Analízis 1 2. ZH Megoldás. 2017. nov. α variáns

1. a) $\forall N>0$ esetén $\exists\, x_N<0$ melyre $\forall x<x_N$ esetén $\Rightarrow f(x)>N$ $\boxed{5}$

$\boxed{6}$ b) $x<-2$-re: $\left|\dfrac{2x^2+2}{|x^2-4|}-2\right| = \dfrac{8}{x^2-4}$ (2)

$\dfrac{8}{x^2-4}<\varepsilon \Longleftrightarrow x^2>\dfrac{8}{\varepsilon}+4$ (2) Tehát $x<x_\varepsilon=-\sqrt{\dfrac{8}{\varepsilon}+4}$

2. $\boxed{8}$ a) $f(x)$ folytonos $\forall x\neq 1$ esetén (2)

(2) $f(1+0)=1/e$, $f(1-0)=a+b$ (2) $\Rightarrow$ $a+b=\dfrac{1}{e}$ (2)

b) $f(x)$ diff-ható $\forall x\neq 1$ esetén (2)

$\boxed{10}$ $f'(x)=e^{-x^2}(1-2x^2)$ (2) $f'(1+0)=-1/e$ (2)

$f'(1-0)=a$ (2) $\Rightarrow$ $a=-1/e$, $b=2/e$ (2)

3. a) $f(x)$ ért. tart: $x^2+3x-1\geq 0 \Longleftrightarrow x<\dfrac{-3-\sqrt{13}}{2}$, $x>\dfrac{-3+\sqrt{13}}{2}$ (4)

értékkészlet $\min f(x)=0$, $[0,+\infty)$ (4)

derivált: $f'(x)=\dfrac{2x+3}{2\sqrt{x^2+3x-1}}$ (4) $\boxed{12}$

b) $m=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\sqrt{1+\dfrac{3}{x}-\dfrac{1}{x^2}}=1$ (2) $\boxed{5}$

(3)

c) $\lim\limits_{x\to\infty}(f(x)-x)=\lim\limits_{x\to\infty}\left(\sqrt{x^2+3x-1}-x\right)$ $\boxed{7}$

$=\lim\limits_{x\to\infty}\dfrac{3x-1}{\sqrt{\ldots}+x}=\lim\limits_{x\to\infty}\dfrac{3-\frac{1}{x}}{\sqrt{1+\frac{3}{x}-\frac{1}{x^2}}+1}=\dfrac{3}{2}$

(3) (2)

4. a) $\lim\limits_{x\to 0}\dfrac{\operatorname{tg}5x}{\sin 2x}=\lim\limits_{x\to 0}\dfrac{\sin 5x}{\cos 5x\sin 2x}$ $\boxed{6}$

$=\lim\limits_{x\to 0}\dfrac{\sin 5x}{\sin 2x}=\lim\limits_{x\to 0}\dfrac{5\cos 5x}{2\cos 2x}=\dfrac{5}{2}.$

(2) (2) (2)

b) $\lim\limits_{x\to 0}\left(\frac{1}{\operatorname{tg}x}-\frac{1}{\sin x}\right)=\lim\limits_{x\to 0}\left(\frac{\sin x-\operatorname{tg}x}{\sin x\operatorname{tg}x}\right)$ (2)

$=\lim\limits_{x\to 0}\frac{\cos x-1}{\sin x}=\lim\limits_{x\to 0}\frac{-\sin x}{\cos x}=0$ $\boxed{7}$

(2) (3)

c) $\lim\limits_{x\to 0}\left(\sqrt{1+x}-x\right)^{\frac{1}{x}}=\lim\limits_{x\to 0}e^{\ln(\sqrt{1+x}-x)/x}$ (2) $\boxed{7}$

$\lim\limits_{x\to 0}\frac{\ln(\sqrt{1+x}-x)}{x}=\lim\limits_{x\to 0}\frac{\frac{1}{\sqrt{1+x}-x}\left(\frac{1}{2\sqrt{1+x}}-1\right)}{1}$ (3)

$=-\frac{1}{2}\lim\limits_{x\to 0}\frac{1}{\sqrt{1+x}-x}=-\frac{1}{2}$. (2) Tehát $1/\sqrt{e}$. (2)

d) $\lim\limits_{x\to -\infty}\frac{e^{-x}\operatorname{sh}5x}{\operatorname{ch}6x}=\lim\limits_{x\to-\infty}\frac{e^{-x}\left(e^{5x}-e^{-5x}\right)}{e^{6x}+e^{-6x}}=$ (2) $\boxed{6}$

$=\lim\limits_{x\to-\infty}\frac{e^{4x}-e^{-6x}}{e^{6x}+e^{-6x}}=\lim\frac{e^{10x}-1}{e^{12x}+1}=-1$ (2)

(2)

5. a) $f'(x)=1-\frac{2}{x^3}$ (1), $\begin{cases} f'(x)>0 \text{ ha } x<0,\ x>\sqrt[3]{2} \\ f'(x)<0, \text{ ha } 0<x<\sqrt[3]{2}\end{cases}$ (4)

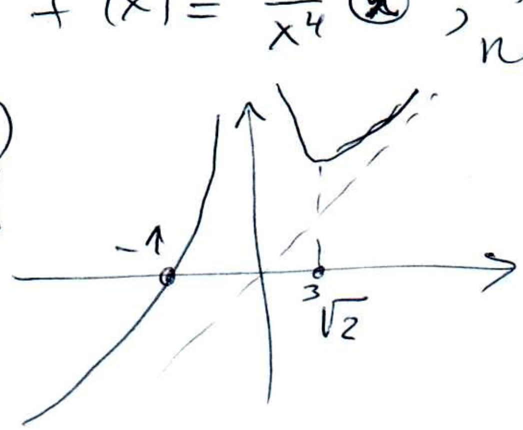
$\boxed{7}$

$x=\sqrt[3]{2}$ lok. min. (2)

$\boxed{8}$ b) $f''(x)=\frac{6}{x^4}$ (2), $f''\geq 0,\ \forall x$ konvex (3), nincs inflexiós pont! (3)

c)

$\boxed{5}$



IMSC feladat Ha 5 gyök lenne akkor
$p'(x) = -22x^{10} + 72x^{23} - 140x^{34} + 230x^{45} = x^{10}(\dots)$-nek
4 előjelváltása lenne és mivel $x^{10} \geq 0$ így
$-22 + 72x^{13} - 140x^{24} + 230x^{35}$-nek 4 előjelváltása van
Ezt még 2-szer ismételve adódik egy
$A + Bx^{11}$ polinom 2 előjelváltással, ami ellentmondás

Analízis 1. 2.ZH Megoldás B (tömör)

1. a) $\forall N<0,\ \exists x_N<0,\ \forall x<x_N$-re: $f(x)<N$ $\boxed{5}$

$\boxed{6}$ b) $x<-2$-re: $\left|\frac{3x^2+5}{x^2-4}-3\right| = \frac{17}{x^2-4}$ (2) $\frac{17}{x^2-4} < \varepsilon \iff$

$x^2 > \frac{17}{\varepsilon}+4$ (2) $\Rightarrow x < x_\varepsilon = -\sqrt{\frac{17}{\varepsilon}+4}$ (2)

2. $\boxed{8}$ a) $f \in C,\ \forall x \neq 1$-re (2)

$f(1+0) = 1/e$ (2), $f(1-0) = a+b$ (2) $\Rightarrow a+b=\frac{1}{e}$ (2)

$\boxed{10}$ b) $f \in D,\ \forall x \neq 1$ (2), $f'(x) = e^{-x^2}(1-2x^2),\ x \geq 1$ (2)

$f'(1+0) = -1/e$ (2) $f'(x) = 2ax,\ f'(1-0) = 2a$ (3)

$\Rightarrow a = -1/2e,\ b = 3/2e$ (1)

3. $\boxed{12}$ a) ÉT: $x < -2-\sqrt{6},\ x > -2+\sqrt{6}$ (4) értékkészlet $[0,\infty)$ (

derivált $f'(x) = \frac{2x+4}{2\sqrt{x^2+4x-2}}$ (4)

b) $\lim\limits_{x\to\infty} \frac{f(x)}{x} = \lim\limits_{x\to\infty} \sqrt{1+\frac{4}{x}-\frac{2}{x^2}} = 1$ (2) (3)
$\boxed{5}$

c) $\lim\limits_{x\to\infty}\left(\sqrt{x^2+4x-2}-x\right) = \lim\limits_{x\to\infty} \frac{4x-2}{\sqrt{\dots}+x} = \lim\limits_{x\to\infty} \frac{4-\frac{2}{x}}{\sqrt{1+\frac{4}{x}-\frac{2}{x^2}}+1}$ (3) (2)
$\boxed{7}$

$= 2$ (2)

4. a) $\boxed{6}$ $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} 5x}{\sin 3x} = \lim\limits_{x\to 0} \dfrac{\sin 5x}{\sin 3x} = \lim \dfrac{5\cos 5x}{3\cos 3x} = \dfrac{5}{3}$ (2)

(2) (2)

$\boxed{7}$ b) $\lim\limits_{x\to 0} \left(\dfrac{1}{\operatorname{tg} 2x} - \dfrac{1}{\sin 2x}\right) = \lim\limits_{x\to 0} \dfrac{\sin 2x - \operatorname{tg} 2x}{\sin 2x \operatorname{tg} 2x}$ (2)

$= \lim\limits_{x\to 0} \dfrac{\cos 2x - 1}{\sin 2x} = \lim\limits_{x\to 0} \dfrac{-2\sin 2x}{2\cos 2x} = 0$

(2) (3)

$\boxed{7}$ c) $\lim\limits_{x\to 0} (\sqrt{1+x} - x)^{\frac{1}{2x}} = \lim\limits_{x\to 0} e^{\ln(\sqrt{1+x}-x)/2x}$ (2)

$\lim\limits_{x\to 0} \dfrac{\ln(\sqrt{1+x}-x)}{2x} = \dfrac{1}{2} \lim\limits_{x\to 0} \dfrac{\frac{1}{\sqrt{1+x}-x}\left(\frac{1}{2\sqrt{1+x}} - 1\right)}{1}$ (3)

$= -\dfrac{1}{4} \Longrightarrow e^{-1/4}$ (2)

$\boxed{6}$ d) $\lim\limits_{x\to -\infty} \dfrac{e^{-2x}(e^{+5x} - e^{-5x})}{e^{6x} + e^{-6x}} = \lim\limits_{x\to \infty} \dfrac{e^{3x} - e^{-7x}}{e^{6x} + e^{-6x}}$ (2)

$= \lim\limits_{x\to -\infty} \dfrac{e^{9x} - e^{-x}}{e^{12x} + 1} = \dfrac{0 - (+\infty)}{0 + 1} = -\infty$ (2) (2)

5. $f(x) = \dfrac{-x^3 + 1}{x^2}$ $\boxed{7}$ a) $f'(x) = -1 - \dfrac{2}{x^3}$ $\begin{cases} > 0, & -\sqrt[3]{2} < x < 0 \\ < 0, & x > 0,\ x < -\sqrt[3]{2} \end{cases}$ (4)

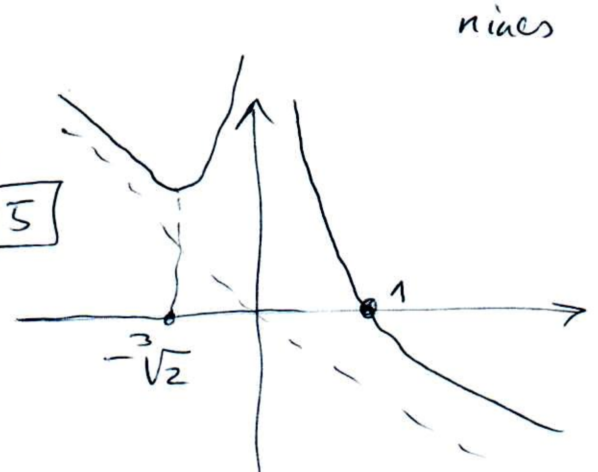
(1)

$x = -\sqrt[3]{2}$ lok. min. (2)

$\boxed{8}$ b) $f''(x) = 6/x^4$ (2), $f'' \geq 0$, $\forall x$ konvex (3)

nincs inflexiós pont! (3)

c) $\boxed{5}$



IMSC: ugyanaz mint az α-ban!