r \neq 1$



ha lenne áram a dielektrikumban → töltés megosztás történne ← lecsillapítaná a saját hatását

- $\mathrm{div}\,\bar{A}$ szabadon választható: mérték választás

$$\mathrm{rot}\,\underbrace{\frac{1}{\mu}\,\underbrace{\mathrm{rot}\,\bar{A}}_{B}}_{H} = \bar{J}$$

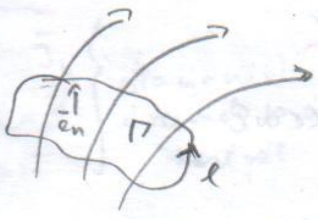
+ mértékválasztás, pl Coulomb mérték $\mathrm{div}\,\bar{A} = \emptyset$

+ peremfeltételek

Γ_B: B_n adott $\quad B_n = \bar{e}_n\,\mathrm{rot}\,\bar{A} \Rightarrow \bar{A}_t$-ra vonatkozó megkötés

Γ_H: H_t adott $\quad \bar{H}_t = \bar{e}_n \times \frac{1}{\mu}\,\mathrm{rot}\,\bar{A} \times \bar{e}_n \Rightarrow \frac{\partial A_n}{\partial n}$-re vonatkozó megkötés

A fizikai jelentése:



$$\Psi = \int_\Gamma \bar{B}\,d\bar{\Gamma} = \int_\Gamma \mathrm{rot}\,\bar{A}\,d\bar{\Gamma} = \oint_\ell \bar{A}\,d\bar{l}$$

Szabad térben a megoldás $\quad [\mathrm{rot}\,u\bar{v} = \mathrm{grad}\,u \times \bar{v} + u\,\mathrm{rot}\,\bar{v}]$

$$\mathrm{rot}\,\frac{1}{\mu}\,\mathrm{rot}\,\bar{A} - \bar{J} + \mathrm{div}\,\bar{A} = \emptyset$$

$$\mathrm{rot}\,\frac{1}{\mu}\,\mathrm{rot}\,\bar{A} = \mathrm{grad}\,\frac{1}{\mu} \times \mathrm{rot}\,\bar{A} + \frac{1}{\mu}\,\mathrm{rot}\,\bar{A}$$

$$\boxed{\Delta\bar{A} = -\mu\bar{J}}$$

$+\bar{A} = \emptyset \quad r \to \infty$

$\mathrm{rot}\,\bar{A} = \bar{B}$

$\mathrm{rot}\,\mathrm{rot}\,\bar{A} = \mathrm{grad}\,\mathrm{div}\,\bar{A} - \Delta\bar{A}$

$$\bar{A}(\bar{r}) = \frac{\mu}{4\pi}\int_V \frac{\bar{J}(\bar{r}')}{|\bar{r} - \bar{r}'|}\,dV' \Rightarrow$$

Biot-Savart

$$B(r) = \frac{\mu}{4\pi}\int \frac{J(\bar{r}') \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}\,dV'$$

4.a) Stacionárius áramok mágneses tere a redukált skalárpotenciállal

saját választás:

$$\mathrm{rot}\,\bar H = \bar J \qquad \bar H = \bar H_s + \bar H_r \;\left.\right\} \Rightarrow \; \mathrm{rot}\,\bar H_r = \emptyset \quad \Rightarrow \bar H_r = -\mathrm{grad}\,\varphi_r$$

$$\mathrm{div}\,\bar B = \emptyset \qquad \mathrm{div}\,\mu(\bar H_s + \bar H_r) = \emptyset$$

$$\mathrm{rot}\,\bar H_s = \bar J$$

$$\bar B = \mu \bar H$$

$$\boxed{-\mathrm{div}\,\mu\,\mathrm{grad}\,\varphi_r = -\mathrm{div}\,\mu \bar H_s}$$

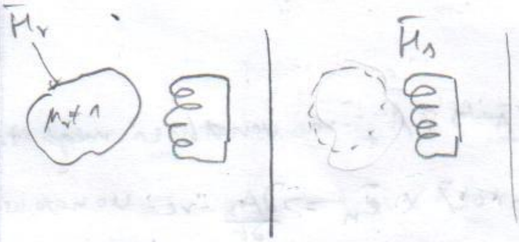
+ peremfeltételek

Γ_D: φ_r adott

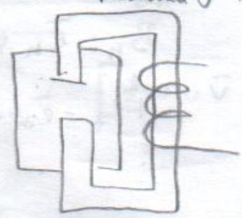
Γ_N: $\mu \dfrac{\partial \varphi_r}{\partial n}$ adott

Egy tipikus választás:

$$\bar H_s(\bar r) = \frac{1}{4\pi}\int \frac{\bar J(\bar r') \times (\bar r - \bar r')}{|\bar r - \bar r'|^3}\, dV'$$



Hogyan változik a tér ha belerakok egy ferromágneses anyagot? redukált skalárpotenciállal



5, kvázistacionárius tér, örvényáramú tér

$$\mathrm{rot}\,\bar H = \bar J + \frac{\partial \bar D}{\partial t} \qquad \bar J = \sigma \bar E \qquad \sigma \ll \omega\varepsilon$$

$$\mathrm{rot}\,\bar E = -\frac{\partial \bar B}{\partial t} \qquad \frac{\partial \bar D}{\partial t} = \mathrm{Re}\{j\omega\varepsilon \bar E e^{j\omega t}\}$$

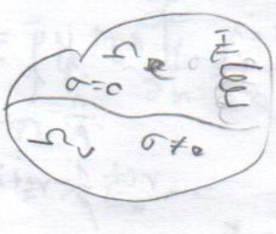
$$\mathrm{div}\,\bar B = \emptyset$$

$$\mathrm{div}\,\bar D = \rho$$

$$\bar B = \mu \bar H \quad \bar D = \varepsilon \bar E$$

$$\bar J = \sigma(\bar E + E_b) + \bar J_b$$

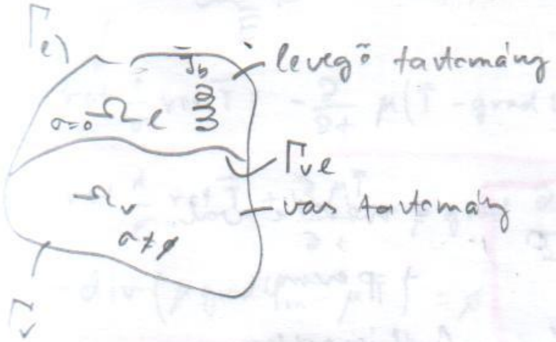
$$w = \tfrac{1}{2}\bar E \bar D + \tfrac{1}{2}\bar H \bar B$$



Ω_v	Ω_e
$\mathrm{rot}\,\bar H = \bar J$	$\mathrm{rot}\,\bar H = \bar J_b$
$\mathrm{rot}\,\bar E = -\frac{\partial \bar B}{\partial t}$	$\mathrm{rot}\,\bar E = -\frac{\partial \bar B}{\partial t}$
$\mathrm{div}\,\bar B = \emptyset$	$\mathrm{div}\,\bar B = \emptyset$
$\bar B = \mu \bar H$	$\bar B = \mu \bar H$
$\bar J = \sigma \bar E$	

Ha a fém mérete összemérhető a behatolási mélységgel + vizsgált térrész a hullám horizon belül van.

f	λ	δ ($\sigma = 10^{6}\frac{S}{m}$)	δ ($\sigma = 5{,}7\cdot 10^{7}\frac{S}{m}$)
50Hz	6000km	7,1cm	3,4mm
100kHz	3km	1,6cm	0,21mm
100MHz	3m	50μm	6,6μm
1GHz	30cm	15μm	2,1μm



Ω_e

$\left.\begin{array}{l} \text{rot}\,\bar{H} = \bar{J}_b \\ \text{div}\,\bar{B} = 0 \\ \bar{B} = \mu\bar{H} \end{array}\right\}$ stacionárius áramok mágneses terével

Hol van az elektromos tér?
→ van de csak a mértéket elhanyagoljuk
(mert nem a hullám jelenséget vizsgáljuk)

Perem felt. (Γ_e)

$\Gamma_{eH}: \bar{e}_n \times \bar{H}$ adott

$\Gamma_{eB}: \bar{e}_n \bar{B} = \bar{B}_n$ adott

Perem felt (Γ_{ve})

- Folytonossági feltétel miatt valami egyenlőség kell.

$\Gamma_{veH}: (\bar{H}_e - \bar{H}_v) \times \bar{e}_n = 0$ H tang. komponense folytonos

$\Gamma_{veB}: B_{en} - B_{nv} = 0$ B norm komponense folytonos

(div J = 0)

Ω_v

$\text{rot}\,\bar{H} = \bar{J}$ — EM tér által létrehozott vezetési áram $\neq \bar{J}_b$

$\text{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$\text{div}\,\bar{B} = 0$

$\bar{B} = \mu\bar{H}$ ($\bar{B} = \mu_0\bar{H} + \mu_0\bar{M}$)

$\bar{J} = \sigma\bar{E}$

Perem felt. (Γ_v)

$\Gamma_{vH}: \bar{e}_n \times \bar{H} = \bar{H}_t$ adott

$\Gamma_{vE}: \bar{e}_n \times \bar{E} = \bar{E}_t$ adott

1, Peremérték - feladat vektorpotenciállal és skalárpotenciállal $\bar{A} - \varphi$ mód

Ω_e $\text{rot}\,\bar{A} = \bar{B}$

$$\boxed{\text{rot}\,\frac{1}{\mu}\,\text{rot}\,\bar{A} = \bar{J}_b}$$

+ mérték választás
pl.: $\text{div}\,\bar{A} = \emptyset$ ún. Coulomb tv.
+ peremfeltételek

Ω_v $\text{rot}\,\bar{A} = \bar{B}$ A mi választásunk hogy vektorpotenciállal írjuk fel a peremérték feladatot

$$\text{rot}\,\bar{E} = -\frac{\partial}{\partial t}\,\text{rot}\,\bar{A} \rightarrow \text{rot}\left(\bar{E} + \frac{\partial \bar{A}}{\partial t}\right) = \emptyset$$

Ha rotáció mentes → leírható egy skalárpotenciállal

$$\bar{E} + \frac{\partial \bar{A}}{\partial t} = -\text{grad}\,\varphi$$

$$\bar{E} = -\text{grad}\,\varphi - \frac{\partial \bar{A}}{\partial t}$$

$$\text{rot}\,\frac{1}{\mu}\,\text{rot}\,\bar{A} = -\sigma\,\text{grad}\,\varphi - \sigma\frac{\partial \bar{A}}{\partial t}$$

$$\boxed{\begin{aligned} &\text{rot}\,\frac{1}{\mu}\,\text{rot}\,\bar{A} + \sigma\,\text{grad}\,\varphi + \sigma\frac{\partial \bar{A}}{\partial t} = 0 \\ &-\text{div}\left(\sigma\,\text{grad}\,\varphi + \sigma\frac{\partial \bar{A}}{\partial t}\right) = 0 \leftarrow \text{div}\,\bar{J} = \emptyset \end{aligned}}$$

+ mérték vál.
+ perem
A, φ ismeretlen

Skalárral leírható az örvényáramok tere + jó mértékválasztással a vektorpotenciál feltoryosó tehát $\tilde{A}$

2, PEF $\bar{T} - \varphi_m$

Ω_e (lásd redukált skalárpotenciállal stacionárius áramok mágneses terére)

$$-\mathrm{div}(\mu \,\mathrm{grad}\, \varphi_m) = -\mathrm{div}(\mu \bar{H}_s)$$

+ peremérték

pl: $\bar{H}_s(\bar{r}) = \frac{1}{4\pi} \int_{\Omega_e} \frac{\bar{J}_b(\bar{r}') \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3} d\Omega$ + folytonossági feltételek

Ω_v

T def: $\mathrm{rot}\, \bar{T} = \bar{J}$ $\quad (\mathrm{div}\, \bar{J} = \emptyset)$

$\mathrm{rot}\, \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$\mathrm{rot}\left(\underbrace{\frac{1}{\sigma} \mathrm{rot} \bar{T}}_{\bar{E}}\right) = -\frac{\partial \bar{B}}{\partial t}$

$\mathrm{rot}\, \bar{H} = \bar{J} = \mathrm{rot}\, \bar{T}$

$\mathrm{rot}(\bar{H} - \bar{T}) = \emptyset \Rightarrow \bar{H} - \bar{T} = \mathrm{grad}\, \varphi_m$

$\bar{H} = \bar{T} - \mathrm{grad}\, \varphi_m$ levegőben a potenciál φ_m

fémben φ_m és $\bar{T}$

$$\mathrm{rot} \frac{1}{\sigma} \mathrm{rot}\, \bar{T} = -\frac{\partial}{\partial t} \mu (\bar{T} - \mathrm{grad}\, \varphi_m)$$

$$\boxed{\begin{aligned} &\mathrm{rot} \frac{1}{\sigma} \mathrm{rot}\, \bar{T} + \frac{\partial \mu \bar{T}}{\partial t} - \mu \,\mathrm{grad} \frac{\partial \varphi_m}{\partial t} = \emptyset \\ &-\mathrm{div}(\mu \,\mathrm{grad}\, \varphi_m - \mu \bar{T}) = \emptyset \end{aligned}}$$

+ mérték választás

+ perem és folyt. feltétel

6, Elektromágneses hullámok

$\mathrm{rot}\, \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

$\mathrm{rot}\, \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$\mathrm{div}\, \bar{B} = \emptyset$

$\mathrm{div}\, \bar{D} = \varrho$

$\bar{B} = \mu \bar{H} \quad \bar{D} = \varepsilon \bar{E}$

$\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$

$w = \frac{1}{2} \bar{E}\bar{D} + \frac{1}{2} \bar{H}\bar{B}$

Γ_E: $\bar{E} \times \bar{e}_n = \bar{E}_t$ adott

Γ_H: $\bar{H} \times \bar{e}_n = \bar{H}_t$ adott

$\bar{H}(t = t_0)$ $\quad \bar{E}(t = t_0)$ adott

$\bar{E}_e(t)$, $\bar{J}_b(t)$ adott

PE7-1 $\quad \bar{A} = \varphi$

(3) $\text{rot}\, \bar{A} = \varphi$

(2) $\text{rot}\left(\bar{E} + \frac{\partial \bar{A}}{\partial t}\right) = \emptyset \qquad \bar{E} + \frac{\partial \bar{A}}{\partial t} = -\text{grad}\,\varphi$

$$\bar{E} = -\text{grad}\,\varphi - \frac{\partial \bar{A}}{\partial t}$$

(1) $\text{rot}\, \frac{1}{\mu}\, \text{rot}\, \bar{A} = \bar{J} - \varepsilon\, \text{grad}\, \frac{\partial \varphi}{\partial t} - \varepsilon \frac{\partial^2 \bar{A}}{\partial t^2}$

$$\boxed{\begin{aligned} &\text{rot}\, \frac{1}{\mu}\, \text{rot}\, \bar{A} + \varepsilon\, \text{grad}\, \frac{\partial \varphi}{\partial t} + \varepsilon \frac{\partial^2 A}{\partial t^2} = \bar{J} \\ (4)\quad &-\text{div}\left(\varepsilon\, \text{grad}\, \varphi + \varepsilon \frac{\partial \bar{A}}{\partial t}\right) = \varrho \end{aligned}}$$

+ mért. vál.
+ perem. felt.

Ha μ és ε állandó a térben

$\text{rot}\, \text{rot}\, \bar{A} + \mu\varepsilon\, \text{grad}\, \frac{\partial \varphi}{\partial t} + \mu\varepsilon \frac{\partial^2 A}{\partial t^2} = \mu \bar{J}$

$\text{grad}\, \text{div}\, \bar{A} - \Delta \bar{A} + \mu\varepsilon\, \text{grad}\, \frac{\partial \varphi}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} = \mu \bar{J}$

Mérték válantás: $\text{div}\, \bar{A} = -\mu\varepsilon \frac{\partial \varphi}{\partial t}$ Lorentz mérték

$$\boxed{\Delta \bar{A} - \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} = -\mu \bar{J}}$$

$-\text{div}\, \text{grad}\, \varphi - \frac{\partial\, \text{div}\, \bar{A}}{\partial t} = \frac{\varrho}{\varepsilon}$

$$\boxed{\Delta \varphi - \mu\varepsilon \frac{\partial^2 \varphi}{\partial t^2} = -\frac{\varrho}{\varepsilon}}$$

$\text{div}\, \bar{J} + \frac{\partial \varrho}{\partial t} = \emptyset$ Folytonossági felt.-nek teljesülnie kell

Elektromágneses hullámok

PEF-1 $\bar{A} - \varphi$

Szabad térben (homogén kitöltésű, végtelen tér)

$$\Delta \bar{A} - \mu\varepsilon \frac{\partial \bar{A}}{\partial t^2} = -\mu \bar{J}$$

jól kell megadni a gerj.-eket: $\operatorname{div} \bar{J} + \frac{\partial \rho}{\partial t} = 0$
nak érvényesülnie kell (folyt. felt.)

$$\Delta \varphi - \mu\varepsilon \frac{\partial \varphi}{\partial t^2} = -\frac{\rho}{\varepsilon}$$

$\Delta u(\bar{r},t) = f(\bar{r},t)\, u(\bar{r},t)$ megoldása

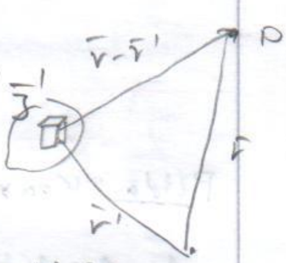
$$\Delta u(\bar{r},t) - \mu\varepsilon \frac{\partial u(\bar{r},t)}{\partial t^2} = f(\bar{r},t) \qquad u\left(\bar{r}, t - \frac{r}{c}\right)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Állítás:

$$\Delta \bar{A} = -\mu \bar{J} \qquad \bar{A}(r) = \frac{\mu}{4\pi} \int_V \frac{\bar{J}(\bar{r}')}{|r - r'|}\, dV'$$

$$\Delta \bar{A} - \frac{1}{c^2}\frac{\partial^2 \bar{A}}{\partial t^2} = -\mu \bar{J} \qquad \bar{A}(\bar{r},t) = \frac{\mu}{4\pi} \int_V \frac{\bar{J}\left(\bar{r}', t - \frac{|\bar{r}-\bar{r}'|}{c}\right)}{|\bar{r}-\bar{r}'|}\, dV'$$



Retardált potenciálok, itt már számít a jel terj. sebessége (relativitás elm.)

$$\Delta \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{\rho}{\varepsilon} \qquad \varphi(\bar{r},t) = \frac{1}{4\pi\varepsilon} \int_V \frac{\rho\left(\bar{r}', t - \frac{|\bar{r}-\bar{r}'|}{c}\right)}{|\bar{r}-\bar{r}'|}\, dV'$$

$$\bar{B}(\bar{r}) = \frac{\mu}{4\pi} \int_V \frac{\bar{J}(\bar{r}') \times (\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}\, dV'$$

PEF-2: $\bar{F} - \varphi_m$ $(J = \emptyset,\ \rho = \emptyset)$

$\bar{D} = \operatorname{rot} \bar{F}$ Elektromos vektorpotenciál

... HF

PE7 3,4 (szinuszos esetben $\bar{E}$, vagy $\bar{H}$ feljtonessága közelítés

3,

$\text{rot}\,\bar{H} = j\omega\varepsilon\bar{E} + \bar{J}$

$\text{rot}\,\bar{E} = -j\omega\mu\bar{H}$

$\text{div}\,\bar{B} = \emptyset$

$\text{div}\,\bar{D} = \varrho$

$\bar{D} = \varepsilon\bar{E}$

$\bar{B} = \mu\bar{H}$

$\text{rot}\,\text{rot}\,\bar{E} = -j\omega\mu\,\text{rot}\,\bar{H}$

$\text{rot}\,\text{rot}\,\bar{E} = \omega^2\mu\varepsilon\bar{E} - j\omega\mu\bar{J}$

$\text{grad}\,\text{div}\,\bar{E} - \Delta\bar{E} - \omega^2\mu\varepsilon\bar{E} = -j\omega\mu\bar{J}$

forrásmentes ($\varrho = \emptyset$) $\text{div}\,\bar{E} = \emptyset$

$\Delta\bar{E} + \omega^2\mu\varepsilon\bar{E} = j\omega\mu\bar{J}$

Forrásmentes esetben ($\bar{J} = 0$)

$\Delta E + \omega^2\mu\varepsilon\bar{E} = 0$

$\downarrow$

$$H = \frac{-1}{j\omega\mu}\,\text{rot}\,\bar{E}$$

Példa: Koaxiális kábel hosszegységre eső kapacitása:

1, Analitikus megoldás

$R_1 = 1\,\text{cm}$

$R_2 = 3\,\text{cm}$

$\varepsilon_r = 3$

$$E_r(r) = \frac{q}{2\pi\varepsilon r}$$

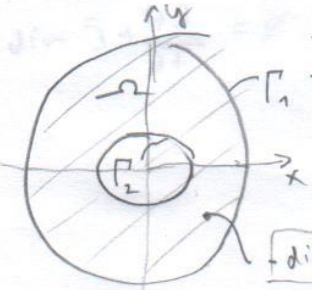
$$U_{1,2} = \int_{R_1}^{R_2} E_r(r)\,dr = \frac{q}{2\pi\varepsilon}\ln\frac{R_2}{R_1}$$

$$q l = C' l\, U_{12}$$

$$C' = \frac{q}{U_{12}} = \frac{2\pi\varepsilon}{\ln\frac{R_2}{R_1}} = 151{,}8\ \frac{\text{pF}}{\text{m}}$$

2, Numerikus megoldás

- Elektrosztatikus feladat



- adott $U_{1,2}$ esetén mennyi q?
- adott U_{12} esetében meghatározzuk az energiát (alternatív m.o.)

$$W = \frac{1}{2}CU^2$$

$$W' l = \frac{1}{2}C' l\,U^2 \rightarrow C' = \frac{2W'}{U^2}$$

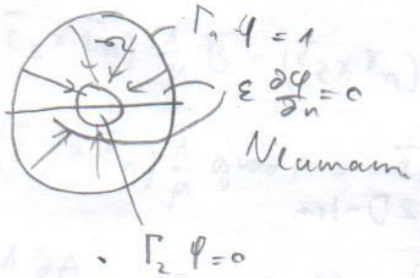
$\boxed{\text{div}\,\varepsilon\,\text{grad}\,\varphi = \emptyset}$ nincs tölt. sűr.

Γ_1 : Diriclet $\varphi = 1V$
Γ_2 : — " — $\varphi = 0V$
$\Big\} \Rightarrow u = 1V$

$$W = \int_V \frac{1}{2} \bar{E}\bar{D}\, dV = \int_V \frac{1}{2} \varepsilon |E|^2 dV = l \underbrace{\int_\Omega \frac{1}{2} \varepsilon |E|^2 d\Omega}_{W'}$$

$$W' = 7{,}6r56 \cdot 10^{-11} \frac{J}{m} \rightarrow C' = \frac{2W'}{1} = 1{,}5211 \cdot 10^{-10} \frac{F}{m}$$

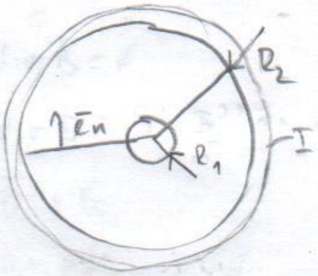
vagy



10 EA | Koaxiális kábel hosszegységre eső induktivitása (külső ind.)

$$\text{Megj.: } L'C' = \frac{\mu}{2\pi}\ln\frac{R_2}{R_1}\,\frac{2\pi\varepsilon}{\ln\frac{R_2}{R_1}} = \mu\varepsilon$$

Analitikus



$$H_r(r) = \frac{I}{2\pi r}$$

$$\Psi = \Psi' l = l\int_{R_1}^{R_2}\frac{\mu I}{2\pi r}\,dr = \frac{l\mu I}{2\pi}\ln\frac{R_2}{R_1}$$

$$\Psi = LI = L'l = \frac{l\mu I}{2\pi}\ln\frac{R_2}{R_1} \Rightarrow L' = \frac{\mu}{2\pi}\ln\frac{R_2}{R_1} = 219{,}75\,\frac{nH}{m}$$

$R_1 = 1\,cm$

$R_2 = 3\,cm$

$\mu = \mu_0$

Stacionárius áramok mágneses tere 2D-ben

$$\frac{\partial}{\partial z} = 0 \Rightarrow \bar{J} = J\bar{e}_z \Rightarrow \bar{A} = A\bar{e}_z \Rightarrow H_z = 0,\ B_z = 0$$

$$\bar{B}(x,y) = B_x(x,y)\bar{e}_x + B_y(x,y)\bar{e}_y$$

$$\bar{H}(x,y) = \frac{1}{\mu}\bar{B}(x,y)$$

PEF:

$$\text{rot}\,\frac{1}{\mu}\,\text{rot}\,\bar{A} = \bar{J}$$

$\bar{A} = A\bar{e}_z \Rightarrow \text{div}\,\bar{A} = 0$

Coulomb mérték

$$\text{rot}\,A\bar{e}_z = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & A \end{vmatrix} =$$

$$\frac{\partial A}{\partial y}\bar{e}_x - \frac{\partial A}{\partial x}\bar{e}_y$$

$$\text{rot}\,\frac{1}{\mu}\,\text{rot}\,A\bar{e}_z = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ \frac{1}{\mu}\frac{\partial A}{\partial y} & -\frac{1}{\mu}\frac{\partial A}{\partial x} & 0 \end{vmatrix} = \left(-\frac{\partial}{\partial x}\frac{1}{\mu}\frac{\partial A}{\partial x} - \frac{\partial}{\partial y}\frac{1}{\mu}\frac{\partial A}{\partial y}\right)e_z$$

$$\boxed{-\operatorname{div}\frac{1}{\mu}\operatorname{grad}A = J_z}$$

$[\operatorname{rot} u\bar v = \operatorname{grad} u \times \bar v + u \operatorname{rot}\bar v]$

$B_n = \bar e_n \bar B = \bar e_n \operatorname{rot}(A\bar e_z) = \bar e_n(\operatorname{grad}A \times \bar e_z) = \operatorname{grad}A(\bar e_z \times \bar e_n) =$

$\operatorname{grad}A \cdot \bar e_z = \frac{\partial A}{\partial z}$ Dirichlet PEF

$\Gamma_D: A = A_0$ adott $\Rightarrow \frac{\partial A}{\partial z}$ adott $\Rightarrow B_n$ adott

2D-ben skalár

$H_t = (\bar e_z \times \bar e_n)\cdot\frac{1}{\mu}\bar B = (\bar e_z \times \bar e_n)\frac{1}{\mu}\operatorname{rot}(A\bar e_z) = (\bar e_z \times \bar e_n)\left(\frac{1}{\mu}\operatorname{grad}A \times \bar e_z\right) =$

$\bar e_z \times (\bar e_z \times \bar e_n)\frac{1}{\mu}\operatorname{grad}A = -\bar e_n\frac{1}{\mu}\operatorname{grad}A = -\frac{1}{\mu}\frac{\partial A}{\partial n}$ Neuman PEF

$\Gamma_N: \frac{1}{\mu}\frac{\partial A}{\partial n} =$ adott $\Rightarrow H_t$ adott

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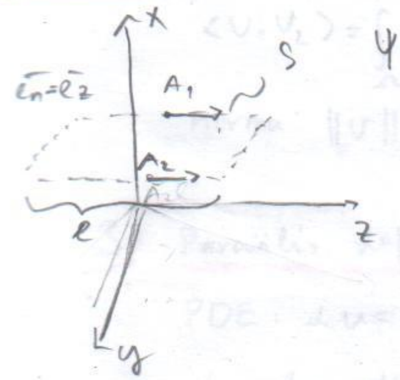


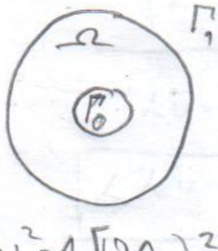
$\Psi = \int_S \bar B d\bar S = \int_S \operatorname{rot}\bar A d\bar S = \oint_\ell \bar A d\bar \ell$

Stokes t.

2D-ben $A = A\bar e_z$

$\Psi = \Psi'\ell = A_2\ell - A_1\ell \Rightarrow \Psi' = A_2 - A_1$

Numerikus modell



$\Omega: -\operatorname{div}\frac{1}{\mu}\operatorname{grad}A = \emptyset$

$\Gamma_0: A = \emptyset$

$\Gamma_1: A = 1$

$\Psi = LI \quad \Psi'\ell = L'\ell I$

$\frac{\Psi'}{I} = L'$

$W = W'\ell = \ell\int_\Omega \frac{1}{2}\mu|H|^2 d\Omega$

$|\bar H|^2 = \frac{1}{\mu^2}\left[\left(\frac{\partial A}{\partial x}\right)^2 + \left(\frac{\partial A}{\partial y}\right)^2\right] =$

$\frac{1}{\mu^2}|\operatorname{grad}A|^2$

$H = \frac{1}{\mu}\operatorname{rot}(A\bar e_z) = \frac{1}{\mu}\operatorname{grad}A \times \bar e_z =$

$\frac{1}{\mu}\left(+\frac{\partial A}{\partial y}\bar e_x - \frac{\partial A}{\partial x}\bar e_y\right)$

$$W' = \int_\Omega \frac{1}{2}\mu \left|\frac{1}{\mu}\operatorname{grad} A\right|^2 d\Omega \quad \Bigg| \quad \Psi' l = L' l I$$

$$W' = \frac{1}{2} L' I^2 = \frac{1}{2} L' \left(\frac{\Psi'}{L'}\right)^2$$

$$\frac{2W'}{(\Psi')^2} = \frac{1}{L'} \Rightarrow L' = \frac{(\Psi')^2}{2W'}$$

$$L' = \frac{1}{2W'} = 219{,}34 \cdot 10^{-9} \frac{H}{m}$$

1, $-\operatorname{div}\operatorname{grad} u = 0$ $\quad \Gamma_{D1}: u = 1$

$\quad \Gamma_{D0}: u = 0$

C': $\quad W'_C = \int_\Omega \frac{1}{2}\varepsilon |\operatorname{grad} u|^2 d\Omega \qquad C' = 2W'_C$

L': $\quad W'_L = \int_\Omega \frac{1}{2}\frac{1}{\mu} |\operatorname{grad} u|^2 d\Omega \qquad L' = \frac{1}{2W'_L}$

$$C'L' = \frac{W'_C}{W'_L} = \frac{\varepsilon \int_\Omega \frac{1}{2} |\operatorname{grad} u|^2 d\Omega}{\frac{1}{\mu}\int_\Omega \frac{1}{2} |\operatorname{grad} u|^2 d\Omega} = \mu\varepsilon$$

Bevezetés a végeselem módszerbe

11EA

I, A súlyozott maradék elve

1, Vektoralgebrai analógia

$\vec{v} = \vec{0} \iff \vec{v} \cdot \vec{a} = 0 \quad \forall \vec{a}$

$\iff \vec{v}\vec{e}_i = 0 \quad i = 1 \dots n$

$\iff \left(\sum_{j=1}^{n} v_j \vec{e}_j \right) \vec{e}_i = \emptyset \quad i = 1 \dots n$

2, Függvények belső szorzata

V : Ω -n négyzetesen integrálható függv. tere

2D:

$v(x,y) \in V$

$\boxed{\langle v_1, v_2 \rangle \triangleq \int_\Omega v_1 v_2^* \, d\Omega}$

belső szorzat

pl 2D

$\langle v_1 v_2 \rangle = \int_\Omega v_1(x,y) v_2^*(x,y) \, dx\,dy$

norma: $\|v\|^2 \triangleq \langle v, v \rangle = \int_\Omega |v|^2 d\Omega$

3 Parciális differenciál egyenlet gyenge alakja

PDE: $\mathcal{L}u = f$

$\mathcal{L}$ – lin. diff. op.

u – skalár mező

f – gerjesztés

$\mathcal{L}u - f = \emptyset$

$\boxed{\langle \mathcal{L}u - f, q \rangle = 0 \quad \forall q} \in W$

w_i a W fgv.-tér bázisa $i = 1, 2 \ldots$

$\langle \mathcal{L}u - f, w_i \rangle = \emptyset \quad i = 1, 2$

$u = \sum_j c_j w_j$

$\langle \mathcal{L} \sum_j c_j w_j - f, w_i \rangle = \emptyset$

$$\boxed{\sum_j c_j \langle \mathcal{L} w_j, w_i \rangle = \langle f, w_i \rangle}$$

4 Elektrosztatika

$$\left.\begin{array}{l} \Omega: -\operatorname{div}(\varepsilon \operatorname{grad} u) = \rho \\ \Gamma_D: \quad u = u_0 \\ \Gamma_N: \quad \dfrac{\partial u}{\partial n} = \emptyset \end{array}\right\} \to \bar{E} = -\operatorname{grad} u$$

a, gyenge alak:

$\mathcal{L}(\cdot) = -\operatorname{div}(\varepsilon \operatorname{grad}(\cdot))$

$f = \rho$

bázis w_i

$$\int_\Omega -\operatorname{div}(\varepsilon \operatorname{grad} u) w_i \, d\Omega = \int_\Omega \overset{\rho}{f} w_i \, d\Omega \quad \forall w_i$$

$\operatorname{div}(p\bar{q}) = (\operatorname{grad} p)\bar{q} + \underline{p \operatorname{div} \bar{q}} \qquad p = w_i$

$\qquad \bar{q} = \varepsilon \operatorname{grad} u$

$\operatorname{div}(w_i \varepsilon \operatorname{grad} u) = \operatorname{grad} w_i \, \varepsilon \operatorname{grad} u + w_i \operatorname{div}(\varepsilon \operatorname{grad} u)$

$$\int_\Omega \varepsilon \operatorname{grad} u \cdot \operatorname{grad} w_i \, d\Omega - \underbrace{\int_\Omega \operatorname{div}(\varepsilon \operatorname{grad} u \, w_i) \, d\Omega}_{I} = \int_\Omega \rho w_i \, d\Omega$$

Gauss: $I = \int_\Gamma \varepsilon \operatorname{grad} u \cdot w_i \, d\bar{\Gamma} = \int_\Gamma \varepsilon w_i \underbrace{\operatorname{grad} u \cdot \bar{n}}_{\frac{\partial u}{\partial n}} \, d\Gamma$

$$= \int_{\Gamma_D} \varepsilon w_i \frac{\partial u}{\partial n} d\Gamma + \underbrace{\int_{\Gamma_N} \varepsilon w_i \frac{\partial u}{\partial n} d\Gamma}_{0}$$

Dirichlet explicit módon →

legyen $w_i\big|_{\Gamma_D} = \emptyset$

$$\int_\Omega \varepsilon \,\mathrm{grad}\, u \;\mathrm{grad}\, w_i \, d\Omega = \int_\Omega \varrho w_i \, d\Omega \qquad u = \sum_j c_j w_j$$

$$\boxed{\sum_{j=1}^{n} c_j \underbrace{\int_\Omega \varepsilon \,\mathrm{grad}\, w_j \;\mathrm{grad}\, w_i \, d\Omega}_{A_{ij}} = \int_\Omega \underbrace{\varrho w_i \, d\Omega}_{b_i} \quad i = 1..n}$$

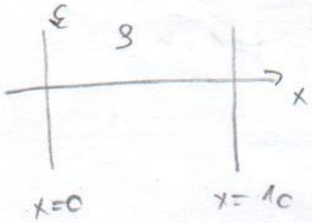
Lin alg. egyenletrend. $\to \underline{\underline{A}}\,\underline{c} = \underline{b}$

Közelítés (diszkretizálás): W véges dim. $w_i;\ i = 1..n$

$$u \approx \tilde{u} = \sum_{j=1}^{n} c_j w_j$$

II. 1D feladat

1,



$\frac{\varrho}{\varepsilon} = 1$

Analitikus m.o.

$u(x) = 15x - \frac{x^2}{2}$

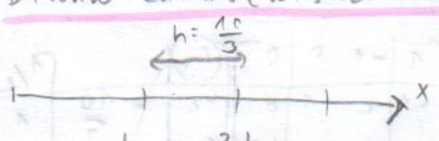
$-\mathrm{div}\,\mathrm{grad}\, u = 1$

$u(0) = 0$

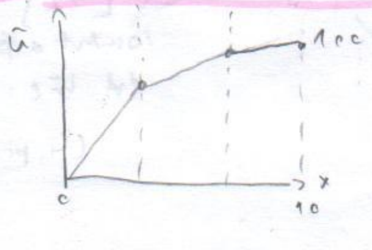
$u(10) = 100$

$u(x) \quad 0 \le x \le 10$

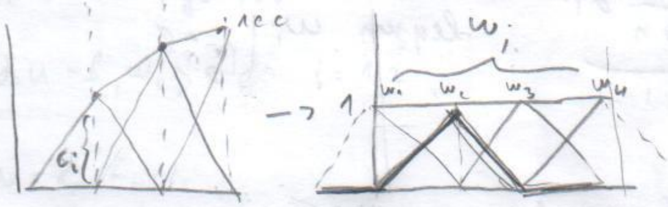
2, Diszkretizálás (kis szakaszokra bontjuk)



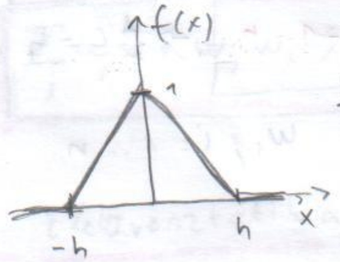
3, Közelítés tartományonként lin.



4, Bázis függvények



$$\hat{u} = \sum_{j=1}^{n} c_j w_i = \sum_{j=1}^{n} c_j w_j$$



$$f(x) = \begin{cases} 1 - \dfrac{|x|}{h} & |x| < h \\ 0 & \end{cases}$$

$$w_j(x) = f(x - (j-1)h)$$

5, Együtthatók számítása

$$\sum_{j=1}^{n} c_i \underbrace{\int_0^{3h} \frac{dw_i}{dx} \frac{dw_i}{dx} dx}_{A_{ij}} = \underbrace{\int_0^{3h} 1 \cdot w_i \, dx}_{b_i} \qquad i = 1 .. n$$

$$A_{11} = \int_0^h \left(-\frac{1}{h}\right)^2 dx = \frac{1}{h} = A_{44}$$

$$A_{22} = \int_0^{2h} \dots = \int_0^h \left(\frac{1}{-h}\right)^2 dx + \int_h^{2h} \left(-\frac{1}{h}\right)^2 dx = \frac{2}{h} = A_{33}$$

$$A_{12} = \int_0^h \left(\frac{1}{h}\right)\left(-\frac{1}{h}\right) dx = \frac{-1}{h} = A_{21}, A_{23}, A_{32}, A_{34}, A_{43}$$

$A_{13} = 0$ ...stb

$$b_1 = \int_0^h 1 \cdot w_1 \, dx = \frac{h}{2} = b_4$$

$$b_2 = \int_0^{2h} w_2 \, dx = h = b_3$$

$$\frac{3}{10} \left[\begin{array}{c|cc|c} 1 & -1 & 0 & 0 \\ \hline -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ \hline 0 & 0 & -1 & 1 \end{array}\right] \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \frac{10}{3} \begin{bmatrix} 1/2 \\ 1 \\ 1 \\ 1/2 \end{bmatrix}$$

Particionálás

ismert a peremérték: 0V, 100V

(4. pont)

6, A Dirichlet-felt figyelembevétele

$$\frac{3}{10}\begin{bmatrix}-1\\0\end{bmatrix}\underset{\overset{\wedge}{0}}{c_1}+\frac{3}{10}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}\begin{bmatrix}c_2\\c_3\end{bmatrix}+\frac{3}{10}\begin{bmatrix}0\\-1\end{bmatrix}\underset{\overset{\wedge}{100}}{c_4}=\frac{10}{3}\begin{bmatrix}1\\1\end{bmatrix}$$

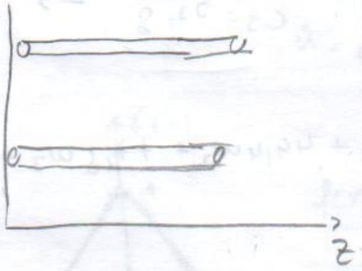
$$\begin{bmatrix}0,6 & -0,3\\-0,3 & 0,6\end{bmatrix}\begin{bmatrix}c_2\\c_3\end{bmatrix}=\begin{bmatrix}\frac{10}{3}\\\frac{10}{3}+30\end{bmatrix}\rightarrow \quad \begin{matrix}c_2=44,4\\c_3=77,8\end{matrix}\rightarrow$$

$$\tilde{u}(x)=0\cdot w_1+44,4w_2+77,8w_3+100w_4$$

12. EA

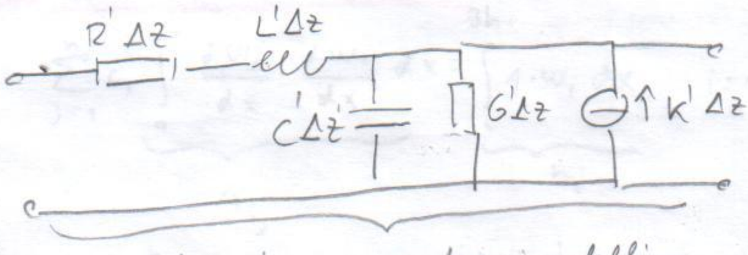
Green-függvények

Bevezetés távvezetékek



- az egyik hossz változás
- a másik statikus tér

Elosztott paraméterű hálózat-ként történő leírás.



Δz hosszúságú szakasz modellje

Szinuszos állandósult állapot

$$-\frac{\partial U}{\partial z} = R' \cdot I + j\omega L' I \qquad u(z,t) = \mathrm{Re}\{ U(z) e^{j\omega t} \}$$

$$\frac{\partial I}{\partial z} = G' U + j\omega C' \cdot U + K'(z)$$

pl. I_a áramforrás a $z = z_0$ helyen $K'(z) = I_a \delta(z - z_0)$

1. egy z szerinti deriváltja $\frac{\partial I}{\partial z}$ helyen a 2 egyenlet

$$\frac{d^2 U}{dz^2} + k^2 (U(z)) = -a K'(z)$$

$\boxed{a = R' + j\omega L'}$ $k^2 = -(R' + j\omega L')(G' + j\omega C')$

+ perem felt.

Def: Minden peremérték-feladathoz definiálhatunk egy Green fv.-t amely a köv tulajdonságokkal rendelkezik:

- 2 független változója van
- kielégíti az eredeti PEF feltételeit
- kiel. a köv diffegyenletet:

$$g(z|z')$$

$$\frac{d^2 g(z|z')}{dz^2} + k^2 g(z|z') = -\delta(z-z')$$

+ peremfeltételek (az eredeti PEF-hez)

$g(z|z')$ -> bármelyik z' esetén kielégíti azt amit az eredeti diffegyenlet elégít ki

Eredeti peremfeltételek:

Példák PEF-re és az ehhez tartozó Green fv.-ek

távvezetékek	Green-fv.-k.
$\frac{d^2U}{dz^2} + k^2(U(z)) = -aK(z)$	$\frac{d^2 g(z\|z')}{dz^2} + k^2 g(z\|z') = -\delta(z-z')$

PEF0

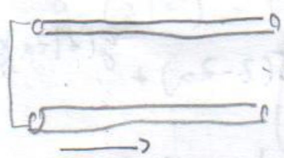
$z \to \infty \quad \frac{dU}{dz} = -jkU \qquad z \to \infty \quad \frac{dg(z|z')}{dz} = -jkg(z|z')$

$z \to -\infty \quad \frac{dU}{dz} = jkU \qquad$ minden állapotban igaz

ha $z \to \infty$, $z' \to$ véges

$u(z) = U^+ e^{-kjz} + U^- e^{+kzj} \qquad z \to -\infty \quad \frac{dg_0(z|z')}{dz} = jkg(z|z')$

PEF 1



$z = 0$

$z = 0 \to U(z) = 0$

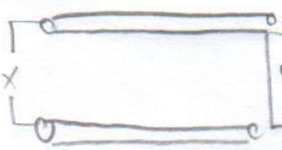
$z \to \infty \to \frac{dU}{dz} = -jkU$

homogén és Dirichlet

$z = 0 \quad g(z|z') = 0$

$z \to \infty \quad \frac{dg_1(z|z')}{dz} = -jkg_1(z|z')$

PEF2



$\frac{dU}{dz} = (R' + j\omega L')I \qquad I = -\frac{1}{R' + j\omega L'}\frac{dU}{dz}$

$z = 0 \quad \frac{dU}{dz} = 0$

$z \to \infty \quad \frac{dU}{dz} = -jkU$

$z = 0 \Rightarrow dg_2(z|z')$ Homogén Neuman

$z \to \infty \Rightarrow \frac{dg_2(z|z')}{dz} = jkg(z|z')$

Mire jó a Green-fv?

$$\frac{d^2U}{dz} + k^2 U = -a k \qquad a = R' + j\omega L' \qquad / \cdot g(z|z')$$

+perem feltételek azonosak

$$\frac{d^2 g(z|z')}{dz^2} + k^2 g(z|z') = -\delta(z-z') \qquad /U(z)$$

$$g(z|z')\frac{d^2U(z)}{dz^2} + k^2 g(z|z')U(z) = -a\, g(z|z')K(z)$$

$$U(z)\frac{d^2g(z|z')}{dz^2} + k^2 U(z) g(z|z') = -U(z)\delta(z-z')$$

$$g(z|z')\frac{d^2U(z)}{dz^2} - U(z)\frac{d^2 g(z|z')}{dz^2} = -a\, g(z|z')K(z) + U(z)\delta(z-z') \Big/ \int_{z_1}^{z_2}$$

$$\int_{z_1}^{z_2} g(z|z')\frac{d^2U(z)}{dz^2} - U(z)\frac{d^2 g(z|z')}{dz}\, dz = -a\int_{z_1}^{z_2} g(z|z')K(z)dz + \int_{z_1}^{z_1} U(z)\delta(z-z')dz$$

$$\underbrace{\left[g(z|z')\frac{dU(z)}{dz} - U(z)\frac{dg(z|z')}{dz}\right]_{z_1}^{z_2}}_{=0 \text{ a perem ért. feladat miatt}} = -a\int_{z_1}^{z_2} g(z|z')K(z)dz + U(z')$$

$$U(z') = a\int_{z_1}^{z_2} g(z|z')K(z)dz$$

Lemma

$$g(z|z') = g(z'|z)$$

$$z' = z_a$$

$$\frac{d^2 g(z|z_a)}{dz^2} + k^2 g(z|z_a) = -\delta(z-z_a) \qquad |\, g(z|z_b)$$

$z' = z_b$

$$\frac{d^2(z|z_b)}{dz^2} + k^2 g(z|z_b) = -\delta(z-z_b) \qquad |\, g(z|z_a)$$

$$g(z|z_b)\cdot\frac{d^2g(z|z_a)}{dz^2} - g(z|z_a)\frac{d^2(z|z_b)}{dz^2} = -g(z|z_b)\delta(z-z_a) + g(z|z_a)\delta(z-z_b)$$

$$\int_{z_1}^{z_2} dz \quad \underbrace{\left[g(z|z_b)\frac{dg(z|z_a)}{dz} - g(z|z_a)\frac{dg(z|z_b)}{dz}\right]_{z_1}^{z_2}}_{=0 \text{ peremfelt. miatt}} = -g(z_a|z_b) + g(z_b|z_a)$$

$g(z_a|z_b) = g(z_b|z_a)$

$$u(z) = a\int_{z_1}^{z_2} g(z|z')K(z')\,dz'$$

Bármilyen $K(z')$ gerj.-re a peremérték feladat megoldását egy integrállal tudnám helyettesíteni.

Tetszőleges gerjesztésre megtudjuk adni a megoldást egy integrál alapján.

$\Downarrow$

$$y(t) = \int_{-\infty}^{\infty} \underset{\substack{\downarrow \\ h(t|\tau)}}{h(t-\tau)}\, u(\tau)\, d\tau \Big\} \text{ impulzusválaszhoz hasonló.}$$

Green fv.-ek meghatározása

PEFC

$$g_0(z|z') = \begin{cases} A e^{-jkz} & z > z' \\ B e^{jkz} & z < z' \end{cases}$$

$$\frac{dg_0(z|z')}{dz} = -jkAe^{-jkz} \quad z > z'$$

$$\frac{dg_0(z|z')}{dz} = -jkBe^{jkz} \quad z' > z$$

$$\frac{d^2 g_0(z|z')}{dz^2} + \underbrace{k^2 g_0(z|z')}_{\emptyset} = -\delta(z|z') \Big/ \int_{z'-\varepsilon}^{z'+\varepsilon}$$

$$\frac{dg(z'+\varepsilon|z')}{dz} - \frac{dg(z'-\varepsilon|z)}{dz} = -1$$

$$-jkAe^{-jkz'} + jkBe^{jkz'} = -1$$

$$A(e^{-jkz'}) = B(e^{jkz'})$$

A $g_0(z|z')$ folytonos a $z = z'$

$$g_0(z|z') = \begin{cases} \frac{1}{2jk} e^{-jk(z-z')} & z' < z \\ \frac{1}{2jk} e^{jk(z-z')} & z' > 0 \end{cases}$$

13. EA

0 Adott egy PEF

$$\frac{d^2u}{dz}+k^2u=-a\,K(z) \quad + \text{ peremfeltételek a } z_1 \text{ és } z_2 \text{ pontokban}$$

1 Definiálok egy Green fv.-t

$$\frac{d^2g(z|z')}{dz^2}+k^2g(z|z')=-\delta(z-z')$$

+ peremfeltételek: megegyeznek az u-ra vonatkozó peremfelt.-el

$(z_1<z'<z_2)$

$\to g(z|z')$

$$u(z)=a\int_{z_1}^{z_2} g(z|z')\,K(z')\,dz'$$

Konvolúció:

$$y(t)=\int_{-\infty}^{\infty} h(t-\tau)\,u(\tau)\,d\tau$$

$\hat{} \; h(t|\tau)$ mások a határok is

$$g_0(z|z')=\begin{cases}\frac{1}{j2k}e^{-jk(z-z')} & z'<z\\ \frac{1}{j2k}e^{jk(z-z')} & z'>z\end{cases}$$

2

$$g_1(z|z')=\begin{cases}Ae^{-jkz} & z>z'\\ B\sin kz & 0<z<z'\end{cases}$$

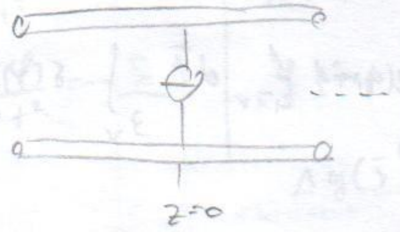
$g_1(z|z')$ folytonos a $z=z'$ helyen $\quad Ae^{-jkz'}=B\sin kz'$

$$\int_{z'-\varepsilon}^{z'+\varepsilon}\frac{d^2g_1(z|z')}{dz^2}dz+\int_{k-\varepsilon}^{k+\varepsilon}k^2g_1(z|z')dz=\int_{z'-\varepsilon}^{z'+\varepsilon}-\delta(z-z')dz$$

$$\left[\frac{dg_1(z|z')}{dz}\right]_{z'-k}^{z'+k}=-1 \longrightarrow -A(jk)e^{-jkz'}+Bk\cos kz'=-1$$

$\Rightarrow A, B$

3 $g_2(z|z') = \begin{cases} \frac{1}{jk} e^{-jkz} \cos kz' & z > z' \\ \frac{1}{jk} e^{-jkz'} \cos kz & 0 < z < z' \end{cases}$



$$g_0(z|0) = \begin{cases} \frac{1}{2jk} e^{-jkz} & z > 0 \\ \frac{1}{2jk} e^{jkz} & z < 0 \end{cases}$$

$$g(z|z') = \begin{cases} \frac{1}{2jk} e^{-jk(z-z')} \end{cases}$$

Pl.

$\Delta \varphi = -\frac{\rho}{\varepsilon}$ $\qquad \varphi(\bar r) = \frac{1}{\varepsilon} \int_V \underbrace{\frac{1}{4\pi|\bar r - \bar r'|}}_{g(\bar r|\bar r')} \rho(\bar r') dV'$

szabad tér

$\lim_{r \to \infty} \varphi = 0$

Elektrosztatika Green fv.e szabad térben

$\Delta \varphi = -\frac{\rho}{\varepsilon}$ $\qquad \Delta g(\bar r|\bar r') = -\delta(\bar r - \bar r')$ $\qquad \Rightarrow g(\bar r|\bar r') = \frac{1}{4\pi(\bar r - \bar r')}$

$\lim_{r \to \infty} \varphi = 0$ $\qquad \lim_{r \to \infty} g(\bar r|\bar r') = 0$

$\Delta g(\bar r|\bar r') = -\delta(\bar r - \bar r')$

$\lim_{r \to \infty} g(r|\bar r) = 0$

1, Peremfeltételek?

igen

2, Diffegyenlet $\bar r \neq \bar r'$ helyen?

$g(\bar r|0) = \frac{1}{4\pi r}$ Gömbi koordinátákban

$g(\bar r|\bar r') = \frac{1}{4\pi|\bar r - \bar r'|}$

$\mathrm{grad}\, g(\bar r|0) = \mathrm{grad} \frac{1}{4\pi r} = -\frac{1}{4\pi r^2} \bar e_r$

$\mathrm{div}\, \mathrm{grad}\, g(\bar r|0) = \mathrm{div} -\frac{1}{4\pi r^2} \bar e_r =$

$= \frac{1}{r^2 \sin\vartheta} \frac{\partial}{\partial r} r^2 \sin\vartheta \left(\frac{-1}{4\pi r^2}\right) = \emptyset$ $\quad r \neq 0$ helyen

3/ Diff egyenlet $\bar{r}=\bar{r}'$ helyen

$g(\bar{r}|0)$ $\quad \bar{r}=0$ helyen

$$\operatorname{div}\operatorname{grad}\frac{1}{4\pi r}=-\delta(r)$$

$$\int_V \operatorname{div}\operatorname{grad}\frac{1}{4\pi r}\,dV=\oint_A \operatorname{grad}\frac{1}{4\pi r}\,d\bar{A}=\int_V -\delta(r)\,dV=-1$$

$V\in\bar{r}=\emptyset$

$$\oint_A -\frac{1}{4\pi r^2}\,\bar{e}_r\,d\bar{A}\overset{?}{=}-1$$

"A" felület rázsugorítása az $r=0$ pontra

$$\oint_A -\frac{1}{4\pi r^2}\,\bar{e}_r\,d\bar{A}=\lim_{\substack{r\to 0\\ A\to}} -\frac{1}{4\pi r^2}\cdot 4\pi r^2=-1$$

4/ $g(\bar{r}|0)\Rightarrow g(\bar{r}|\bar{r}')$

$$\frac{1}{4\pi r}\Rightarrow\frac{1}{4\pi|\bar{r}-\bar{r}'|}$$

Stacionárius áramok mágneses tere

Descartes-koordinátákban

$$\Delta\bar{A}=-\mu\bar{J}\qquad \Delta A_x=-\mu J_x$$

$$\lim_{r\to\infty}\bar{A}=\emptyset\qquad \Delta A_y=-\mu J_y$$

$$\Delta A_z=-\mu J_z$$

$$A_x(\bar{r})=\mu\int_V\frac{1}{4\pi|\bar{r}-\bar{r}'|}J_x(\bar{r}')\,dV'$$

$$A_y(\bar{r})=\mu\int_V\frac{1}{4\pi|\bar{r}-\bar{r}'|}J_y(\bar{r}')\,dV'$$

$\vdots$

$$\bar{A}(\bar{r})=\mu\int_V\frac{1}{4\pi|\bar{r}-\bar{r}'|}\bar{J}(\bar{r}')\,dV'$$

Hullámegjenlet szinuszos állandósult állapotban, szabad térben

$$\Delta \bar{A} - \mu\varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} = -\mu \bar{J}$$

$$\Delta \varphi - \mu\varepsilon \frac{\partial^2 \varphi}{\partial t^2} = -\frac{\varrho}{\varepsilon}$$

Szinuszos állandósult állapot

$$\bar{A}(\bar{r},t) = \mathrm{Re}\{\bar{A}(\bar{r})\, e^{j\omega t}\}$$

$$\varphi(\bar{r},t) = \mathrm{Re}\{\varphi(\bar{r})\, e^{j\omega t}\}$$

$$\Rightarrow \quad \Delta \bar{A} + \omega^2\mu\varepsilon \bar{A} = -\mu \bar{J}$$

$$\Delta \varphi + \omega^2 \mu\varepsilon \varphi = \frac{-\varrho}{\varepsilon}$$

$$\Delta g(\bar{r}|\bar{r}') + k^2 g(\bar{r}|\bar{r}') = -\delta(\bar{r}-\bar{r}')$$

+ sugárzási feltételek

$$\lim_{r\to\infty} r\left(\frac{dg(\bar{r}|\bar{r}')}{dr} + jk\, g(\bar{r}|\bar{r}')\right) = 0$$

$$g(\bar{r}|\bar{r}') = \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|}$$

1, Peremfeltételek?

$$g(\bar{r}|0) = \frac{e^{-jkr}}{4\pi r}$$

$$\frac{dg(\bar{r}|0)}{dr} = -jk\frac{e^{-jkr}}{4\pi r} - \frac{e^{-jkr}}{4\pi r^2}$$

$$\lim_{r\to\infty} r\left(-jk\frac{e^{-jkr}}{4\pi r} - \frac{e^{-jkr}}{4\pi r^2} + jk\frac{e^{-jkr}}{4\pi r}\right) = 0$$

3, Diff egyenlet $\bar{r} = \bar{r}'$

$$\mathrm{div\,grad}\, g(\bar{r}|0) + k^2 g(\bar{r}|0) = -\delta(\bar{r})$$

2, Diff egyenlet $\bar{r} \neq \bar{r}'$

$$g(\bar{r}|0) = \frac{e^{-jkr}}{4\pi r}$$

$$\mathrm{grad}\frac{e^{-jkr}}{4\pi r} = \left[\frac{e^{-jkr}}{4\pi r^2} - jk\frac{e^{-jkr}}{4\pi r}\right]\bar{e}_r$$

$$\mathrm{div\,grad}\frac{e^{-jkr}}{4\pi r} = \frac{1}{r^2 \sin\vartheta}\frac{\partial}{\partial r} r^2 \sin\vartheta\left(-\frac{e^{-jkr}}{4\pi r^2} - jk\frac{e^{-jkr}}{4\pi r}\right) = -k^2\frac{e^{-jkr}}{4\pi r}$$

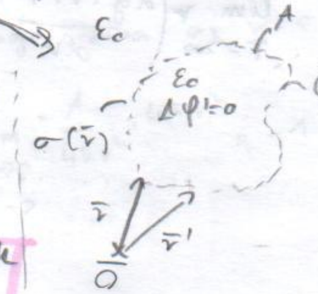
14.EA | Elektrosztat. peremérték-feladatok megoldása integrálegyenletekkel

- elektródák (σ)
- homogén szigetelés (ε)
- tértöltés ρ ($\rho=0$)

I, Alapelv

$$\left.\begin{array}{l}\Delta\varphi=0\\ \varphi(\infty)=0\end{array}\right\} \to \text{egyértelműen megoldható}$$

helyettesíthető egy ekvivalens képpel ami egyszerűbben kezelhetővé teszi a feladatot



$$\left.\begin{array}{l}\Delta\varphi'=0\\ \varphi'(\infty)=0\end{array}\right\} \varphi'=\varphi \text{ kívül}$$

$$\varphi'(r)=\oint_A \underbrace{\frac{1}{4\pi\varepsilon_0|\bar r-\bar r'|}}_{g(\bar r,\bar r')}\sigma(\bar r')\,dA'$$

Elektrosztatika integrál egyenlete

$$\boxed{\Phi_1=\oint_A g(\bar r,\bar r')\sigma(\bar r')\,dA' \qquad \bar r\in A,\ \bar r\in V}$$

(elsőfajú Fredholm)

$$Q_1=\oint_A \sigma_1(r)\,dA \longrightarrow \left(C=\frac{Q}{\Phi_1}\dots\right)$$

II, Numerikus megoldás (Momentum módszer)

1, $\sigma(\bar r)=\sum_{i=1}^{N} c_i v_i(\bar r)$ $\quad v_i(\bar r)$: i-dik bázisfv.

v_i és v_j független $i\neq j$

$[c_i=?\quad i=1,2\dots N]$

2, Tesztelés (pont-kollokáció)

$$\Phi_1=\oint g(\bar r_j,\bar r')\left(\sum_{i=1}^{N} c_i v_i(\bar r')\right)dA \qquad \bar r_j;\ j=1,2\dots N$$

3, lin. algebrai egyenlet rdsz.

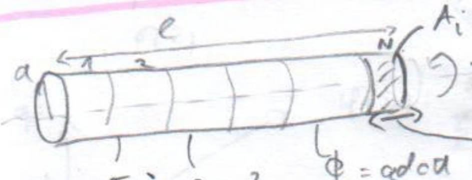
$$\phi_j = \sum_{i=1}^{N} c_i \oint_A g(\bar{r}_j, \bar{r}') V_i(\bar{r}') dA' \qquad j = 1, 2 \ldots N$$

$$S_{ji} = \oint g(\bar{r}_j, \bar{r}') V_i(\bar{r}') dA$$

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix} = \underline{\underline{S}} \cdot \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}$$

$$\hat{c}_i = \ldots \qquad i = 1, 2 \ldots N$$

1. Példa: Henger elektróda



$l >> a$

z forgásszim.

$\Delta l = \frac{l}{N}$ felosztjuk N részre

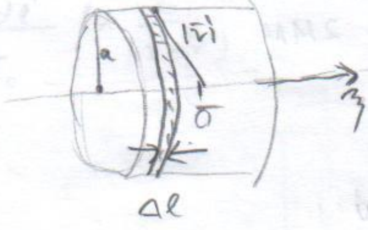
$$V_i(\bar{r}) = \begin{cases} 1 & \bar{r} \in A_i \\ 0 & \bar{r} \notin A_i \end{cases}$$

$$S_{ii} = \oint_A g(\bar{r}_i, \bar{r}') \, 1 \, dA' = \oint_A \frac{1}{4\pi\varepsilon_0 |-\bar{r}'|} \underbrace{1 \cdot dA'}_{d\xi 2a\pi}$$

$\bar{r}_i = \bar{0}$, $|-\bar{r}'| = \sqrt{\xi^2 + a^2}$

$S_{ji} = ?$

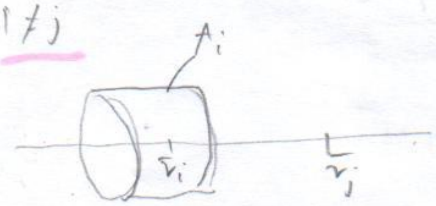
$j = i$



$$= \int_{\xi = -\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} \frac{1}{4\pi\varepsilon_0 \sqrt{\xi^2 + a^2}} 2a\pi \, d\xi =$$

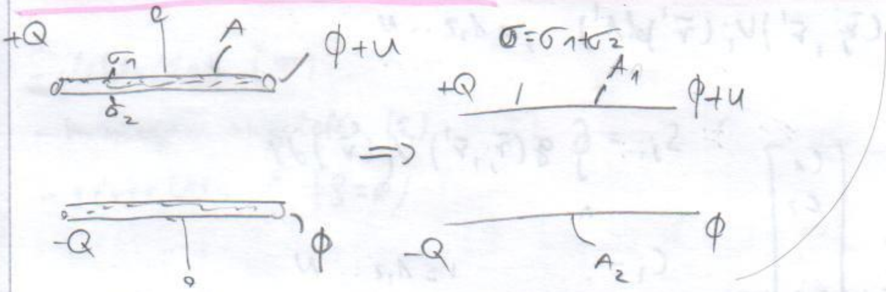
$$\frac{a}{2\varepsilon_0} \ln \frac{\frac{\Delta l}{2} + \sqrt{a^2 + \left(\frac{\Delta l}{2}\right)^2}}{-\frac{\Delta l}{2} + \sqrt{a^2 + \left(\frac{\Delta l}{2}\right)^2}}$$

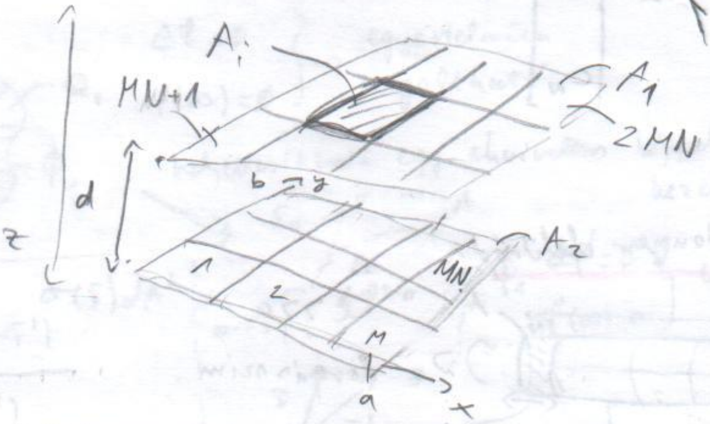
$i \neq j$



$$S_{ji} = \oint_{A_i} g(\bar{r}_j, \bar{r}') \, 1 \cdot dA_i' = \oint_{A_i} \frac{1}{4\pi\varepsilon_0 \underbrace{|\bar{r}_j - \bar{r}'|}_{\approx |\bar{r}_j - \bar{r}_i|}} dA_i' = \frac{2a\pi\Delta l}{4\pi\varepsilon_0 |\bar{r}_j - \bar{r}_i|}$$

2. Példa : Síkkondenzátor





$$V_i(\bar{r}) = \begin{cases} 1 & r \in A_i \\ 0 & r \notin A_i \end{cases}$$

$$\phi + U = \sum_{i=1}^{2MN} \left(\oint_{A_i} g(\bar{r}_j, \bar{r}') V_i(\bar{r}') dA' \right) c_i$$

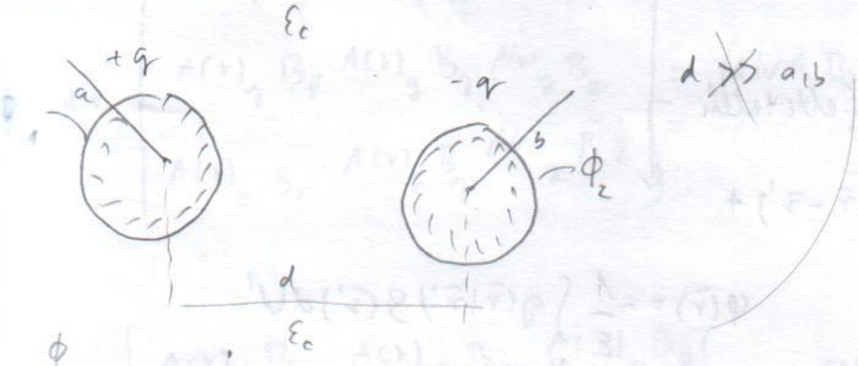
$j = MN+1 .. 2MN$

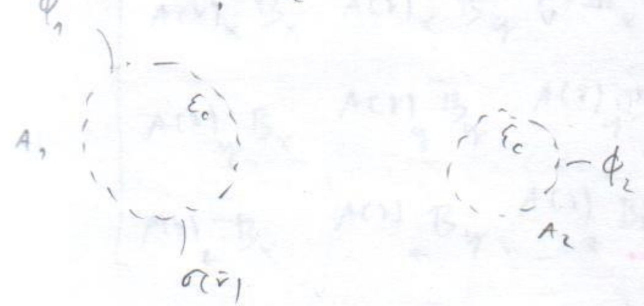
$$\phi = \sum_{i=0}^{2MN} \left(\oint g(\bar{r}_j, \bar{r}') V_i(\bar{r}') dA' \right) c_i$$

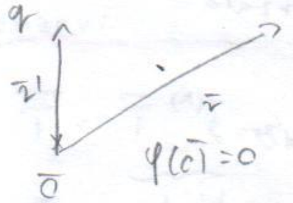
$j = 1, 2 .. MN$

$$\left. \begin{aligned} Q &= \sum_{i=MN+1}^{2MN} c_i \underbrace{\Delta A_i}_{\frac{ab}{MN}} \\ -Q &= \sum_{i=1}^{MN} c_i \Delta A_i \end{aligned} \right\} 2MN+2 \text{ egyenlet}$$

3, Vezeték pár







$$\varphi(\bar{r}) = \frac{q}{2\pi\varepsilon_0} \frac{|\bar{r}'|}{|\bar{r}-\bar{r}'|} \qquad \ln\frac{|\bar{r}'|}{r_0} + \ln\frac{r_0}{|\bar{r}-\bar{r}'|}$$

$\varphi(\bar{O}) = 0$

$$\varphi(\bar{r}) = \oint_l \frac{\sigma(\bar{r}')\,dl'}{2\pi\varepsilon_0} \ln\frac{|\bar{r}'|}{|\bar{r}-\bar{r}'|} = \underbrace{\oint_l \frac{\sigma(\bar{r}')\,dl'}{2\pi\varepsilon_0} \ln\frac{|\bar{r}'|}{r_0}}_{K} +$$

$$\oint \frac{\sigma(\bar{r}')\,dl'}{2\pi\varepsilon_0} \ln\frac{r_0}{|\bar{r}-\bar{r}'|}$$

15. EA

$$\Delta\varphi = \frac{-\varrho}{\varepsilon} + \text{peremfeltételek}$$

$$\Delta g(\bar{r}|\bar{r}') = -\delta(\bar{r}-\bar{r}') +$$

$$\Downarrow$$

$$g(\bar{r}|\bar{r}') \qquad \varphi(\bar{r}) = -\frac{1}{\varepsilon}\int_V g(\bar{r}|\bar{r}')\varrho(\bar{r}')dV'$$

Diadikus Green-függvények

Diád (bevezetés): két vektor diadikus szorzata

$$\bar{\bar{D}} = \bar{A}\bar{B} \qquad \hat{\bar{\bar{D}}} = \bar{B}\bar{A}$$

$\hat{}$ transzponált

Descartes-koordinátákban:

$$\bar{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

$$\bar{A}\bar{B} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \begin{bmatrix} B_x & B_y & B_z \end{bmatrix} = \begin{bmatrix} A_xB_x & A_xB_y & A_xB_z \\ A_yB_x & A_yB_y & A_yB_z \\ A_zB_x & A_zB_y & A_zB_z \end{bmatrix}$$

Diádokkal végzett műveletek:

$$\bar{\bar{D}}\cdot\bar{C} = \bar{A}(\bar{B}\bar{C}) \qquad \operatorname{div}\bar{\bar{D}} = \operatorname{div}\bar{A}(\bar{r})\bar{B}$$

$$\bar{C}\cdot\bar{\bar{D}} = (\bar{C}\bar{A})\bar{B} \qquad \operatorname{rot}\bar{\bar{D}} = \operatorname{rot}\bar{A}(\bar{r})\bar{B}$$

$$\bar{\bar{D}}\times\bar{C} = \bar{A}(\bar{B}\times\bar{C})$$

$$\bar{\bar{D}} = \bar{\bar{D}}(\bar{r}) = \bar{A}(\bar{r})\bar{B}$$

$$\mathrm{div}\begin{bmatrix} A(r)_x B_x & A(r)_x B_y & A(r)_x B_z \\ A(r)_y B_x & A(r)_y B_y & A(r)_y B_z \\ A(r)_z B_x & A(r)_z B_y & A(r)_z B_z \end{bmatrix} = (\mathrm{div} A\, B_x \;\; \mathrm{div} A\, B_y \;\; \mathrm{div} A\, B_z)$$

$$\mathrm{rot}\begin{bmatrix} A(r)_x B_x & A(r)_x B_y & A(r)_x B_z \\ A(r)_y B_x & A(r)_y B_y & A(r)_y B_z \\ A(r)_z B_x & A(r)_z B_y & A(r)_z B_z \end{bmatrix} = \begin{matrix} \mathrm{rot}(A)_x B_x & \mathrm{rot}(A)_x B_y & \mathrm{rot}(A)_x B_z \\ \mathrm{rot}(A)_y B_x & \mathrm{rot}(A)_y B_y & \mathrm{rot}(A)_y B_z \\ \mathrm{rot}(A)_z B_x & \mathrm{rot}(A)_z B_y & \mathrm{rot}(A)_z B_z \end{matrix}$$

Tenzor:

$$\underbrace{\overline{\overline{T}} = \overline{T}^{(1)}\bar{e}_1}_{\text{diád}} + \overline{T}^{(2)}\bar{e}_2 + \overline{T}^{(3)}\bar{e}_3$$

$$\begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix} = \begin{bmatrix} T_{xx} \\ T_{yx} \\ T_{zx} \end{bmatrix}[1\;0\;0] + \begin{bmatrix} T_{xy} \\ T_{yy} \\ T_{zy} \end{bmatrix}[0\;1\;0] + \begin{bmatrix} T_{xz} \\ T_{yz} \\ T_{zz} \end{bmatrix}[0\;0\;1]$$

$$\mathrm{rot}\,\overline{\overline{T}} = \mathrm{rot}\,\overline{T}^{(1)}\bar{e}_1 + \mathrm{rot}\,\overline{T}^{(2)}\bar{e}_2 + \mathrm{rot}\,\overline{T}^{(3)}\bar{e}_3$$

$$\mathrm{div}\,\overline{\overline{T}} = \mathrm{div}\,\overline{T}^{(1)}\bar{e}_1 + \mathrm{div}\,\overline{T}^{(2)}\bar{e}_2 + \mathrm{div}\,\overline{T}^{(3)}\bar{e}_3$$

def: egység diád:

$$\overline{\overline{I}} = \bar{e}_1\bar{e}_1 + \bar{e}_2\bar{e}_2 + \bar{e}_3\bar{e}_3$$

$$\overline{\overline{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{A}(\bar{r}) = \mu \int \overline{\overline{G}}(\bar{r}|\bar{r}')\,\bar{J}(\bar{r}')\,dV'$$

Descartes:

$$\overline{\overline{G}}(\bar{r}|\bar{r}') = \begin{bmatrix} g_0(\bar{r}|\bar{r}') & 0 & 0 \\ 0 & g_0(\bar{r}|\bar{r}') & 0 \\ 0 & 0 & g_0(\bar{r}|\bar{r}') \end{bmatrix} \qquad g_0(\bar{r}|\bar{r}') = \frac{1}{4\pi|\bar{r}-\bar{r}'|}$$

$$\bar{B}(\bar{r}) = \mathrm{rot}(\bar{A}) = \mu \int \mathrm{rot}\, \bar{\bar{G}}(\bar{r}|\bar{r}')\, \bar{J}(\bar{r})\, dV'$$

$$\bar{B}(r) = \mu \int_V \left[\mathrm{rot}\, G(\bar{r}|\bar{r}')\right]^x J_x + \left[\mathrm{rot}\, G(\bar{r}|\bar{r}')\right]^y J_y + \left[\mathrm{rot}\, G(\bar{r}|\bar{r}')\right]^z J_z\, dV$$

$$J_x = \underline{e}_x \cdot \bar{J}$$

Példa

$$\Delta \bar{A} + k^2 \bar{A} = -\mu \bar{J} \qquad k^2 = \omega^2 \mu \varepsilon$$

$$\Delta g(\bar{r}|\bar{r}') + k^2 g(\bar{r}|\bar{r}') = -\delta(\bar{r}-\bar{r}') \Rightarrow g(\bar{r}|\bar{r}') = \frac{e^{-jk|\bar{r}-\bar{r}'|}}{4\pi |\bar{r}-\bar{r}'|}$$

$$\bar{A}(\bar{r}) = \mu \int_V \bar{\bar{G}}(\bar{r}|\bar{r}')\, \bar{J}(\bar{r}')\, dV$$

$$\bar{\bar{G}} = \begin{bmatrix} g & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & g \end{bmatrix}$$

Példa

$$\Delta \bar{A} = -\mu \bar{J} + \text{perem felt.}$$

$$\Delta \bar{\bar{G}}(\bar{r}|\bar{r}') = -\delta(\bar{r}-\bar{r}')\, \bar{\bar{I}} +$$

$$\Delta \bar{\bar{G}}(\bar{r}|\bar{r}')\, \bar{e}_1 = \delta(\bar{r}-\bar{r}')\, \bar{\bar{I}}\, \bar{e}_1$$

$$\Delta \bar{A} - \mu \varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} = -\mu \bar{J} + \text{perem felt.}$$

$$\Delta \bar{\bar{G}}(\bar{r}|\bar{r}') - \mu \varepsilon \frac{\partial^2 \bar{\bar{G}}(\bar{r}|\bar{r}')}{\partial t^2} = -\delta(\bar{r}-\bar{r}')\, \bar{\bar{I}}$$

Időbeli véges-differencia módszer, Finite Difference Time Domain FDTM

16. EA

Visszatekintés

① Fizikai való → egyértelműen megoldható peremérték-feladatok megoldása

② PEF numerikus megoldása
- FEM térbeli diszkretizáció – elektrodinamika minden területen hasonló
  - jól kezeli a geometriát
  - nagy geometriáknál nem jók.
- Integrál egyenletek módszere (Method of moment módszerével) → plusz forrással új egyenlet – Green függvények – diszkretizálandó tart. a forrás tartományára koncentr.
  - szabad térben jó
  - bonyolult. geometriánál nem jó
- FDTD – Maxwell egyenletek diszkretizációja – hullámegyenletekhez jó

FDTD

- lineáris anyagok
- időben változó megoldás

$$\mathrm{rot}\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \quad / \cdot \mathrm{div}$$

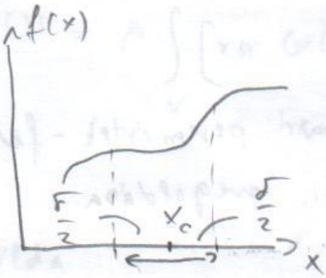
$$\mathrm{div}\,\mathrm{rot}\,\bar{H} = 0 = \underbrace{\mathrm{div}\,\bar{J}}_{-\frac{\partial \rho}{\partial t}} + \frac{\partial}{\partial t}\mathrm{div}\,D \rightarrow \mathrm{div}\,\bar{D} + \text{időtől független tag} = \rho + \text{időtől független tag}$$

$$\mathrm{rot}\,\bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad / \mathrm{div}$$

$$\mathrm{div}\,\mathrm{rot}\,\bar{E} = 0 = -\frac{\partial}{\partial t}\mathrm{div}\,\bar{B} \rightarrow \mathrm{div}\,\bar{B} + \text{időtől független} = 0$$

FDTD: időtől függő megoldásár alkalmaz, csak az első-kettő Maxwell egyenletre koncentrál

Centrális másodrendű véges differencia



$$f\left(x_0+\frac{\delta}{2}\right)=f(x_0)+\frac{\delta}{2}f'(x_0)+\left(\frac{\delta}{2}\right)^2\frac{1}{2!}f''(x_0)+\frac{1}{3!}\left(\frac{\delta}{2}\right)^3 f'''(x_0)$$

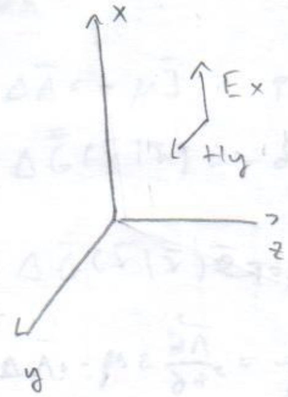
$$f\left(x_0-\frac{\delta}{2}\right)=f(x_0)-\frac{\delta}{2}f'(x_0)+\left(\frac{\delta}{2}\right)^2\frac{1}{2!}f''(x_0)-\frac{1}{3!}\left(\frac{\delta}{2}\right)^3 f'''(x_0)$$

$$f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)=\delta f'(x_0)+\frac{1}{24}\delta^3 f'''(x_0)+\ldots$$

$$\left.\frac{d f(x)}{dx}\right|_{x=x_0}=f'(x_0)\approx\frac{f\left(x_0+\frac{\delta}{2}\right)-f\left(x+\frac{\delta}{2}\right)}{\delta}+\underbrace{\frac{1}{24}\delta^2 f'''(x_0)+\ldots}$$

a hiba δ^2 szerint tűnik el

1D-s eset



$$\frac{\partial}{\partial x}=0,\ \frac{\partial}{\partial y}=0 \Rightarrow E_y=E_z=0$$

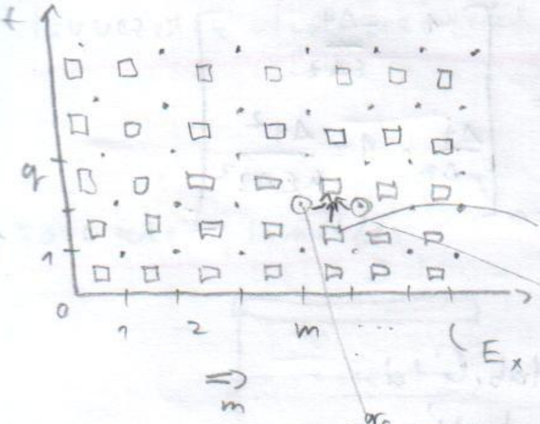
$$H_x=H_z=0$$

$$\operatorname{rot}\bar{H}=+\frac{\partial\bar{D}}{\partial t}=+\varepsilon\frac{\partial\bar{E}}{\partial t}$$

$$\operatorname{rot}\bar{H}=\begin{vmatrix}\bar{e}_x & \bar{e}_y & \bar{e}_z\\ 0 & 0 & \frac{\partial}{\partial z}\\ 0 & H_y & 0\end{vmatrix}=-\frac{\partial H_y}{\partial z}\bar{e}_x$$

$$\boxed{\begin{aligned}(2)\ -\frac{\partial H_y}{\partial z}&=\varepsilon\frac{\partial E_x}{\partial t}\\ (1)\ -\frac{\partial E_x}{\partial z}&=\mu\frac{\partial H_y}{\partial t}\end{aligned}}$$

duális rács



$q\Delta t \Rightarrow E_x^q[m]$

$H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2}\right]$

3 pont lin. kombinációjaként előállítható a 4. → 3 kezdőértékből az egész rács felírható → hullám terjedés

$E_x^q[m]$

(1) $$-\frac{E_x^q[m+1]-E_x^q[m]}{\Delta z}=\mu\left[m+\frac{1}{2}\right]\frac{H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right]-H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2}\right]}{\Delta t}$$

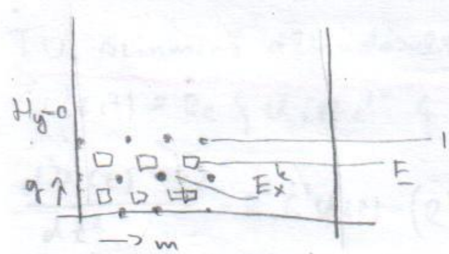
(2) $$\frac{H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right]-H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2}\right]}{\Delta z}=\varepsilon[m]\frac{E_x^{q+1}[m]-E_x^q[m]}{\Delta t}$$

$$H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right]=H_y^{q-\frac{1}{2}}\left[m+\frac{1}{2}\right]-\frac{\Delta t}{\mu\left[m+\frac{1}{2}\right]\Delta z}\left(E_x^q[m+1]-E_x^q[m]\right)$$

$$E_x^{q+1}[m]=E_x^q[m]-\frac{\Delta t}{\varepsilon[m]\Delta z}\left(H_y^{q+\frac{1}{2}}\left[m+\frac{1}{2}\right]-H_y^{q+\frac{1}{2}}\left[m-\frac{1}{2}\right]\right)$$

- Nem kell egyenletrendszert felírni
- nem kell a múltbeli értékeket tárolni → Mint az előrelépő Euler módszer.
- sok a numerikus hiba a végtelen felé → stabilitási gondok.

- fontos a rácsfelbontás



DI rendszer rendszeregyenlete:

$$\Rightarrow E_x^{k+1}=E_x^k-\frac{\Delta t}{\varepsilon\Delta z}\left(H_y^k-0\right) \qquad \Rightarrow H_y^{k+1}=\frac{\Delta t}{\mu\Delta z}E_x^k+\left(1+\frac{\Delta t^2}{\mu\varepsilon\Delta z^2}\right)H_y^k$$

$$H_y^{k+1}=H_y^k-\frac{\Delta t}{\mu\Delta z}\left(0-E_x^{k+1}\right)$$

$$\begin{bmatrix} E_x \\ H_y \end{bmatrix} = \underline{x} \qquad x[k+1] = \underline{\underline{A}}\,\underline{x}[k] \qquad \underline{\underline{A}} = \begin{bmatrix} 1 & \dfrac{-\Delta t}{\varepsilon \Delta z} \\ \dfrac{\Delta t}{\mu \Delta z} & 1 - \dfrac{\Delta t^2}{\mu \varepsilon \Delta z^2} \end{bmatrix}$$

$$\det(\lambda \underline{\underline{I}} - A) = 0$$

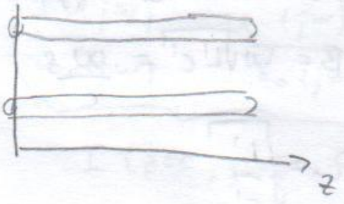
$$\lambda^2 + \left(\frac{\Delta t^2}{\mu \varepsilon \Delta z^2} - 2\right)\lambda + 1 = 0$$

$$\underbrace{\frac{c}{\sqrt{\varepsilon_r \mu_r}}}_{v} \cdot \frac{\Delta t}{\Delta z} \leq 2 \qquad \text{stabilitási krit.}$$

17. EA

Távvezeték tranziensek számítása - ált. időfv. gerj. figyelembe vétele

Távvezeték: ismétlés



$$-\frac{\partial u}{\partial z} = R'i + L'\frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z} = G'u + C'\frac{\partial u}{\partial t}$$

+ perem felt.

$$-\frac{\partial^2 u}{\partial z^2} = R'\frac{\partial i}{\partial z} + L'\frac{\partial}{\partial t}\frac{\partial i}{\partial z}$$

$$-\frac{\partial^2 u}{\partial t^2} = -R'\left(G'u + C'\frac{\partial u}{\partial t}\right) - L'\left(G'\frac{\partial u}{\partial t} + C'\frac{\partial^2 u}{\partial t^2}\right)$$

$$\frac{\partial^2 u}{\partial z^2} - R'G'u - (R'C' + L'G')\frac{\partial u}{\partial t} - L'C'\frac{\partial^2 u}{\partial t^2} = 0$$ homogén differenciálegyenlet, a gerj. a kezdeti ért. és a peremfelt.-ből adódik

$$-\frac{du}{dz} = R'i + L'\frac{di}{dt}$$

+ Perem felt.

$R' = 0 \quad G' = 0$ ideális táv. vez.

nagy frekvencián a táv. vez. veszteségmentes mert R', G' eltörpül $j\omega L'$, $\frac{1}{j\omega C}$ hez képest.

TV. szinuszos állandósult állapotban (ω adott)

$u(z,t) = \mathrm{Re}\{U(z)e^{j\omega t}\}$; $i(z,t) = \mathrm{Re}\{I(z)e^{j\omega t}\}$

$$\frac{d^2U(z)}{dz^2} = -R'G'U(z) - (R'C' + L'G')j\omega U(z) + L'C'(j\omega)^2 U(z)$$

$I(z) = -\frac{dU(z)}{dz}\cdot\frac{1}{R' + j\omega L'}$ + perem. felt.

$$u(z) = U^+ e^{-\gamma z} + U^- e^{\gamma z} \qquad \gamma^2 = (R' + j\omega L')(G' + j\omega C')$$

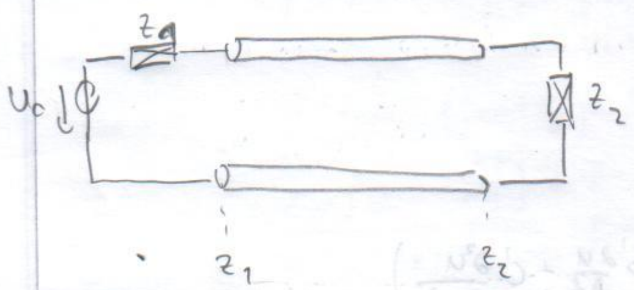
$$I(z) = \frac{U^+}{Z_0} e^{-\gamma z} - \frac{U^-}{Z_0} e^{\gamma z} \qquad Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

ideális esetben:

$$\gamma^2 = -\omega^2 L'C' = (j\beta)^2$$

$$\beta = \omega\sqrt{L'C'} = \frac{\omega}{c}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$



$$U(z=z_1) = U_0 - Z_1 I(z=z_1)$$

$$U(z=z_2) = I(z=z_2) Z_2$$

Tetszőleges időbeli változás

1, Stabilis rendszer

$$\frac{\partial^2 u}{\partial z^2} - R'G'u - (R'C' + L'G')\frac{\partial u}{\partial t} + L'C'\frac{\partial^2 u}{\partial t^2} = 0$$

$$U(z, j\omega) = \mathcal{F}\{u(z,t)\} = \int_{-\infty}^{\infty} u(z,t)\, e^{-j\omega t}\, dt$$

$$I(z, j\omega) = \mathcal{F}\{i(z,t)\}$$

$$\frac{\partial^2 U(z,j\omega)}{\partial z^2} - \underbrace{\left[R'G' + j\omega(R'C' - L'G') + (j\omega)^2 L'C'\right]}_{\gamma^2(j\omega)} U(z,j\omega) = 0$$

Gerj. véges stabilitás az átviteli fv. felírásának feltétele

$$U(z,j\omega) = U^+(j\omega)e^{-\gamma z} + U^-(j\omega)e^{\gamma z} \rightarrow -\frac{\partial U(z,j\omega)}{\partial z} = (R' + j\omega L')I(z,j\omega)$$

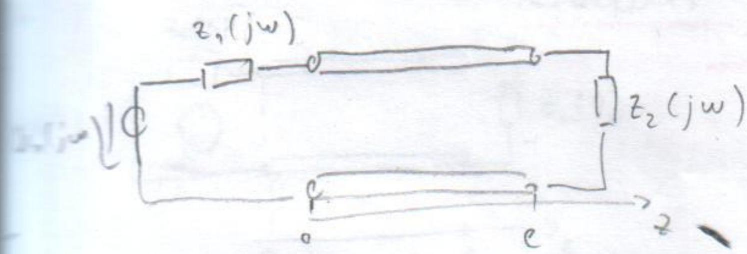
$$Z_0(j\omega) = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$I(z,j\omega) = \frac{U^+}{Z_0(j\omega)} e^{-\gamma(j\omega)z} - \frac{U^-(j\omega)}{Z_0(j\omega)} e^{+\gamma(j\omega)z}$$

+PEF

$$\gamma(j\omega) = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

Formálisan mint a szinusznál de itt ω-tól függő fv. is



$Z_1(j\omega), Z_2(j\omega)$ értelmezése: lezárás átviteli karakt.

$U_0(j\omega) = \mathcal{F}\{u_0(t)\}$

$$-U_0(j\omega) + \underbrace{\left(\frac{U^+(j\omega)}{Z_0(j\omega)} - \frac{U^-(j\omega)}{Z_0(j\omega)}\right)}_{I(z=0,\, j\omega)} Z_1(j\omega) + \underbrace{U^+(j\omega) + U^-(j\omega)}_{U(z=0,\, j\omega)} = 0$$

$$\underbrace{\left[\frac{U^+(j\omega)e^{-\gamma\ell}}{Z_0(j\omega)} - \frac{U^-(j\omega)e^{\gamma\ell}}{Z_0(j\omega)}\right]}_{I(z=\ell,\, j\omega)} Z_2(j\omega) = \underbrace{U^+(j\omega)e^{-\gamma\ell} + U^-(j\omega)e^{\gamma\ell}}_{U(z=\ell,\, j\omega)}$$

$$U^+(j\omega) = U_0(j\omega)\frac{Z_0(j\omega)}{Z_1(j\omega)+Z_0(j\omega)}\left(\frac{1}{1 - r_1(j\omega)r_2(j\omega)e^{-2\gamma\ell}}\right)$$

$$U^-(j\omega) = U^+(j\omega)\, r_2(j\omega)$$

$$r_1(j\omega) = \frac{Z_1(j\omega) - Z_0(j\omega)}{Z_1(j\omega) + Z_0(j\omega)}$$

$$r_2(j\omega) = \frac{Z_2(j\omega) - Z_0(j\omega)}{Z_2(j\omega) + Z_0(j\omega)}$$

$$U(z, j\omega) = U_0(j\omega)\frac{Z_0(j\omega)}{Z_1(j\omega)+Z_0(j\omega)} \cdot \frac{e^{-\gamma z} + r_2(j\omega)e^{-\gamma(2\ell - z)}}{1 - r_1(j\omega)r_2(j\omega)e^{-2\gamma\ell}}$$

$$I(z, j\omega) = U_0(j\omega)\frac{1}{Z_1(j\omega)+Z_0(j\omega)} \frac{e^{-\gamma z} - r_2(j\omega)e^{-(2\ell - z)\gamma}}{1 - r_1(j\omega)r_2(j\omega)e^{-2\gamma\ell}}$$

Fel lehet írni az U, I F. transzformáltját minden z pontra

18. EA Távvezeték tranziensek

$$-\frac{\partial u}{\partial z} = R'i + L'\frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial z} = G'u + C'\frac{\partial u}{\partial t}$$

+ Perem feltételek + kezdeti felt.

- veszteség mentes $R'=0$, $G'=0$
- bekapcsolási jelenség
  - $u(t,z)=0$ $t<0$
  - $i(t,z)=0$ $t<0$ $\Rightarrow$ $u(z,0)=\emptyset$, $i(z,0)=0$
  - perem felt. $\emptyset$ $t<0$

$$-\frac{\partial u}{\partial z} = L'\frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial z} = C'\frac{\partial u}{\partial t}$$

$u(z,0)=0$, $i(z,0)=0$
+ perem feltételek

Laplace $\Rightarrow$ $U(z,s) = \mathcal{L}\{u(z,t)\} = \int_{-0}^{\infty} u(z,t)e^{-st}\,dt$

$I(z,s) = \mathcal{L}\{i(z,t)\}$

$$-\frac{dU(z,s)}{dz} = sL'I(z,s)$$

$$-\frac{dI(z,s)}{dz} = sC'U(z,s)$$

$$\frac{d^2U(z,s)}{dz^2} - s^2L'C'U(z,s) = 0$$

$$I(z,s) = -\frac{1}{sL'}\frac{dU(z,s)}{dz} \text{ + perem felt.}$$

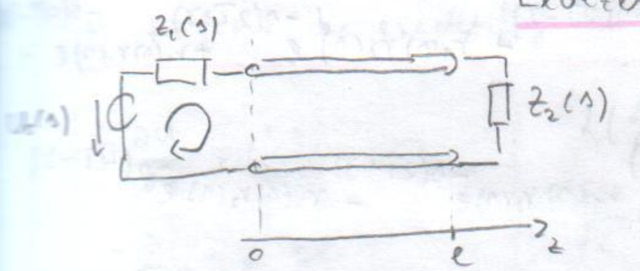
$$U(z,s) = U^+(s)e^{-\frac{s}{v}z} + U^-(s)e^{\frac{s}{v}z}$$

$v = \frac{1}{\sqrt{L'C'}}$ ha nem veszteséges lenne v s-től függene (leesengőjel, burkoló)

$$I(z,s) = \frac{U^+(s)}{Z_0(s)}e^{-\frac{s}{v}z} - \frac{U^-(s)}{Z_0(s)}e^{\frac{s}{v}z}$$

$Z_0 = \sqrt{\frac{L'}{C'}}$ s-től független (mivel veszteség mentes)

Levezetés



Perem felt.: $$-U_0(s) + Z_1(s) I(z=0,s) + U(z=0,s) = \emptyset$$
$$U(z=l,s) = Z_2(s) I(z=l,s)$$

$$-U_0(s) + \frac{Z_1(s)}{Z_0}\left(U^+(s) - U^-(s)\right) + U^+(s) + U^-(s) = 0$$

$$U^+(s) e^{-s\frac{l}{v}} + U^-(s) e^{s\frac{l}{v}} = \frac{Z_2(s)}{Z_0}\left(U^+(s) e^{-s\frac{l}{v}} - U^-(s) e^{s\frac{l}{v}}\right) \qquad \frac{l}{v} = T$$

ennyi idő alatt megy végig a jel a vezetéken

reflexiós tényező
$$r_2(s) = \frac{U^- e^{sT}}{U^+ e^{-sT}}$$ (nem azonos a szinuszos gerjesztéssel)

$$U^+(1 + r_2(s)) = \frac{Z_2(s)}{Z_0} U^+(1 - r_2(s))$$

$$\frac{1 + r_2(s)}{1 - r_2(s)} = \frac{Z_2(s)}{Z_0} \Rightarrow r_2(s) = \frac{Z_2(s) - Z_0}{Z_2(s) + Z_0}$$

$$-U_0(s) + \frac{Z_1(s)}{Z_0} U^+(s)\left(1 - r_2(s) e^{-2sT}\right) + U^+\left(1 + r_2 e^{-2sT}\right) = 0$$

Sor rendezés után

$$U^+(s) = U_0(s) \frac{Z_0}{Z_1(s) + Z_0} \frac{1}{1 - r_1(s) r_2(s) e^{-2sT}} \qquad r_1(s) = \frac{Z_1(s) - Z_0}{Z_1(s) + Z_0}$$

$$U^-(s) = U^+(s) e^{-2sT} r_2(s) = U_0(s) \frac{Z_0}{Z_1(s) + Z_0} \frac{r_2(s) e^{-2sT}}{1 - r_1(s) r_2(s) e^{-2sT}}$$

$$U(z,s) = U_0(s) \frac{Z_0}{Z_1(s) + Z_0}\left(\frac{e^{-s\tau}}{1 - r_1(s) r_2(s) e^{-2sT}} + \frac{r_2(s) e^{+s\tau} e^{-2sT}}{1 - r_1(s) r_2(s) e^{-2sT}}\right) \qquad \tau = \frac{z}{v}$$

$$I(z,s) = U_0(s) \frac{1}{Z_1(s) + Z_0}\left(\frac{e^{-s\tau}}{1 - r_1(s) r_2(s) e^{-2sT}} - \frac{r_2(s) e^{+s\tau} e^{-2sT}}{1 - r_1(s) r_2(s) e^{-2sT}}\right)$$

Mértani sor:

$$\frac{A}{1-q} = A + Aq + Aq^2 \dots \qquad q = r_1(s) r_2(s) e^{-2sT}$$

$$U(z,s) = U_0(s)\frac{Z_c}{Z_1(s)+Z_0}\left(e^{-s\tau} + r_2(s)e^{-s(2T-\tau)} + r_1(s)r_2(s)\,e^{-s(2T+\tau)} + r_1(s)r_2(s)\,e^{-s(4T-\tau)} + \dots\right)$$

$$I(z,s) = U_c(s)\frac{1}{Z_1(s)+Z_0}\left(e^{-s\tau} - r_2(s)e^{-s(2T-\tau)} + r_1(s)r_2(s)\,e^{-s(2T+\tau)} - r_1(s)r_2^2(s)\,e^{-s(4T-\tau)}\right)$$

$$U(z,s) = \cancel{U^0(s)} + \sum_{i=1}^{\infty} U_i^A(s)\,e^{-s(2(i-1)T-\tau)} + U_i^B(s)\,e^{-s(2iT-\tau)}$$

$$U_0(s) = U_0(s)\frac{Z_c}{Z_1(s)+Z_0}, \quad U_i^A(s) = U^0(s)\left(r_1(s)r_2(s)\right)^{i-1}, \quad U_i^B(s) = U^0(s)\,r_1^{(i-1)}(s)\,r_2^{i}(s)$$

$$u(z,t) = \sum_{i=1}^{\infty} u_i^A\left(t-2(i-1)T-\tau\right) + u_i^B\left(t-2iT+\tau\right)$$

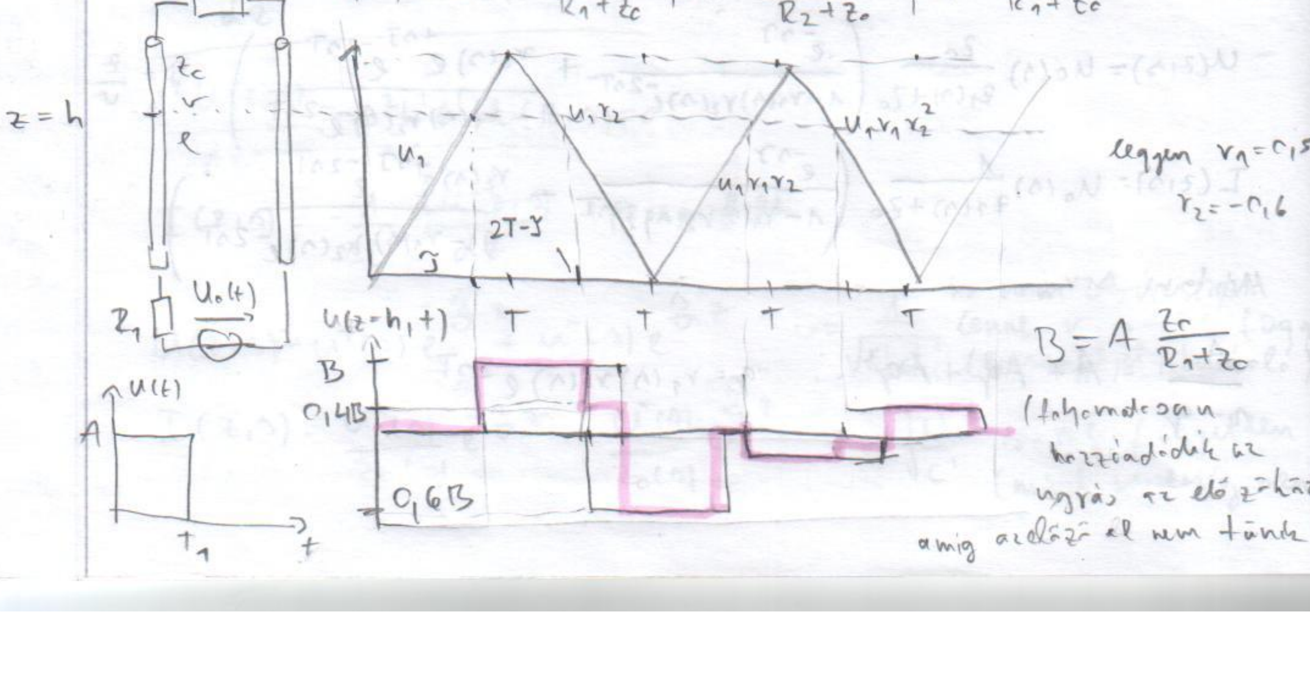
$$u_i^A(t) = \mathcal{L}^{-1}\left\{U_i^A(s)\right\} = \varepsilon(t)\,u_i^A(t)$$

$$u_i^B(t) = \mathcal{L}^{-1}\left\{U_i^B(s)\right\}$$

- 1x torzítatlanul elmegy a jel a végéig
- torzítva reflektálódik
- megint reflektálódik a végén ...

Frekvencia független lezárás, ideális távvezeték => Menetdiagram

$$u_1(t) = U_0(t)\frac{Z_c}{R_1+Z_c}, \quad r_2 = \frac{R_2-Z_0}{R_2+Z_0}, \quad r_1 = \frac{R_1-Z_0}{R_1+Z_c}$$



legyen $r_1 = 0,5$, $r_2 = -0,6$

$$B = A\frac{Z_c}{R_1+Z_0}$$

(fokozatosan hozzáadódik az ugrás az előzőhöz) amíg az előző el nem tűnik

Mi történik ha eredetileg nem volt energiamentes a tv.? (ideális)

$$-\frac{\partial u}{\partial z} = L' \frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial z} = C' \frac{\partial U}{\partial t}$$

$u(z, t=0) \neq 0 \quad u(z, t=-0)$

$i(z, t=0) \neq 0$

$$\mathcal{L}\left(\frac{df}{dt}\right) = sF(s) - f(0)$$

$$-\frac{dU}{dz} = sL'I - L'i(z, t=-0) \quad \text{Legyen } i(z, t=-0) = 0$$

$$-\frac{dI}{dz} = sC'U - C'u(z, t=-0)$$

$$-\frac{d^2U}{dz^2} = -s^2L'C'U + sL'C'u(z, t=-0)$$

$$\frac{d^2U}{dz^2} - \left(\frac{s}{v}\right)^2 U = -sL'C'u(z, t=-0)$$

$$U(z, s) = U^+(s)e^{-\frac{s}{v}z} + U^-(s)e^{\frac{s}{v}z} + U_{ih}(s, z)$$

Diff egyenlet m.o.

$u(z, t=-0) = A$

$U_{ih}(z, s) = B$

$$-\left(\frac{s}{v}\right)^2 B = -sL'C'u(z, t=-0) \Rightarrow B = -\frac{L'C'}{s}v^2 u(z, t=-0)$$

EM inverz feladatok

Fizika: ok → köv

forrás $\bar{J}, \rho \to \bar{E}, \bar{B}$

paraméterek ε, μ, σ

Maxwell-e

Inverz: mért tér → forrás? param?

Alapfogalmak:

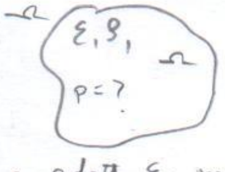
- modell: x függvény: obj. egyértelmű jellemzése
- modell tér: $x \in X$: $\forall x$
- adat: y: mérhető kimenet
- adat tér: $y \in Y$ $\forall y$
- direkt feladat: $y = D\{x\}$ ← direkt op. egyértelmű
- inverz feladat: $\hat{x} = D^{-1}\{y\}$ $\hat{x} \in X \Rightarrow D\{\hat{x}\} = y$ ← nem mindig létezik

Példák

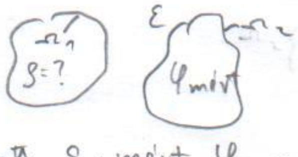
1, elektrosztatika

$-\operatorname{div} \varepsilon(\bar{r}) \operatorname{grad} \varphi(\bar{r}) = \rho(\bar{r})$

- adott $\varepsilon, \rho \to \varphi$ direkt



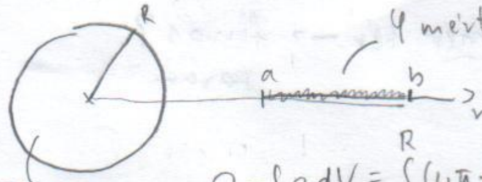
- adott ε, mért $\varphi \to \rho = ?$ inverz



forrás rekonstrukció

- adott ρ, mért φ anyagparam. rekonstrukció

forrás rekonstrukció



$\varphi(r)$ $r\in[a,b]$

$$Q=\int_V \rho\, dV=\int_0^R (4\pi r^2 dr)\,\rho(r)$$

$\boxed{\rho=\rho(r) \quad r\in[0,R]}$

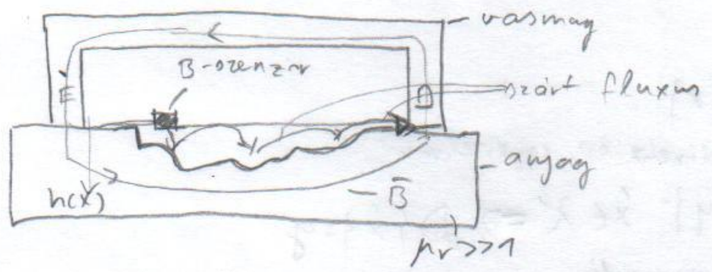
modell

$$\varphi(r)=\frac{Q}{4\pi\varepsilon_0}\frac{1}{r}$$

Direkt ✓
inverz: megold., unicitás ∅

$$\varphi(r)=\int_0^R \frac{\xi^2}{\varepsilon_0 r}\,\rho(\xi)\,d\xi$$

2, fluxusmérés - anyagvizsg.



$h(x)$: modell
$\bar{B}(x)$: adat

Gyengén meghatározott inverz feladat

$y=D\{x\}$ y mért $\hat{x}=?$ $(D\{\hat{x}\}=y)$

jól meghatározottság:

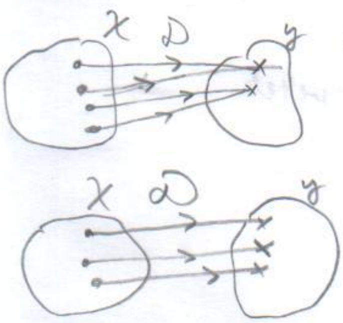
1, $\hat{x}$ létezik (egzisztencia)

2, $\hat{x}$ egyértelmű (unicitás)

3, $\hat{x}(y)$ folytonos

$\hat{x}(y')$ $\{\|y-y'\|\to 0\}\Rightarrow\{\|x-x'\|\to 0\}$
$\hat{x}(y)$

$\hat{x} = ?$ $\quad (\mathcal{D}\{\hat{x}\} = y)$



Szubjektív

Bijektív

injektív

Gyengén meghat $\Rightarrow \emptyset$ jól meghat

HADAMARD 1923

Példa (gömb) $\quad \varphi(r) = \int_0^R \frac{\xi^2}{\varepsilon r} \rho(\xi) d\xi$

FREDHOLM 1 fajú IE

1, egzisztencia ✓

pl: a konst ρ_0 vagyis mindenképpen tudunk adni

2, unicitás $\emptyset$

ρ megoad, $\rho' = \rho + \rho^*$ ha ?

$$\int \frac{\xi^2}{\varepsilon r} \rho^*(\xi) d\xi = \emptyset$$

3, $\rho \to \varphi \qquad \|\varphi - \varphi'\| \to \emptyset \overset{?}{\Longrightarrow} \|\rho - \rho'\| \to 0$

$\rho' = \rho + A \sin pr \to \varphi'$

$$\varphi - \varphi' = \underbrace{\int_0^R \frac{\xi^2}{\varepsilon r} A \sin p\xi \, d\xi}_{\overset{!}{0}} \qquad p \to \infty$$

$\|\varphi - \varphi'\| \to 0$

$\rho - \rho' = A \sin pr \not\to 0$

folytonosság $\emptyset$ sérül

Regularizálás

Cél: gyengén meghat. „csökkentése"

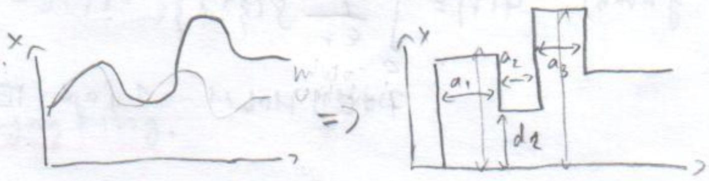
Mód: „a priori" ismeret / feltételezés figyelembe vétele

a, Dimenzió - kontroll

$$\underset{\infty\ \text{Dim}}{x} \longrightarrow \underset{N\ \text{Dim}}{\underline{x}}$$

– véges bázis: $x = \sum_{i=1}^{N} b_2 x_i$ (rögzített bázis fv.), $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$

– paraméterezés:



$\left.\begin{matrix} a_1, a_2 \ldots \\ d_1, d_2 \ldots \end{matrix}\right\} N$ db

– mintavételezés: $\int \to$ diszkrét inverz feladat

$$\hat{x} \triangleq \arg\min_{x \in X} \|y - D\{x\}\|$$ (minta)

$$\hat{\underline{x}} = \arg\min_{\underline{x} \in \mathbb{X}} \|y - D\{x\}\|$$

– véges iteráció:

$$\hat{x} = \arg\min_{x \in X} \|y - D\{x\}\|$$ véges számú iteráció

b, Additív büntető fv. alkalmazása (TIKHONOV 1977)

$$\hat{x} = \arg\min_{x \in X} \|y - D\{x\}\|$$

$$\hat{x}^* = \arg\min_{x \in X} \left(\|y - D\{x\}\| + \alpha \underbrace{F(x)}_{\text{büntető funkcionál}} \right)$$

$0 < \alpha < \infty$ reg paraméter

Pl: „Simaság": $F(x) = \int_x |x'|^2 \, dx$

$\left.\begin{matrix} \alpha \downarrow: \text{a hűség az adatokhoz} \\ \alpha \uparrow: \text{az előzetes elvárások} \end{matrix}\right\}$ kompromisszum

Sík hullámok

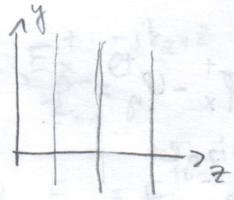
20. EA Bilitz - optimalizálás
e-tananyag

21. EA

- szinuszos állandósult áll: (kiterjeszthető kis idő fv-re is)
- vannak kitüntetett síkok ahol $\bar{E}$ és $\bar{H}$ ugyanaz
- homogén anyaggal kitöltött, végtelen kiterjedésű, veszt. mentes közeg

$$\Delta \bar{E} + \omega^2 \mu \varepsilon \bar{E} = 0$$

$$\frac{\partial}{\partial x} = 0 , \quad \frac{\partial}{\partial y} = 0$$



a, legyen $\bar{E} = E_x \bar{e}_x$

$$\underbrace{\cancel{\frac{\partial^2 E_x}{\partial x^2}}}_{=0} + \underbrace{\cancel{\frac{\partial^2 E_x}{\partial y^2}}}_{=0} + \frac{\partial^2 E_x}{\partial z^2} + \underbrace{\omega^2 \mu \varepsilon}_{k_z^2} E_x = 0 \qquad k_z = \omega\sqrt{\mu \varepsilon} = \frac{\omega}{c}$$

$$E_x = E_x^+ e^{-jk_z z} + E^- e^{jk_z z}$$

$$\mathrm{rot}\,\bar{E} = -j\omega\mu H \qquad \bar{H} = H_y \bar{e}_y \qquad H_y = \frac{E_x^+}{Z_0} e^{-jk_z z} - \frac{E_x^-}{Z_0} e^{jk_z z}$$

b, legyen $\bar{E} = E_y \bar{e}_y$ $\qquad Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$

$$\frac{\partial^2 E_y}{\partial z^2} + k_z^2 E_y = 0$$

$$E_y = E_y^+ e^{-jk_z z} + E_y^- e^{jk_z z}$$

$$H_x = \frac{-E_y^+}{Z_0} e^{-jk_z z} + \frac{E_y^-}{Z_0} e^{jk_z z}$$

c, legyen $\bar{E} = E_z \bar{e}_z \rightarrow \bar{H} = 0$

Nem lehet hogy z irányú komponense lehet ebben az esetben (nem is EM hullám)

Polarizáció: $E_x^+ = |E_x^+| e^{j\varphi_x^+}$

$E_y^+ = |E_y^+| e^{j\varphi_y^+}$

pl: • $|E_y^+| = 0$ x irányba lineárisan polarizált

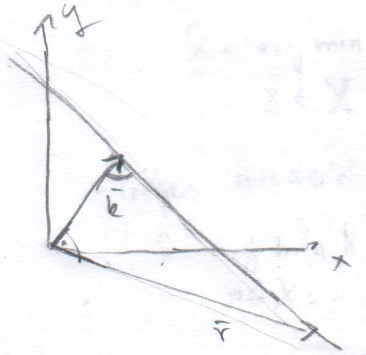
• $|E_x^+| = |E_y^+|$ és $\varphi_x^+ - \varphi_y^+ = 90°$ cirkulárisan polarizált

• Eliptikusan polarizált

Komplex Poyting-vektor $\bar{S} = \frac{1}{2} \bar{E} \times \bar{H}^*$

Síkhullám tekinthető gyakorlati esetben leggyakrabban előforduló, távolt térben, véges tartományon (ezzel közelíthető)

Ferdén terjedő síkhullám



tualajd.:

$E \perp \bar{H}$, $\bar{E} \perp \bar{k}$ terj. irány

E_x H_y E_x $k_z \bar{e}_z$

E_y H_x E_y $k_z \bar{e}_z$

$\bar{S} \parallel \bar{k}$ $\bar{k}$-ra merőleges síkokban $\bar{E}$ és $\bar{H}$ ugyanaz

$$\bar{k} = k_x \bar{e}_x + k_y \bar{e}_y + k_z \bar{e}_z$$

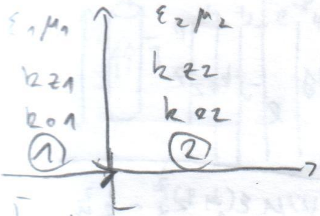
$$|\bar{k}|^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon$$

$$\bar{E}(x,y,z) = \bar{E}_0 e^{-j(k_x x + k_y y + k_z z)} = \bar{E}_0 e^{-j\bar{k}\bar{r}}$$

$$\bar{H}(x,y,z) = \bar{H}_0 e^{-j\bar{k}\bar{r}}$$

Reflexió

ismétlés: merőleges beesés



$\bar{k} = k_z e_z$

$k_{01} = \omega\sqrt{\varepsilon_1\mu_1}$ $\quad k_{z1} = k_{01}$

$k_{02} = \omega\sqrt{\varepsilon_2\mu_2}$ $\quad k_{z2} = k_{02}$

1 $E_y = 0$

$$E_x = E_x^+ e^{-jk_{z1}z} + E_{x1}^- e^{jk_{z1}z}$$

$$H_y = \frac{E_x^+}{Z_{01}} e^{-jk_{z1}z} - \frac{E_{x1}^-}{Z_{01}} e^{jk_{z1}z}$$

2 $$E_x = E_{x2}^+ e^{-jk_{z2}z} + \underbrace{\cancel{E_{x2}^- e^{jk_{z2}z}}}_{=0}$$

$$H_y = \frac{E_{x2}^+}{Z_{02}} e^{-jk_{z2}z} - \underbrace{\cancel{\frac{E_{x2}^-}{Z_{02}} e^{jk_{z2}z}}}_{=0}$$

Perem feltételek: E_{x1}^+ adott

$E_{x2}^- = 0$ sugárzási felt.

Folyt. felt. $z = 0$ ban

$\bar{H}_{t1} = \bar{H}_{t2}$

$\bar{E}_{t1} = \bar{E}_{t2}$

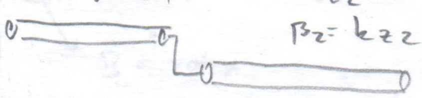
$$\left.\begin{aligned} E_{x1}^+ + E_{x1}^- &= E_{x2}^+ \\ \frac{E_{x1}^+}{Z_{01}} - \frac{E_{x1}^-}{Z_{01}} &= \frac{E_{x2}^+}{Z_{02}} \end{aligned}\right\} \Rightarrow$$

$$E_{x1}^- = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{02}} E_{x1}^+$$

$$E_{x2}^+ = E_{x1}^+ + E_{x1}^- = \frac{2 Z_{02}}{Z_{02} + Z_{01}} E_{x1}^+$$

$\beta_1 = k_{z1}$, Z_{01} ? Z_{02}

$\beta_2 = k_{z2}$



Ferde beesés

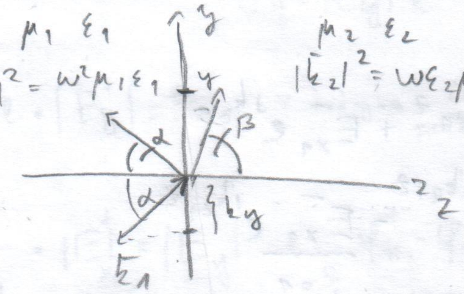
$z = 0$ -ban $E_{t1} = E_{t2}$

(1) $\bar{E}_{01} e^{-jk_{x1}x} e^{-jk_{y1}y} e^{-jk_{z1}z}$

(2) $\bar{E}_{02} e^{-jk_{x2}x} e^{-jk_{y2}y} e^{-jk_{z2}z}$

$\mu_1 \ \varepsilon_1$ $\quad$ $\mu_2 \ \varepsilon_2$

$|k_1|^2 = \omega^2\mu_1\varepsilon_1$ $\quad$ $|k_2|^2 = \omega\varepsilon_2\mu_2$

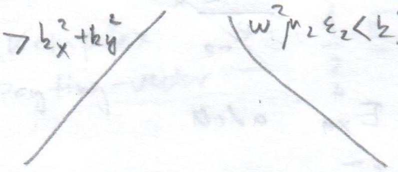


$\left.\begin{matrix} k_{x1} = k_{x2} \\ k_{y1} = k_{y2} \end{matrix}\right\} \Rightarrow \begin{matrix} k_x \\ k_y \end{matrix}$ $\quad k_{z1} = \sqrt{\omega\mu_1\varepsilon_1 - k_x^2 - k_y^2}$

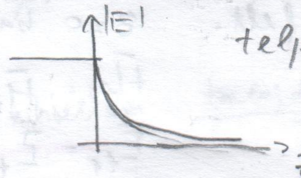
$k_{z2} = \sqrt{\omega^2\mu_2\varepsilon_2 - k_x^2 - k_y^2}$

$k_{z2} = \sqrt{\omega^2\mu_2\varepsilon_2 - k_x^2 - k_y^2} =$

$\omega^2\mu_2\varepsilon_2 > k_x^2 + k_y^2$ $\quad$ $\omega^2\mu_2\varepsilon_2 < k_x^2 + k_y^2$



$\sqrt{\omega^2\mu_2\varepsilon_2 - k_x^2 - k_y^2}$ valós $\quad$ $j\sqrt{\omega^2\mu_2\varepsilon_2 - k_x^2 - k_y^2}$

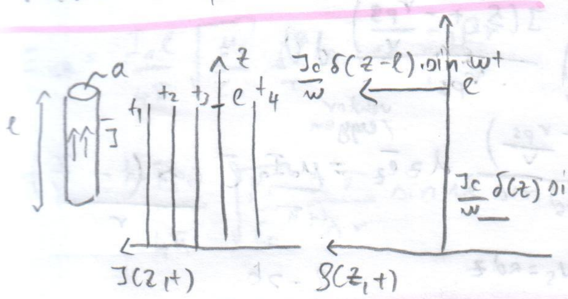


teljes visszaverődés

$\sin\alpha = \frac{k_y}{\omega\sqrt{\mu_1\varepsilon_1}}$ $\quad$ $\frac{\sin\alpha}{\sin\beta} = \frac{\omega\sqrt{\mu_2\varepsilon_2}}{\omega\sqrt{\mu_1\varepsilon_1}} = \frac{\sqrt{\mu_2\varepsilon_2}}{\sqrt{\mu_1\varepsilon_2}} =$

$\sin\beta = \frac{k_y}{\omega\sqrt{\mu_2\varepsilon_2}}$ $\quad$ $= \frac{\sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}}} = \frac{n_2}{n_1}$ $\quad$ $\mu_1 = \mu_2 = \mu_0$

Hertz - dipólus (pont dipólus)



$$J(z,t)=\begin{cases} J_0\cos\omega t & \text{ha } 0<z<l \\ 0 & \text{egyébként}\end{cases}$$

$$J(z,t)=J_0\left[\varepsilon(z)-\varepsilon(z-l)\right]\cos\omega t$$

Folyt. egyenlet:

$$\operatorname{div}\bar{J}+\frac{\partial\rho}{\partial t}=0$$

csak z. szerinti komp.

$$\frac{\partial J}{\partial z}+\frac{\partial\rho}{\partial t}=0$$

$$\frac{\partial\rho}{\partial t}=-\frac{\partial}{\partial z}J_0\left[\varepsilon(z)-\varepsilon(z-l)\right]\cos\omega t$$

$$\frac{\partial\rho}{\partial t}=-J_0\left[\delta(z)-\delta(z-l)\right]\cos\omega t$$

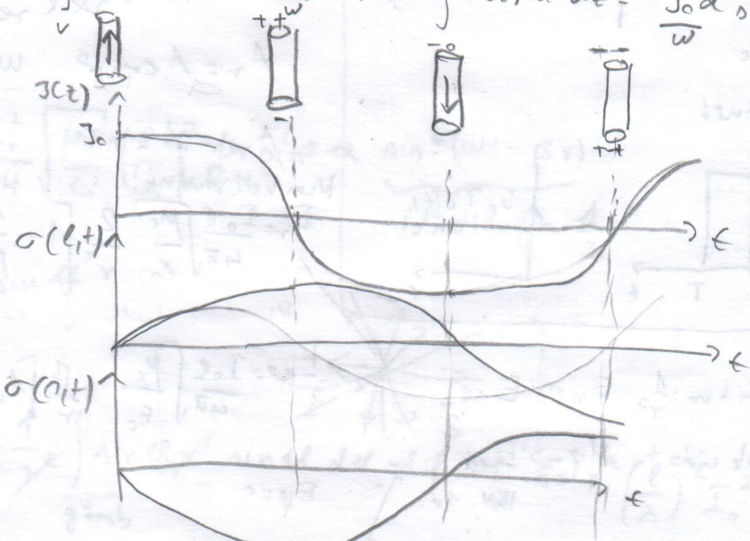
$$\rho(z,t)=-\frac{J_0}{\omega}\left[\delta(z)-\delta(z-l)\right]\sin\omega t$$

alaplap össztöltés: $Q_{al}=\int_V\rho(0,t)\,dV=-\frac{J_0}{\omega}\sin\omega t\int_{-0}^{+0}\delta(z)\cdot a\cdot dz=-\frac{J_0 a}{\omega}\sin\omega t$ (felület)

felületi tölt. sűr. $\frac{Q_{al}}{a}$

$$\sigma(0,t)=-\frac{J_0}{\omega}\sin\omega t$$

$$\sigma(l,t)=\frac{J_0}{\omega}\sin\omega t$$



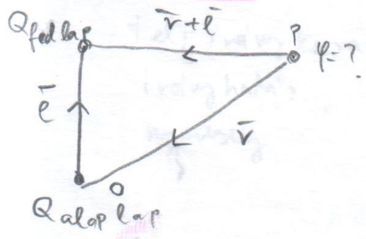
$$\left.\begin{aligned}\rho &\to \varphi\\ \bar{J}&\to\bar{A}\end{aligned}\right\}\to \begin{aligned}\bar{E}&=-\operatorname{grad}\varphi-\frac{\partial A}{\partial t}\\ \bar{B}&=\operatorname{rot}\bar{A}\end{aligned}$$

Skalár pot

retardálás

$$\varphi(P)=\frac{1}{4\pi\varepsilon}\left[\frac{\frac{J_0 a}{\omega}\sin\omega\left(t-\frac{|\bar{r}+\bar{l}|}{v}\right)}{|\bar{r}+\bar{l}|}-\frac{\frac{J_0 a}{\omega}\sin\left(\omega t-\frac{r}{v}\right)}{r}\right]$$

($J_0 a = I_0$)



$$\varphi(P)=\frac{I_0}{4\pi\varepsilon\omega}\,\bar{l}\,\operatorname{grad}_0\left[\frac{1}{r}\sin\omega t\left(t-\frac{r}{v}\right)\right]$$

└ 0 pont szerinti

$l\to 0$

$I_0\to\infty$

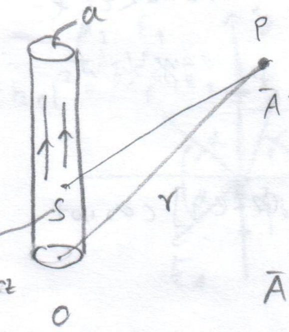
$\bar{p}=I_0\bar{l}$

$$\boxed{\varphi(P)=\frac{\bar{p}}{4\pi\varepsilon\omega}\operatorname{grad}_P\left[\frac{1}{r}\sin\omega\left(t-\frac{r}{v}\right)\right]}$$

P pont szerinti képz.

Dipól antennák kívüli pot potenciálja

Vektor pot.



futópont az integráláshoz

$\bar{A} = ?$

$$\bar{A}(P,t) = \frac{\mu}{4\pi}\int_V \frac{\bar{J}\left(S, t - \frac{r_{ps}}{v}\right)}{r_{ps}}\, dV_s =$$

$$\mu \frac{J_0 a}{4\pi}\int_0^{e} \frac{\cos\omega\left(t - \frac{r_{ps}}{v}\right)}{r_{ps}}\, dz\, \bar{e}_z = \frac{\mu_0 I_0}{4\pi}\, \bar{e}\, \frac{\cos\left(t - \frac{r}{v}\right)}{r}$$

vektor / egyen

$r_{ps} \approx r \quad e \ll r \quad dV_s = a\,dz$

$e \to 0$

$I_0 \to \infty$

$$\bar{A}(P,t) = \mu_0 \frac{\bar{p}}{4\pi}\frac{\cos\left(t - \frac{r}{v}\right)}{r}$$

$\bar{p} = I_0 \bar{e} = \text{const}$

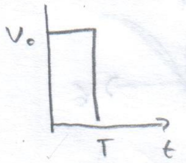
Lorentz mérték

$$\operatorname{div}\bar{A} + \varepsilon\mu\frac{\partial\varphi}{\partial t} = \emptyset$$

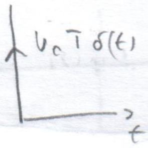
valóban teljesül.

$e \updownarrow I_0 \approx \bar{p} = I_0 \bar{e}$

hasonló



$\approx$



$$\bar{E} = -\operatorname{grad}\varphi - \frac{\partial\bar{A}}{\partial t}$$

$\bar{B} = \operatorname{rot}\bar{A}$ gömb koord. r.

$\bar{A} = A_r\bar{e}_r + A_\vartheta\bar{e}_\vartheta$

$A_r = A\cos\vartheta \qquad \omega\left(t - \frac{r}{v}\right) = \omega t - \frac{\omega}{v}r = \omega t - \beta r$

$A_\vartheta = -A\sin\vartheta$

Nem kell tudni!!:

$$E_r = \frac{I_0 e}{4\pi}\sqrt{\frac{\mu_c}{\varepsilon_c}}\frac{2}{r^2}\left[1 - \frac{1}{\beta r}\right]\cos\vartheta\, e^{-j\beta r}$$

$$E_\vartheta = \frac{I_0 e}{4\pi}\sqrt{\frac{\mu_c}{\varepsilon_c}}\, j\frac{\beta}{r}\left[1 - \frac{1}{\beta^2 r^2} - \frac{j}{\beta r}\right]\sin\vartheta\, e^{-j\beta r}$$

Sztatikus: $\frac{1}{r^3}$ E_r is E_ϑ

Indukciós t. $H_\varphi \to E_\vartheta$

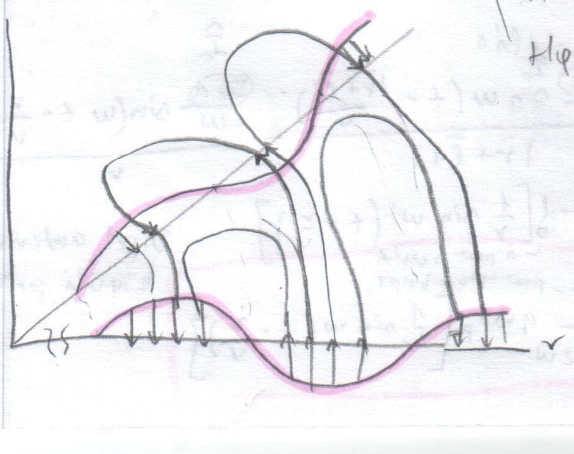
Ind. tv.

$E_\varphi = 0$

$H_r = 0$

$H_\vartheta = 0$

$$H_\varphi = \frac{I_0 e}{4\pi}\frac{j\beta}{r}\left[1 - \frac{j}{\beta r}\right]\sin\vartheta\, e^{-j\beta r}$$



Sugárzó tér, távoltér $\frac{1}{r}$

$$E_\vartheta = \frac{I_0 l}{4\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{j\beta}{r}\sin\vartheta\, e^{-j\beta r}$$

$$E_\vartheta = \frac{-I_0 l}{4\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{2\pi}{\lambda r}\sin\vartheta \sin(\omega t-\beta r)$$

$$\boxed{\begin{aligned} E_\vartheta &= -\frac{I_0}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{l}{\lambda}\frac{1}{r}\sin(\omega t-\beta r)\sin\vartheta \\ H_\varphi &= -\frac{I_0}{2}\frac{l}{\lambda}\frac{1}{r}\sin(\omega t-\beta r)\sin\vartheta \end{aligned}}$$

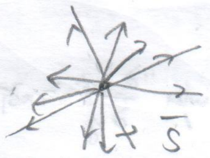
1, E_ϑ és H_φ fázisban

2, $\frac{E_\vartheta}{H_\varphi} = Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ \Omega$

Elsugárzott telj.

$$S(r,\vartheta,t) = E_\vartheta H_\varphi = \frac{I_0^2}{4}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{l}{\lambda}\right)^2\frac{1}{r^2}\sin^2\vartheta\ \underbrace{\sin^2(\omega t-\beta r)}_{\text{időátlaga } \frac{1}{2}}$$

$$S_{iá} = \frac{I_0^2}{8}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{l}{\lambda}\right)^2\frac{1}{r^2}\sin\vartheta$$



$$P = \oint_A \bar{S}_{iá}\, da = \frac{I_0^2}{8}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{l}{\lambda}\right)^2\frac{1}{\cancel{r^2}}\int_{\text{gömb}}\sin\vartheta\,\cancel{r^2}\sin\vartheta\, d\vartheta\, d\varphi = \frac{1}{2}\,80\pi^2\left(\frac{l}{\lambda}\right)^2 I_0^2$$

$$\int_{\vartheta=0}^{\pi}\sin\vartheta\, d\vartheta = \frac{4}{3}$$

átnézni:

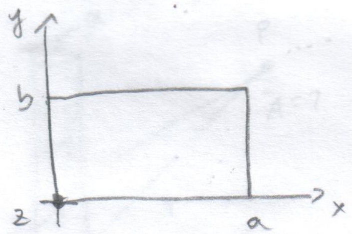
irány karakterisztika

telj irány karakterisztika

irányhatás

nyereség

23. EA | Csőtápvonal



transzverzális sík: x, y, sík
longitudinális irány: z irány

1, E_x, E_y, H_x, H_y kifejezhető E_z-vel és H_z-vel

$\frac{\partial}{\partial t} \to j\omega$ $\quad e^{-\gamma z}$ z függés $\quad \frac{\partial}{\partial z} \to \gamma \quad e^{j\omega t - \gamma z}$

I. $\mathrm{rot}\, \bar{H} = \varepsilon \frac{\partial E}{\partial t}$

$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$ (1) $\quad \Big| \quad \frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x$ (4)

$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$ (3) $\quad -\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$ (2)

①② $\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \quad \Big| \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$

①② $$E_x = \frac{1}{\omega^2\varepsilon\mu + \gamma^2}\left[-j\omega\mu \frac{\partial H_z}{\partial y} - \gamma \frac{\partial E_z}{\partial x}\right]$$

$E_x = E_x(H_z, E_z = 0) + E_x(E_z, H_z = 0)$

3, 4 $$E_y = \frac{1}{\omega^2\varepsilon\mu + \gamma^2}\left[j\omega\mu \frac{\partial H_z}{\partial x} - \gamma \frac{\partial E_z}{\partial y}\right]$$

1, 2 $$H_x = \frac{1}{\omega^2\varepsilon\mu + \gamma^2}\left[j\omega\varepsilon \frac{\partial E_z}{\partial y} - \gamma \frac{\partial H_z}{\partial x}\right]$$

3, 4 $$H_y = \frac{1}{\omega^2\varepsilon\mu + \gamma^2}\left[-j\omega\varepsilon \frac{\partial E_z}{\partial x} - \gamma \frac{\partial H_z}{\partial y}\right]$$

II. A longitudinális komp-re vonatkozó hullám-e megoldása

$$\Delta \bar{E} = \varepsilon\mu \frac{\partial E^2}{\partial t^2}$$ 3 skalár egyenlet, mi csak a z komp.-re írjuk fel

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E_z}{\partial t^2} \quad \frac{\partial^2}{\partial z^2} \to \gamma^2 \quad \frac{\partial^2}{\partial t^2} \to -\omega^2$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = -(\gamma^2 + \omega^2\varepsilon\mu) E_z$$

$$\Delta_t E_z = -\underbrace{(\gamma^2 + \omega^2\varepsilon\mu)}_{s} E_z$$ saját ért. egyenlet

$$\Delta_t E_z = s E_z$$ differenciál operátor: a peremfeltételekkel együtt van meghatározva

Megoldás szorzat szeparációval

$E_z = X(x)Y(y)$ alakban keressük a szorzatot

$$Y \frac{\partial^2 X}{\partial x^2} + X \frac{\partial^2 Y}{\partial y^2} = -(\gamma^2 + \omega^2\varepsilon\mu) XY$$

$$\underbrace{\frac{1}{X}\frac{d^2 X}{dx^2}}_{\text{csak } x\text{-től függ}} + \underbrace{\frac{1}{Y}\frac{d^2 Y}{dy^2}}_{\text{csak } y\text{-tól}} = -(\gamma^2 + \omega^2\varepsilon\mu)$$

Összegük konst → mindkettőnek „szeparáció" konst. kell lenni

$$\frac{1}{X}\frac{d^2 X}{dx^2} = -k_x^2 \qquad \frac{1}{Y}\frac{d^2 Y}{dy^2} = -k_y^2$$

$$\gamma^2 + \omega^2\mu\varepsilon = \underbrace{k_x^2 + k_y^2}_{\text{peremfelt. hat. meg}}$$

$$\frac{\partial^2 X}{\partial x^2} = -k_x^2 X$$

$$\left.\begin{aligned} X(x) &= C_1 \sin(k_x X) + C_2 \cos(k_x X) \\ Y(y) &= C_3 \sin(k_y y) + C_4 \cos(k_y y) \end{aligned}\right\} \to \quad E_z(x,y) = \left(C_1 \sin(k_x X) + C_2 \cos(k_x X)\right) \cdot \left(C_3 \sin(k_y y) + C_4 \cos(k_y y)\right)$$

Peremfeltételek

$$E_{tang}\Big|_{perem}=0 \qquad E_z(x=0,y)=0 \quad \text{ha} \quad C_2=0$$

$$E_z(x,y=0)=0 \quad \text{ha} \quad C_4=0$$

$$C_1C_3=C$$

$$E_z(x,y)=C\sin(k_x x)\sin(k_y y)$$

$E_z(x=a,y)=0$ ha $k_x a=m\pi$ minden n-hez és m-hez

$k_x=\dfrac{m\pi}{a}$ tartozik egy megoldás

$E_z(x,y=b)=0$ ha $k_y b=n\pi$

$k_y=\dfrac{n\pi}{b}$

saját fv. = hullám forma = módus

mivel $H_z=0$ TM_{mn} módus (Transzverzális Mágneses)

TE: Transzv. Elektromos

Hasonlóan: $E_z=0$ csak H_z van

$$H_z=C\cos k_x x\cos k_y y \quad k_x=\frac{m\pi}{a} \quad k_y=\frac{n\pi}{b}$$

A TM_{mn} teljes tér

$$E_x=C\frac{j\omega\mu k_y}{\omega^2\varepsilon\mu+\gamma^2}\cos k_x x\sin k_y y$$

$$E_y=-C\frac{j\omega\mu k_x}{\omega^2\varepsilon\mu+\gamma^2}\sin k_x x\cos k_y y$$

$$H_x=C\frac{\gamma k_x}{\omega^2\varepsilon\mu+\gamma^2}\sin k_x x\cos k_y y$$

$$H_y=C\frac{\gamma k_y}{\omega^2\varepsilon\mu+\gamma^2}\cos k_x x\sin k_y y$$

$$H_z=C_6\cos k_x x\cos k_y y$$

A diszperziós egyenlet $\gamma(\omega)$ vagy $\beta(\omega)$

$$\boxed{\gamma^2 + \omega^2 \varepsilon\mu = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

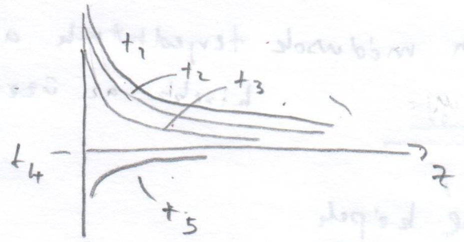
$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \varepsilon\mu}$$

$$\omega^2 \varepsilon\mu < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$\gamma = \alpha$

$$f(x,y,z,t) = \operatorname{Re} F(x,y)\, e^{j\omega t - \gamma z}$$

$$f = F(x,y)\, e^{-\alpha z} \cos \omega t$$



E_t és H_t között 90-os fázis kül.

$\operatorname{Re} \bar{S}_{kompl} = 0$

nincs teljesítmény szállítás

$$\omega^2 \varepsilon\mu > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$\gamma = j\beta \quad \alpha = 0$

$$f(x,y,z,t) = \operatorname{Re}\{F(x,y)\, e^{j(\omega t - \beta z)}\} = F(x,y) \cos(\omega t - \beta z)$$

csillapítatlan haladó hullám

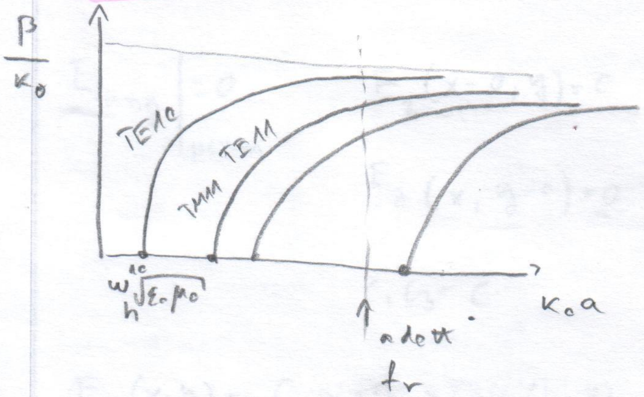
Határfrekvencia:

$$\omega_h^{nm} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\, \frac{1}{\sqrt{\varepsilon\mu}}$$

Célszerű a legkisebb határ frek.-ú módust használni

$$f_h^{mn} = \frac{1}{2\sqrt{\varepsilon\mu}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

diszperziós görbék:



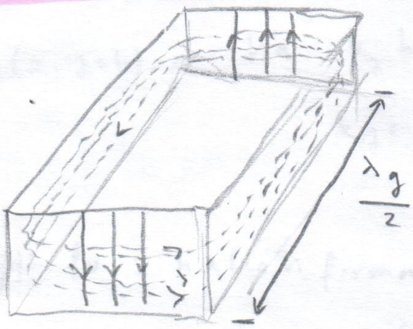
$k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ a szabadtéri fázis tényező

$k_0 a = \omega\sqrt{\varepsilon_0\mu_0}\, a$ (??)

$$\omega = \frac{k_0 a}{\sqrt{\varepsilon_0\mu_0}}$$

Mindazon módusok terjedhetnek amelyek határfr.-a kisebb az üzemi frekvenciá (amivel gerjesztjük)

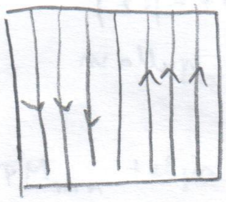
Erővonal képek



λ_g a csőben a tengely mentén mérhető hullámhossz.

$\lambda_g > \lambda_0$ λ_0 a szabadtéri törésmutató. $n = \frac{\lambda_0}{\lambda_g} < 1$

TE_{10} csak E_y H_x H_z



TE20

TM11