AN1. INF. PPZH2. — 1 — (2016. 12. 13.

1, i, $\boxed{15}$ $$\frac{d}{dx}\left(\cos(x^2)\ln(2-3x)\right) = -2x\sin(x^2)\ln(2-3x) - \frac{3\cos(x^2)}{2-3x}$$ (7p)

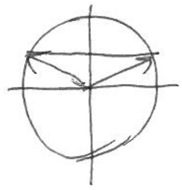
ii, $$\frac{d}{dx}\left(\frac{2^{\operatorname{tg} x}}{\sin x}\right) = \frac{\ln 2\cdot 2^{\operatorname{tg} x}\cdot\left(\frac{1}{\cos^2 x}\right)\cdot\sin x - 2^{\operatorname{tg} x}\cos x}{\sin^2 x}$$ (8p)

2, $\boxed{25}$ f folytonos, $[0, \frac{\pi}{2}]$ korlátos, zárt, tehát Weierstrass II. tétele értelmében f felveszi szélsőértékeit az intervallumon. (3)

f deriválható, tehát ha x lokális szélsőérték-hely, akkor $f'(x)=0$.

$\boxed{8}$

$f'(x) = \frac{1}{2} - \sin x = 0$ (3) $\Rightarrow \sin x = \frac{1}{2} \Rightarrow \begin{cases} x_1 = \frac{\pi}{6} + 2k_1\pi \\ x_2 = \frac{5\pi}{6} + 2k_2\pi \end{cases}$ $(k_i \in \mathbb{Z})$ (3)



A $[0, \frac{\pi}{2}]$ intervallumon f'-nek egyetlen zérushelye: $x_0 = \frac{\pi}{6}$ (2)

Vizsgálandó pontok: $x_0 = \frac{\pi}{6}$, $x_1 = 0$, $x_2 = \frac{\pi}{2}$ (végpontok) (2)

$$f(x_0) = \frac{\pi}{12} + \cos\frac{\pi}{6} = \frac{\pi}{12} + \frac{\sqrt{3}}{2} = \frac{\pi + 6\sqrt{3}}{12} = \frac{\pi + \sqrt{108}}{12} > \frac{3+10}{12} = \frac{13}{12} > 1$$ (2)

$$f(x_1) = f(0) = 0 + \cos(0) = 1$$ (2)

$$f(x_2) = f\left(\frac{\pi}{2}\right) = \frac{\pi}{4} + \cos\left(\frac{\pi}{2}\right) = \frac{\pi}{4} < \frac{4}{4} = 1$$ (2)

Tehát a minimum: $f(x_2) = f\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$ (3), a maximum: $f\left(\frac{\pi}{6}\right) = \frac{\pi}{12} + \frac{\sqrt{3}}{2}$ (3)

3, i, $\boxed{15}$ $$\lim_{x\to 0}\left(\frac{1}{x} + \frac{1}{1-e^x}\right) = \lim_{x\to 0}\frac{1-e^x+x}{x(1-e^x)} \overset{\text{„}\frac{0}{0}\text{", L'H}}{=} \lim_{x\to 0}\frac{1-e^x}{1-e^x-xe^x} \overset{\text{„}\frac{0}{0}\text{", L'H}}{=}$$ (4) (4)

$$= \lim_{x\to 0}\frac{-e^x}{-2e^x - xe^x} = \lim_{x\to 0}\frac{-1}{-2-x} = \frac{1}{2}$$ (4) (3)

(Ki lehet használni azt is, hogy $\lim_{x\to 0}\frac{e^x-1}{x} = (e^x)'\big|_{x=0} = e^0 = 1$)

3, ii, $\boxed{15}$ $f(x):=\ln(1-\cos^2 x)-\ln(x^2)=\ln\dfrac{1-\cos^2 x}{x^2}=\ln\dfrac{\sin^2 x}{x^2}=2\ln\left(\dfrac{\sin x}{x}\right)$ 3

$$\lim_{x\to 0} f(x)=\lim_{x\to 0} 2\ln\left(\frac{\sin x}{x}\right) \overset{6}{=} 2\ln\Big(\underbrace{\lim_{x\to 0}\frac{\sin x}{x}}_{1}\Big) \overset{4}{=} 2\ln 1 = \underline{\underline{0}}$$ 2

ln folytonos

4, $\boxed{30}$ $f(x)=(x+2)^2(x-7)$ zérushelyek: $x_1=-2,\ x_2=7$

$\boxed{12p.}$

$f'(x)=2(x+2)(x-7)+(x+2)^2=(x+2)(2x-14+x+2)=(x+2)(3x-12)=$

$=3(x+2)(x-4)$ 5

x	$x<-2$	-2	$-2<x<4$	4	$4<x$
f'	+	0	−	0	+
f	↗	lok. max	↘	lok. min	↗

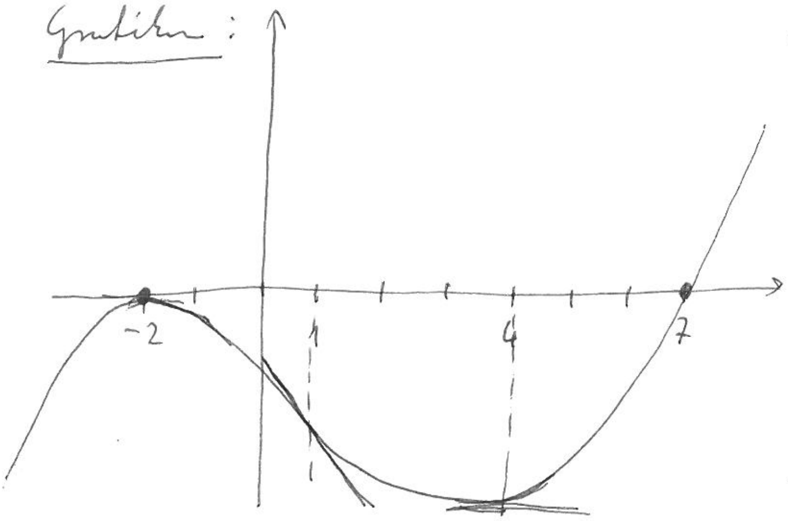
7

$f''(x)=3(x-4)+3(x+2)=6(x-1)$ 4

x	$x<1$	1	$1<x$
f''	−	0	+
f	∩	infl. pont	∪

5

Grafikon:



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5, (IMSC) $\boxed{7}$ $f(x)=x-e\ln x$

$f(e)=e-e\underbrace{\ln e}_{1}=0$ 1

$f'(x)=1-\overset{1}{\dfrac{e}{x}}=\dfrac{x-e}{x}\ \overbrace{>0,\ \text{ha } x>e}^{1}$

$\Rightarrow \overbrace{\forall x>e \text{ esetén } f(x)>0.}^{2}$

(Alkalmazzuk Lagrange-tételt $[e,x]$-re: $\dfrac{f(x)-f(e)}{x-e}=\dfrac{f(x)}{x-e}=f'(\xi)>0$

$3>e$, tehát $f(3)=3-e\ln 3>0 \Rightarrow 3>e\ln 3=\ln(3^e)$

$\underset{e^x\nearrow}{\Rightarrow}$ $e^3>3^e$ ✓ (Megj.: $e^3=20.08>3^e=19.81$) 2