Inverz Rodriguez képlet

$$\overline{Rot}(t,\varphi) = C_\varphi \bar{I} + (1-C_\varphi)[t \circ t] + S_\varphi [t\times]$$

$$\begin{bmatrix} C_\varphi + (1-C_\varphi)t_x^2 & (1-C_\varphi)t_x t_y - S_\varphi t_z & (1-C_\varphi)t_x t_z + S_\varphi t_y \\ (1-C_\varphi)t_y t_x + S_\varphi t_z & C_\varphi + (1-C_\varphi)t_y^2 & (1-C_\varphi)t_y t_z - S_\varphi t_x \\ (1-C_\varphi)t_z t_x - S_\varphi t_y & (1-C_\varphi)t_z t_y + S_\varphi t_x & C_\varphi + (1-C_\varphi) + t_z^2 \end{bmatrix} = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix}$$

$$3C_\varphi + (1-C_\varphi)(t_x^2 + t_y^2 + t_z^2) = l_x + m_y + n_z$$

$$C_\varphi = \frac{l_x + m_y + n_z - 1}{2}$$

$$S_\varphi \xrightarrow{atan2} \varphi$$

$$2S_\varphi t_x = m_z - n_y$$

$$2S_\varphi t_y = n_x - l_z$$

$$2S_\varphi \cancel{t}_z = l_y - m_x$$

$$t_x = \sqrt{\frac{l_x - C_\varphi}{1 - C_\varphi}} \cdot \mathrm{sign}(m_z - l_y)$$

$\vdots$

$$t_y = \sqrt{\frac{m_y - C_\varphi}{1 - C_\varphi}} \cdot \mathrm{sign}(n_x - l_z)$$

$$t_z = \sqrt{\frac{n_z - C_\varphi}{1 - C_\varphi}} \cdot \mathrm{sign}(l_y - m_x)$$

$C_\varphi \approx 1$, szingularitás

$\varphi \approx 0$ => előre beállított t tengelyt választunk

Euler szögek z, y, z forgatás

φ, ϑ, ψ

Direkt Euler, inverz Euler

$$\mathrm{Euler}(\varphi,\vartheta,\psi) = S_\varphi C_\vartheta C_\psi + C_\varphi C_\psi \ldots$$

$$\begin{bmatrix} C_\varphi C_\vartheta C_\psi - C_\varphi S_\psi & -C_\varphi C_\vartheta S_\psi - S_\varphi C_\psi & C_\varphi S_\vartheta \\ S_\varphi C_\vartheta C_\psi + C_\varphi S_\psi & -S_\varphi C_\vartheta S_\psi + C_\varphi C_\psi & S_\varphi S_\vartheta \\ -S_\vartheta C_\psi & S_\vartheta S_\psi & C_\vartheta \end{bmatrix} = \begin{matrix} l_x & n_x & n_x \\ l_y & \ldots & \\ & & \ldots \end{matrix}$$

szingularitás $\quad 0,0,1$

$\qquad\qquad\quad 0,0,-1$

$$T_\varphi = \frac{n_y}{n_x} \xrightarrow[2x]{\text{atan}} \varphi$$

$$\left.\begin{aligned} S_\vartheta &= S_\varphi n_y + C_\varphi n_x \\ C_\vartheta &= n_z \end{aligned}\right\} \xrightarrow{\text{atan2}} \vartheta$$

S_ψ

$C_\psi \quad \ldots$

RPY $\quad Z \; y \; X$ $\qquad$ Matlab eps

$\varphi \; \vartheta \; \psi$

Homogén koord

$$\begin{bmatrix} \bar{\bar{A}}_1 & \bar{r}_1 \\ \bar{0}^T & 1 \end{bmatrix} \begin{bmatrix} \bar{\bar{A}}_2 & \bar{r}_2 \\ \bar{0}^T & 1 \end{bmatrix} = \begin{bmatrix} \bar{\bar{A}}_1 \bar{\bar{A}}_2 & \bar{\bar{A}}_1 \bar{r}_2 + \bar{r}_1 \\ \bar{0}^T & 1 \end{bmatrix}$$

$$\bar{\bar{T}}\,\bar{\bar{T}}^{-1} = \begin{bmatrix} \bar{\bar{A}} & \bar{r} \\ \bar{0}^T & 1 \end{bmatrix} \begin{bmatrix} \bar{\bar{A}}_{inv} & \bar{r}_{inv} \\ \bar{0} & 1 \end{bmatrix} = \begin{bmatrix} \bar{\bar{I}}_3 & \bar{0} \\ \bar{0}^T & 1 \end{bmatrix}$$

$$\bar{\bar{A}}\,\bar{\bar{A}}_{inv} = \bar{\bar{I}}_3 \Rightarrow \bar{\bar{A}}_{inv} = \underbrace{\bar{\bar{A}}^{-1}} = \bar{\bar{A}}^T$$

Rodriguez φ helyett $-\varphi$

csak ortonormált mátrixokra

Mivel jobbsodrású is, determináns $= 1$

$$\bar{\bar{A}}\,\bar{r}_{inv} + \bar{r} = \underline{0} \Rightarrow \bar{r}_{inv} = -\bar{\bar{A}}^{-1}\bar{r} = -\bar{\bar{A}}^{T}\bar{r}$$

Denavit Hart

$$T_{i-1,i} = Rot(z_{i-1}, \vartheta_i)\,Trans$$

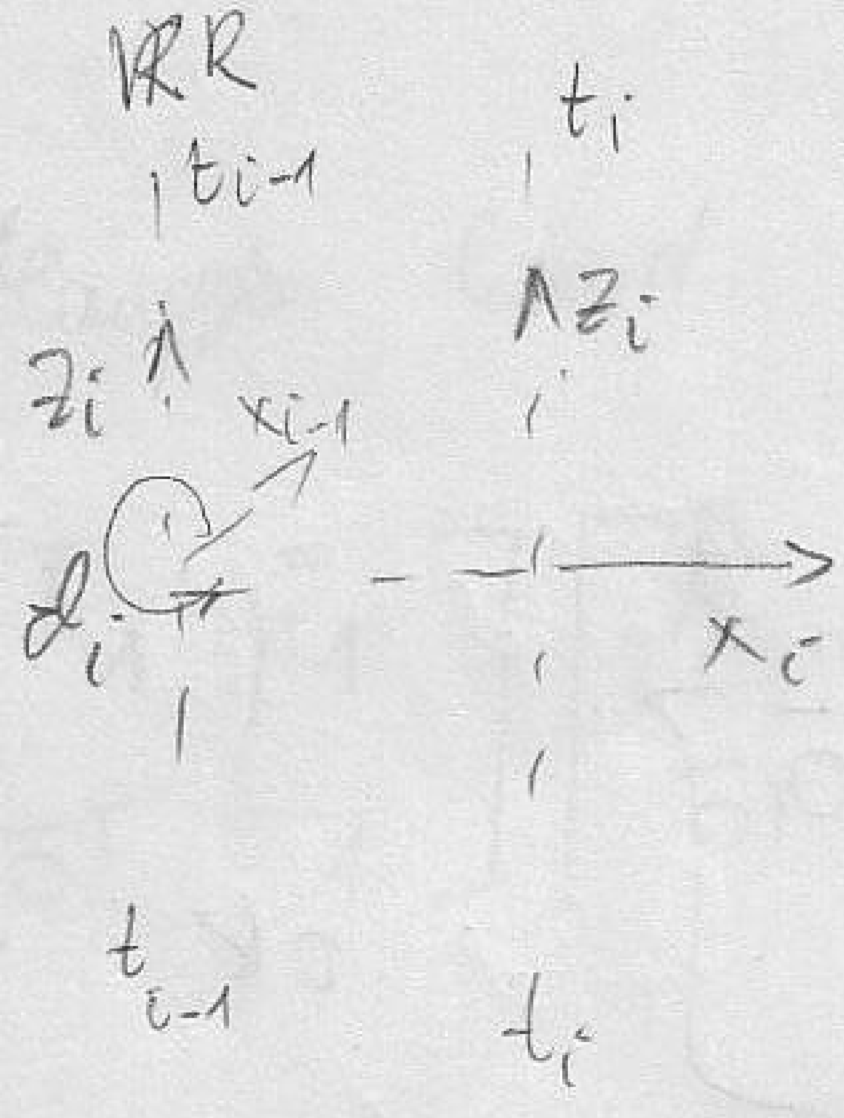
$$\bar{\bar{T}} = \begin{bmatrix} \bar{\bar{A}} & \bar{r} \\ \dots & \end{bmatrix} = \begin{bmatrix} \bar{\bar{I}}_3 & \bar{r} \\ & \dots \end{bmatrix} \begin{bmatrix} \bar{\bar{A}} & \bar{0} \\ & \dots \end{bmatrix} \quad \text{ok}$$

~~[illegible]~~

megcserélve

nem lehet megadni

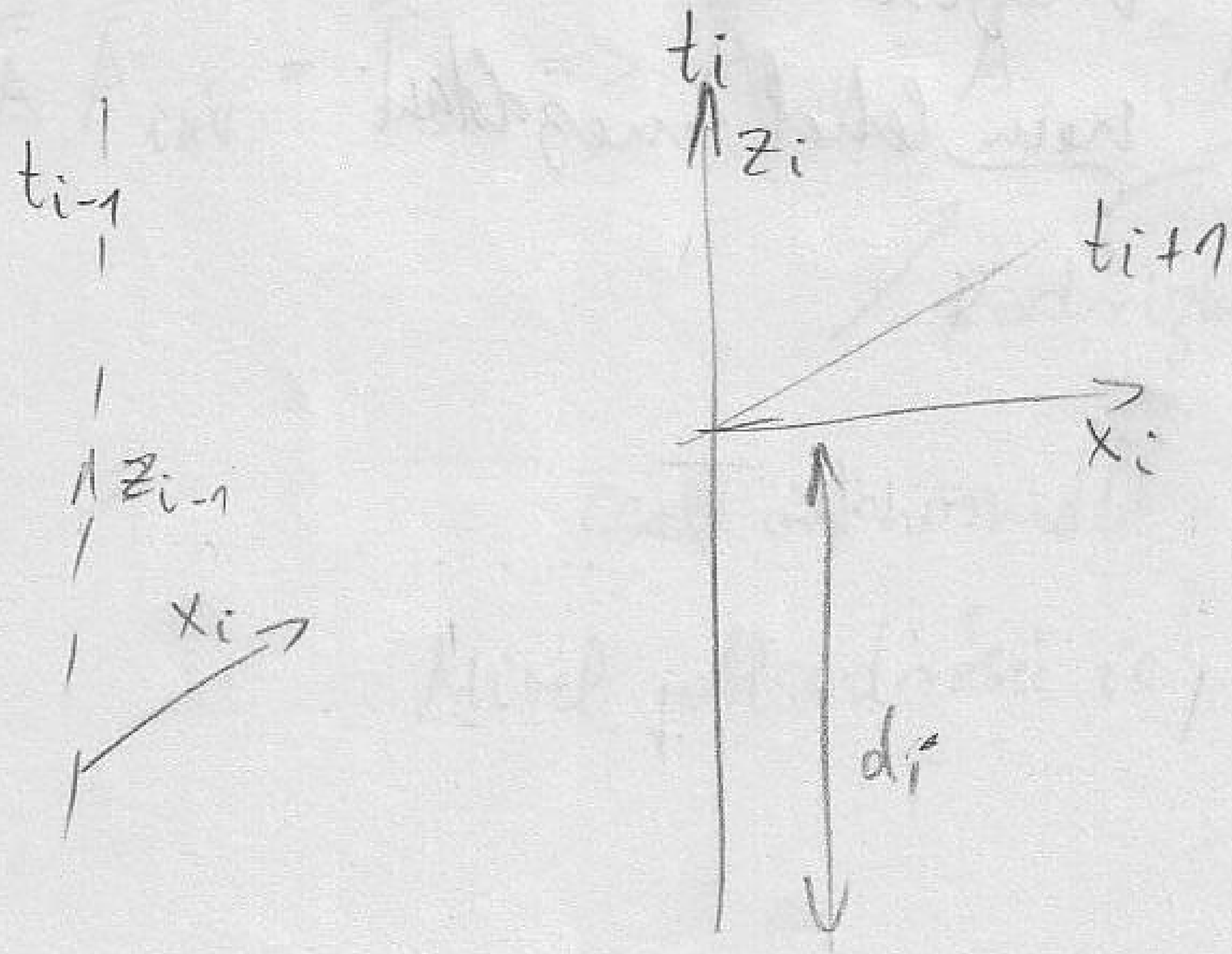
√... Denavit Hartenberg általános alak



$d_i = 0$

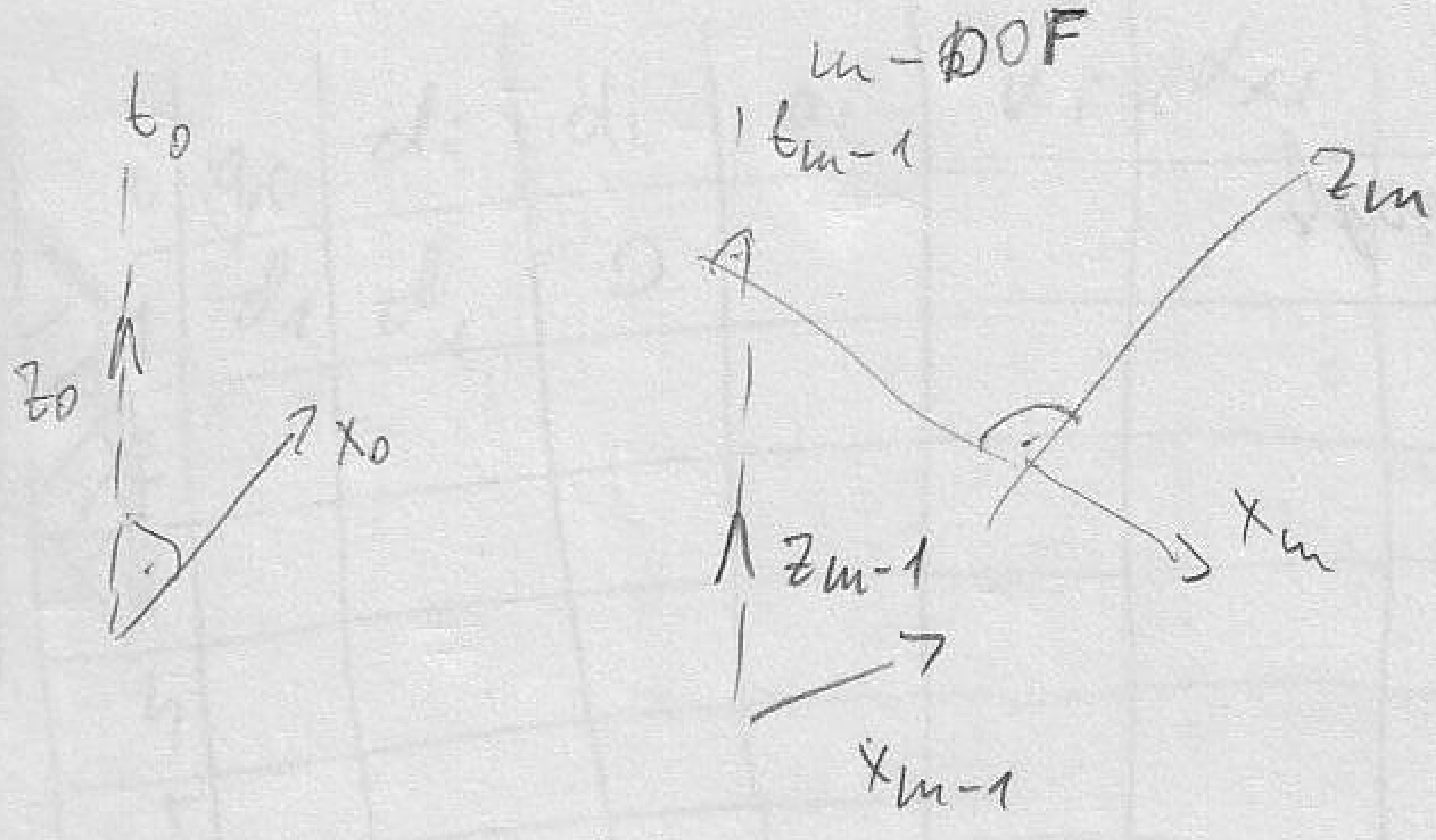
$\alpha_i = 0$

RTR



$\alpha = 0°$

⑥



n-DOF

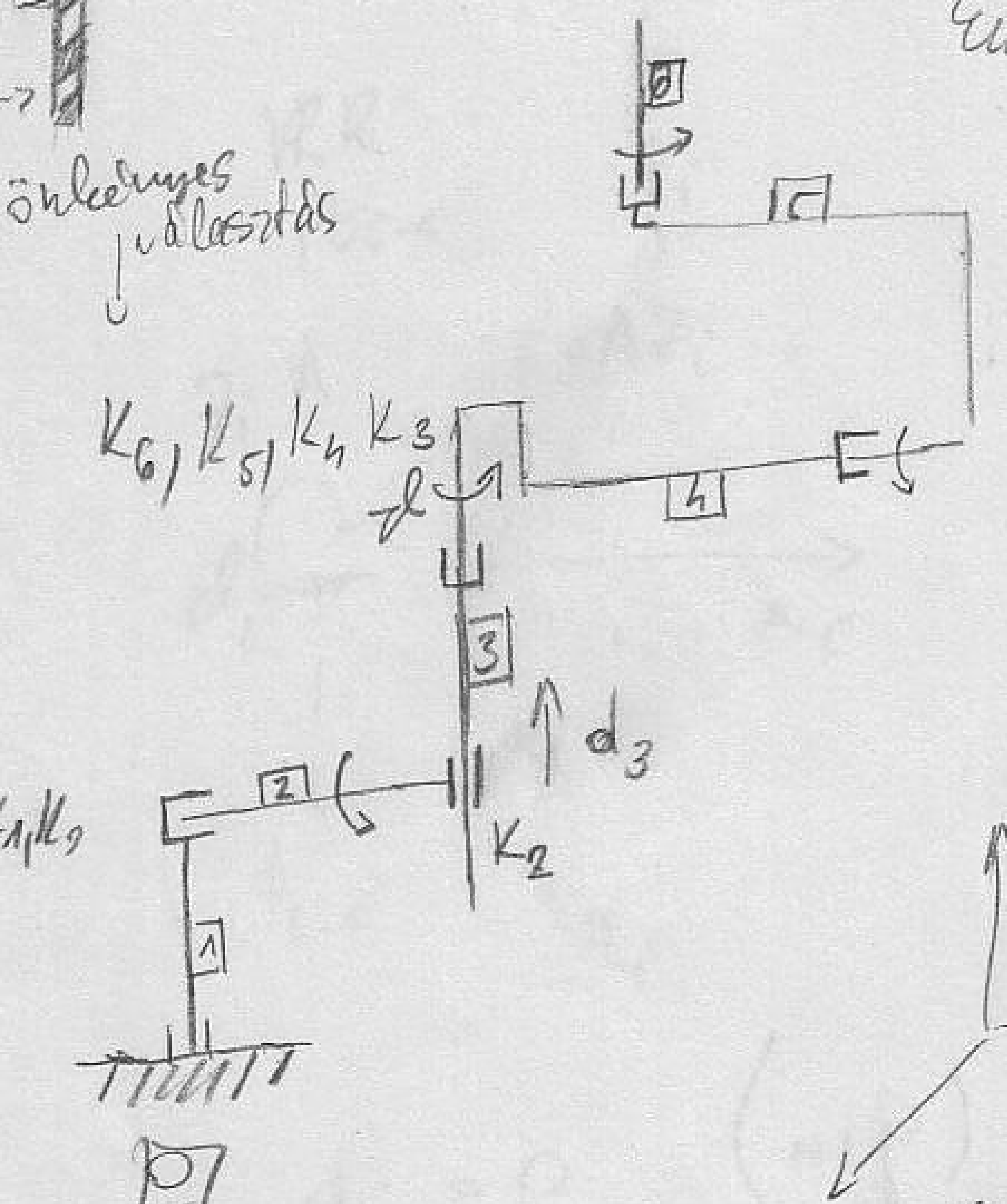
$$\bar{\bar{T}}_{0,n}(\bar{q}) = \bar{\bar{T}}_{01}(q_1)\,\bar{\bar{T}}_{12}(q_2)\cdots\bar{\bar{T}}_{n-1,n}(q_n)$$

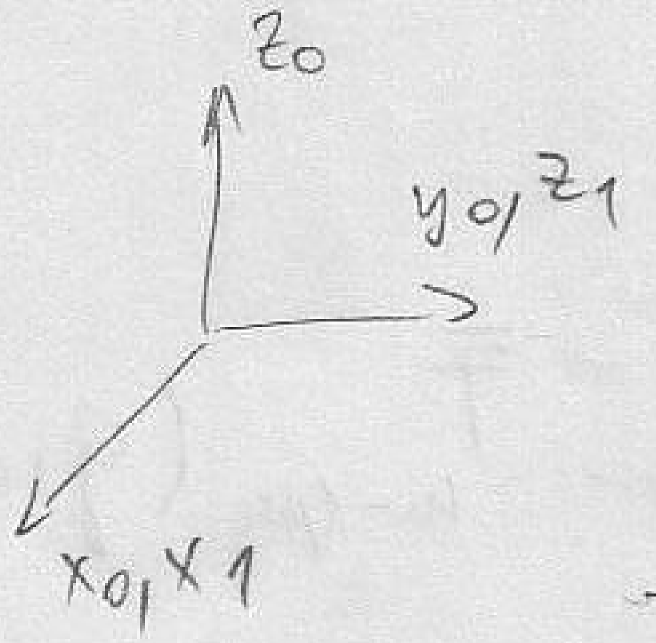
$$\begin{tikzcd}
K_B \arrow[r, "\bar{\bar{T}}_{B0}"] \arrow[rrr, bend right, "\bar{\bar{T}}_{BE}"'] & K_0 \arrow[r, "\bar{\bar{T}}_{0n}(\bar{q})"] & K_n \arrow[r, "\bar{\bar{T}}_{nE}"] & K_E
\end{tikzcd}$$

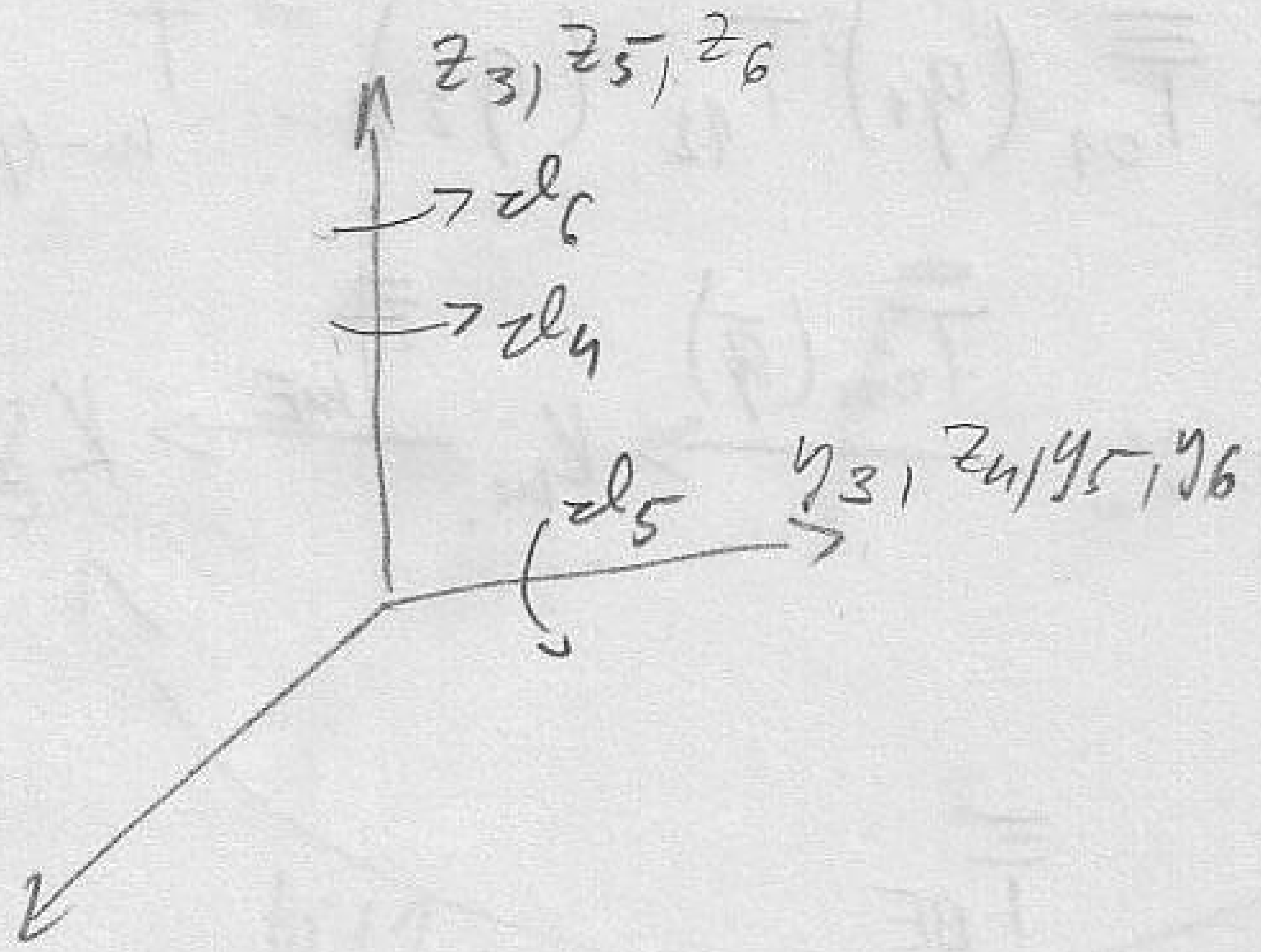
$$\bar{\bar{T}}_{BE} = \bar{\bar{T}}_{B0}\,\bar{\bar{T}}_{0n}(\bar{q})\,\bar{\bar{T}}_{nE} \Rightarrow \boxed{\bar{\bar{T}}_{0n}(\bar{q}) = \bar{\bar{T}}_{B0}^{-1}\,\bar{\bar{T}}_{BE}\,\bar{\bar{T}}_{nE}^{-1}}$$

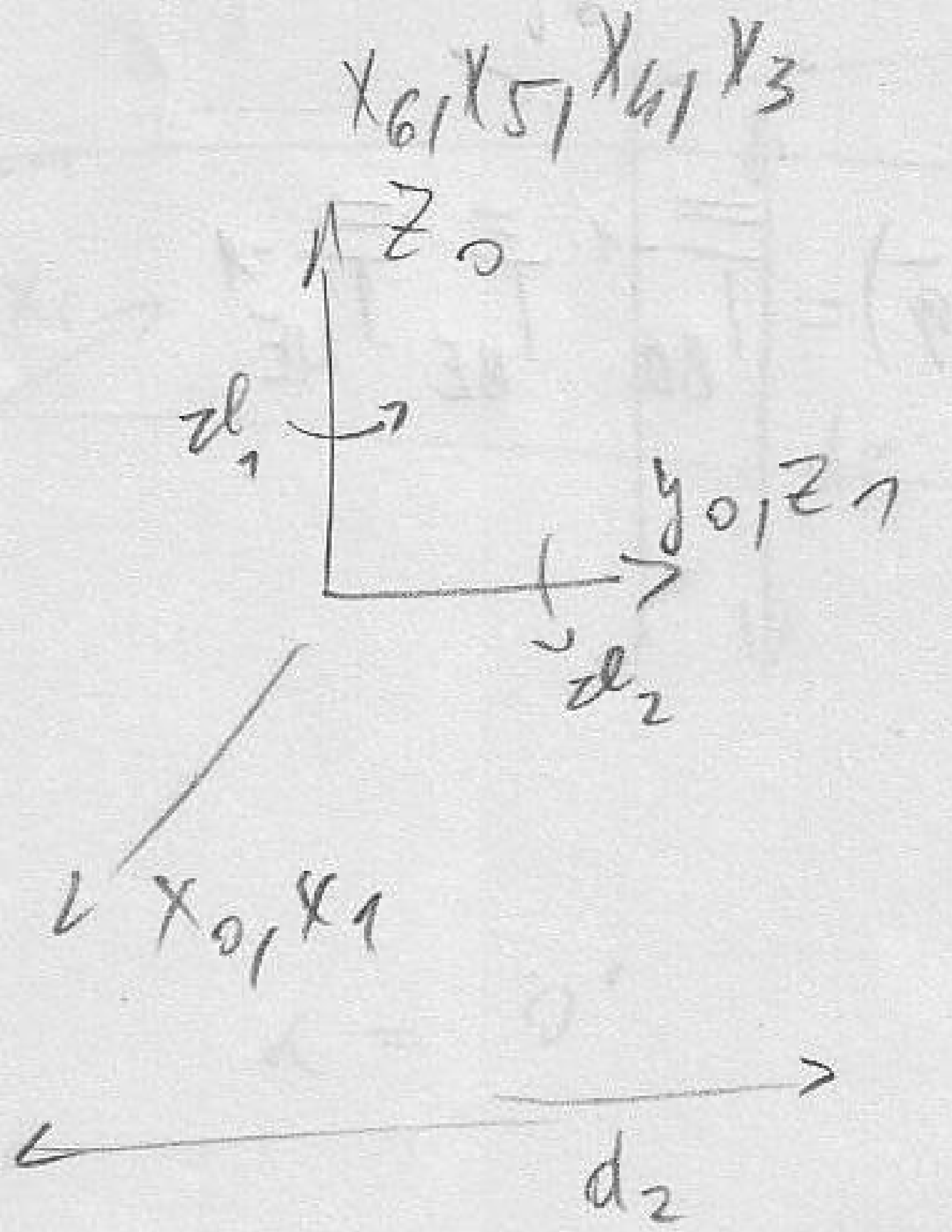
Stanford robot : RRT $\underbrace{RRR}_{\text{Euler szögek}}$

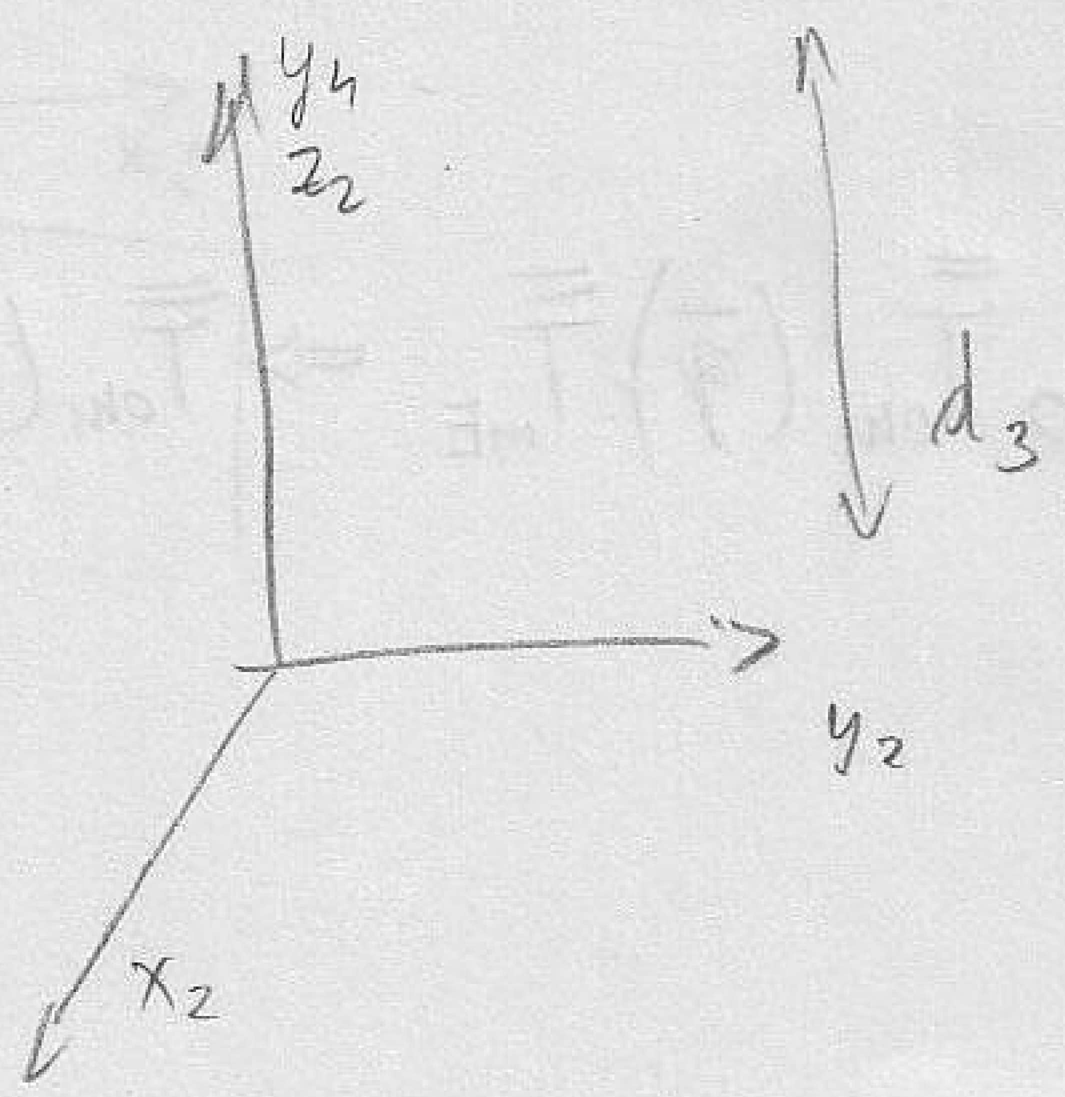
önkényes választás











i	q_i	ϑ_i	d_i	a_i	α_i	$S_{\alpha i}$	$C_{\alpha i}$
1	ϑ_1	ϑ_1	0				
2							
3							
4							
5							
6							

$$T_{i-1,i} = \begin{bmatrix} C_{\vartheta_i} & -S_{\vartheta_i} C_{\alpha_i} & C_{\vartheta_i} S_{\alpha_i} & a_i C_{\vartheta_i} \\ S_{\vartheta_i} & C_{\vartheta_i} C_{\alpha_i} & -C_{\vartheta_i} S_{\alpha_i} & a_i S_{\vartheta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{01} = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ⓡ $C_{\theta_i} = C_{q_i} = C_i$

$$\bar{\bar{T}}_{12} = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overline{\overline{T}}_{34} = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overline{\overline{T}}_{45} = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overline{\overline{T}}_{56} = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

z_2 tengely körüli forgatás

$$\overline{\overline{T}}_{06} = \overline{\overline{T}}_{03}\,\overline{\overline{T}}_{36}$$

$$\overline{\overline{T}}_{36} = \text{Euler}(q_4, q_5, q_6)$$

$\varphi \quad \theta \quad \psi$

$$T_{02} = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & -d_2S_1 \\ S_1C_2 & C_1 & S_1S_2 & d_2C_1 \\ -S_2 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overline{\overline{T}}_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{03} = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & d_3C_1S_2 - d_2S_1 \\ S_1C_2 & C_1 & S_1S_2 & d_3S_1S_2 + d_2C_1 \\ -S_2 & 0 & C_2 & d_3C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} l_x & m_x & \\ & \vdots & \end{bmatrix}$$

$$l_x = C_1C_2(C_4C_5C_6 - S_4S_6) - S_1(S_4C_5C_6 + C_4C_6) +$$

$$+ C_1C_2(-S_5C_6)$$

⋮

HF

$$\bar{\bar{A}}_{06} = \bar{\bar{A}}_{03}\,\bar{\bar{A}}_{36} \Rightarrow \bar{\bar{A}}_{36} = \bar{\bar{A}}^{T}_{03}(q_1, q_2, q_3)\bar{\bar{A}}_{06} \xrightarrow{\text{inv. Euler}} q_4, q_5, q_6$$

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$$\bar{r}_{06} = \bar{r}_{03} \to \left.\begin{aligned} p_x &= d_3 C_1 C_2 - d_2 S_1 \\ p_y &= d_3 S_1 S_2 + d_2 C_1 \end{aligned}\right\} \to q_1, \dots q_3$$

Direkt-geometriai feladat:

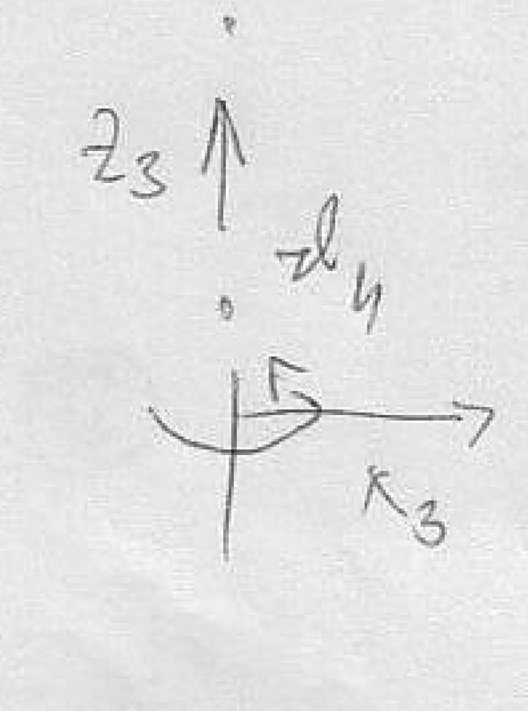
$$\bar{q} \to \bar{\bar{T}}_{06}(\bar{q})$$

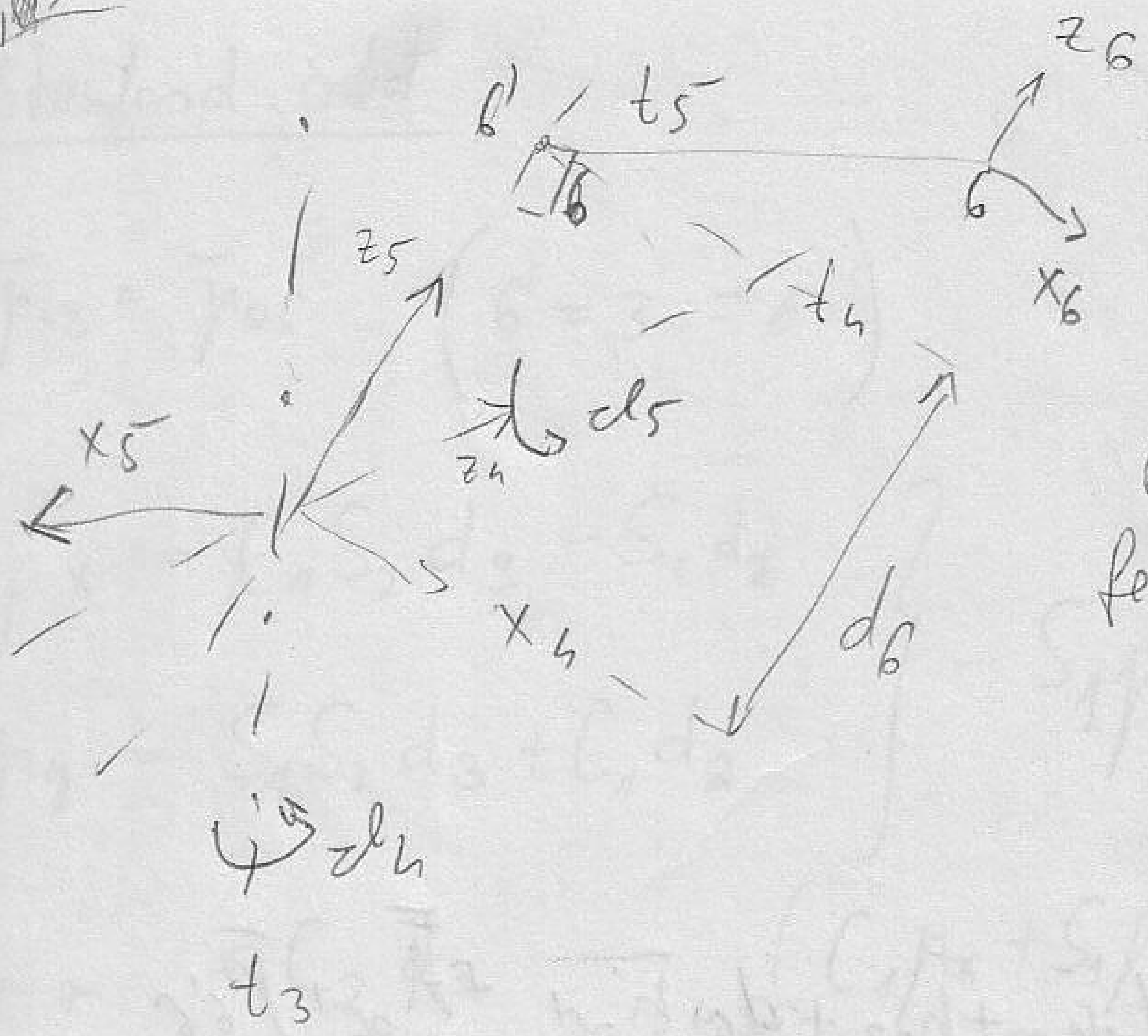
Inverz geometria

$$\bar{\bar{T}}_{06}(\bar{q}) \to \bar{q}$$

Meg tudjuk-e oldani Newton-Raphson nélkül?

1, módszer: ha az utolsó három csukló rotációs, és a tengelyek egy közös ponton mennek át, akkor az inverz geometriai feladat 2 részfeladatra bontható: pozicionálás
orientálás





(t_5 tengelyen egy fiktív irányt is célszerű felvenni)

	q_i	ϑ_i	d_i	a_i	α_i
4	ϑ_4	ϑ_4	d_4	0	α_4
5	ϑ_5	ϑ_5	0	0	α_5
6'	ϑ_6	ϑ_6	d_6'	0	0

$$\overline{\overline{T}}_{06} = \overline{\overline{T}}_{03} \begin{bmatrix} * & * & * & 0 \\ * & * & * & 0 \\ * & * & * & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} * & * & * & 0 \\ * & * & * & 0 \\ * & * & * & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} * & * & 0 & 0 \\ * & * & 0 & 0 \\ 0 & 0 & 1 & d_6' \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} * & * & 0 & 0 \\ * & * & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_6' \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

előbb eltolás, aztán forgatás

„ez mindig megtehető"

speciálisan most igaz

$$\begin{bmatrix} \overline{A}_{66} & \overline{p}_{6'6} \\ \overline{0}^T & 1 \end{bmatrix}$$

$$\bar{\bar{A}}_{06} = \bar{\bar{A}}_{06'} \bar{\bar{A}}_{6'6}$$

likjerrük   ismerjük

$$d_4 \bar{n}_{03} + \bar{r}_{03} \qquad d_4 \bar{n}_{03} + \bar{r}_{03} + d_{6'} \bar{n}_{06'} + \bar{\bar{A}}_{06'} \bar{r}_{6'6}$$

$$\bar{r}_{06} = d_4 \bar{n}_{03} + \bar{r}_{03} + d_{6'} n_{06'} + \bar{\bar{A}}_{06'} \bar{r}_{6'6}$$

① $\bar{r}_{03} + d_4 \bar{n}_{03} = \bar{r}_{06} - d_{6'} \bar{n}_{06'} - \bar{\bar{A}}_{06'} \bar{r}_{6'6} \longrightarrow q_1, q_2, q_3$

② $\bar{\bar{A}}_{06'} = \bar{\bar{A}}_{03} \bar{\bar{A}}_{36'} \Rightarrow \bar{\bar{A}}_{36'} = \bar{\bar{A}}_{03}^{T} \bar{\bar{A}}_{06'}$

$$\bar{\bar{A}}_{36'} = \begin{bmatrix} L_x & M_x & N_x \\ L_y & M_y & N_y \\ L_z & M_z & N_z \end{bmatrix}$$

stanford robot

$\overline{p}_{03} = \overline{p}_{06} \quad (6' = 3 = 6)$

$$\left.\begin{aligned} p_x &= C_1 S_2 d_3 - S_1 d_2 \\ p_y &= S_1 S_2 d_3 + C_1 d_2 \end{aligned}\right\} -S_1 p_x + C_1 p_y = d_2$$

$$p_z = C_2 d_3 \qquad (C_1 p_x + S_1 p_y) C_2 - p_z S_2 = 0$$

$$d_3 = (C_1 p_x + S_1 p_y) S_2 + p_z C_2 \qquad (1x) \quad q_3$$

$$\overline{\overline{A}}_{06} = \overline{\overline{A}}_{03} \overline{\overline{A}}_{36}$$

$$\overline{\overline{A}}_{36} = \overline{\overline{A}}_{03}^T \overline{\overline{A}}_{06} = \begin{bmatrix} L_x & M_x & N_x \\ L_y & M_y & N_y \\ L_z & M_z & N_z \end{bmatrix} = \text{Euler}(..,..,..)$$

$\Downarrow$ inverz Euler

$$T_4 = \frac{N_y}{N_x} \xrightarrow{\text{atan}} q_4 \quad (\overset{2\pi,0}{\downarrow}\, 2x)$$

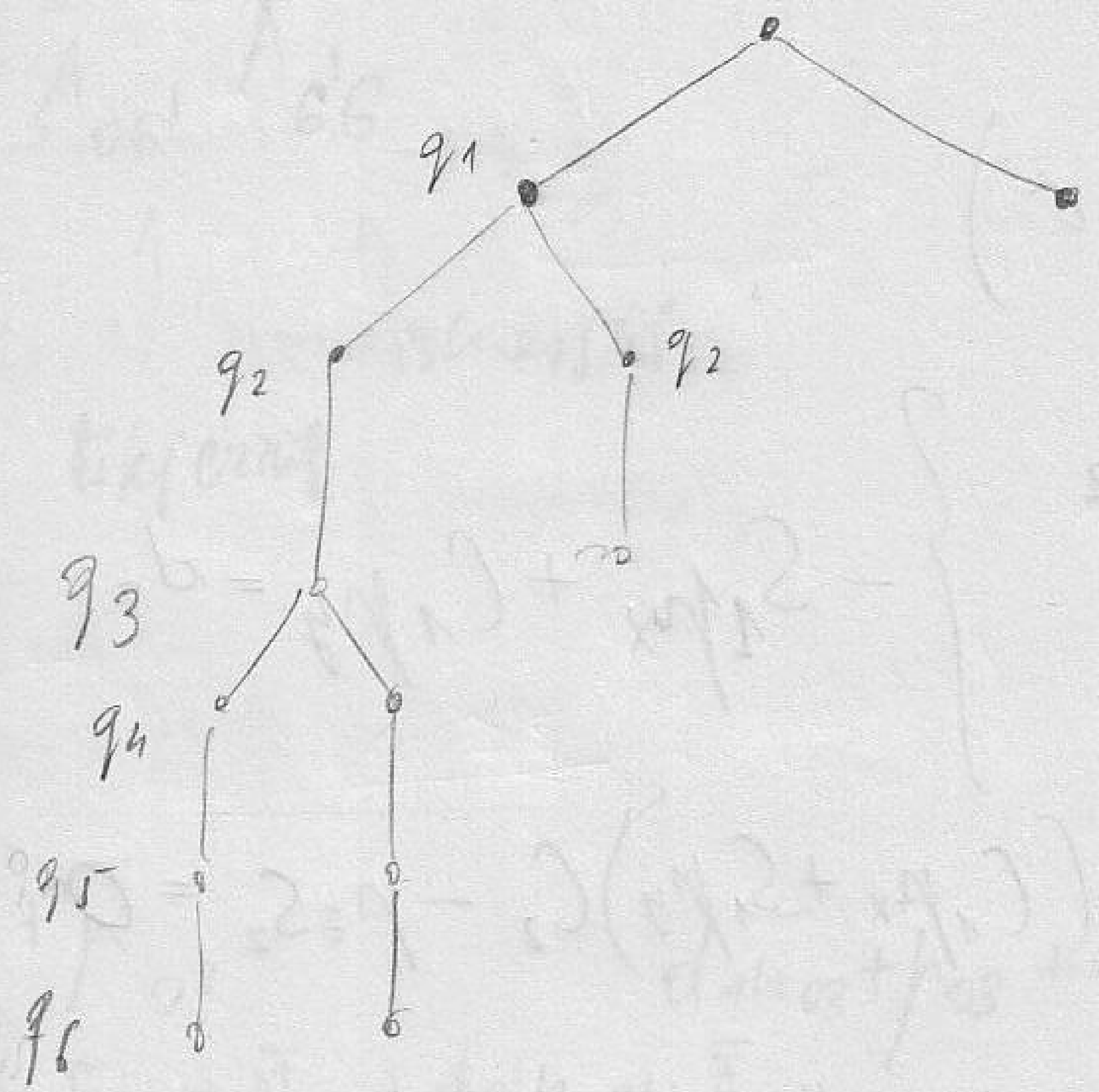
$$S_5 = S_4 N_y + C_4 N_x$$

$$C_5 = N_z$$

$$S_6 = -S_4 L_x + C_4 L_y$$

$$\longrightarrow (1x)\ q_6$$

$$C_6 = -S_4 M_x + C_4 M_y$$



összesen gyors m.o.: valós időben az elő „20"
csuklóértékekhez közelieket választunk.

(szingularitás, d-rend, memória)

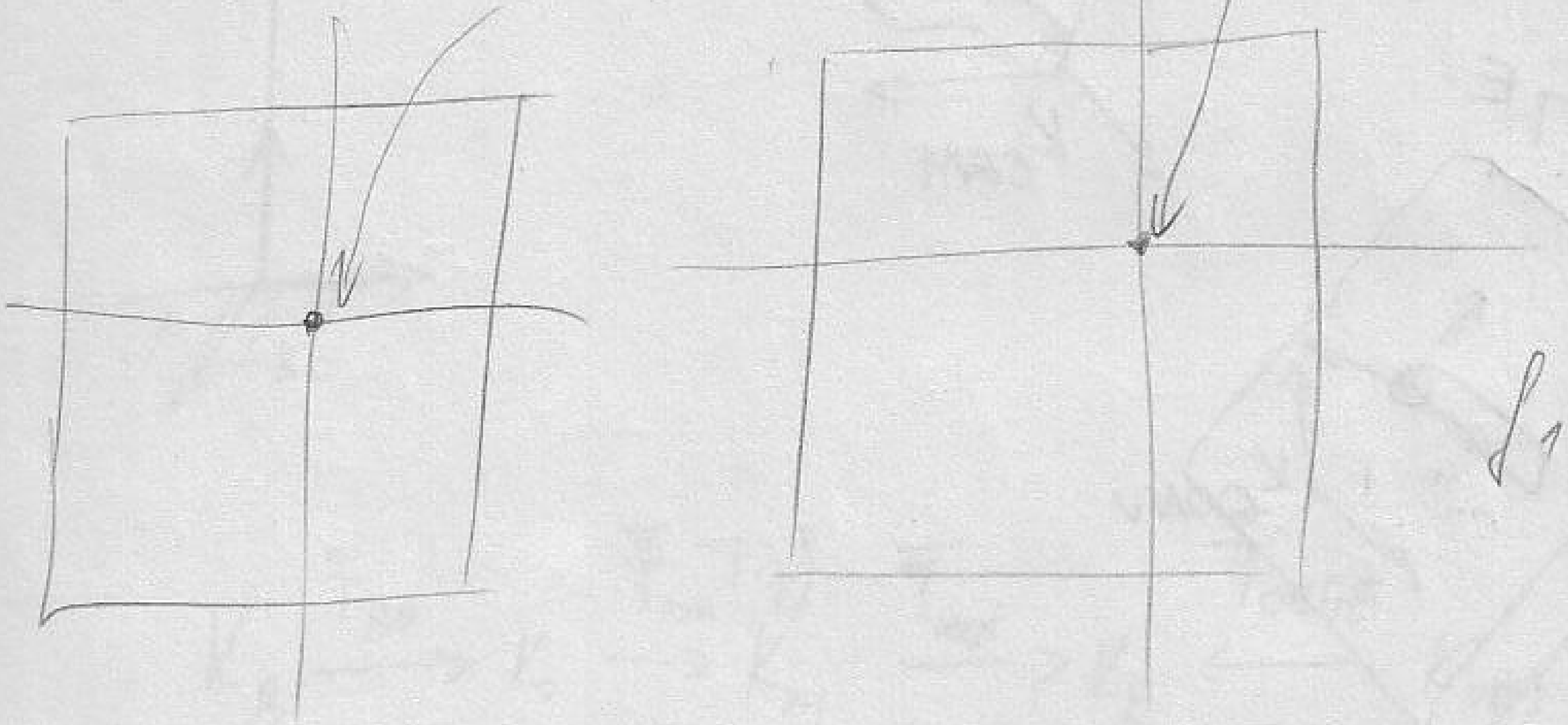
Általános szerkezetű robot

$$\overline{\overline{T}}_{01}\overline{\overline{T}}_{12}\dots\overline{\overline{T}}_{56} = \overline{\overline{T}}_{06} \qquad \Big/ \cdot \overline{\overline{T}}_{01}^{-1}$$

$$\underbrace{\overline{\overline{T}}_{12}\dots\overline{\overline{T}}_{56}}_{\text{nem függ } q_1\text{-től}} = \underbrace{\overline{\overline{T}}_{01}^{-1}}_{f(q_1)}\underbrace{\overline{\overline{T}}_{06}}_{\text{num.}}$$

Constans: C

$f_1(q_1)$



$f_1(q_1) = C \rightarrow q_1$

(Többnyire utolsó 3 tengely egy ponton megy át)

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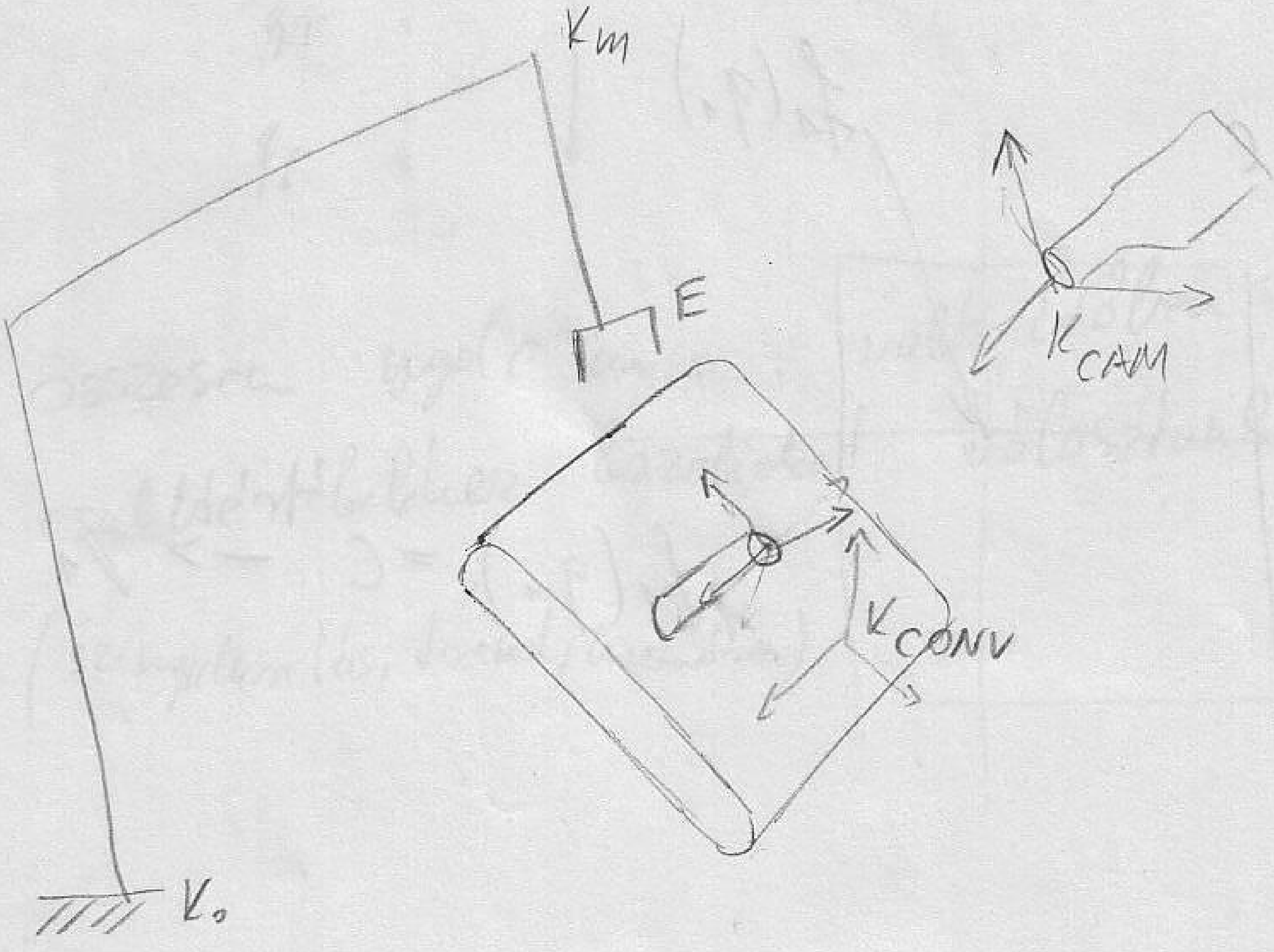
IB310

m. keretben felírva

$$\begin{pmatrix} {}^{m}\overline{v}_m \\ {}^{m}\overline{\omega}_m \end{pmatrix} = \underbrace{\overline{\overline{J}}(\overline{q})}_{\text{Jacobi mátrix}}\,\dot{\overline{q}}$$

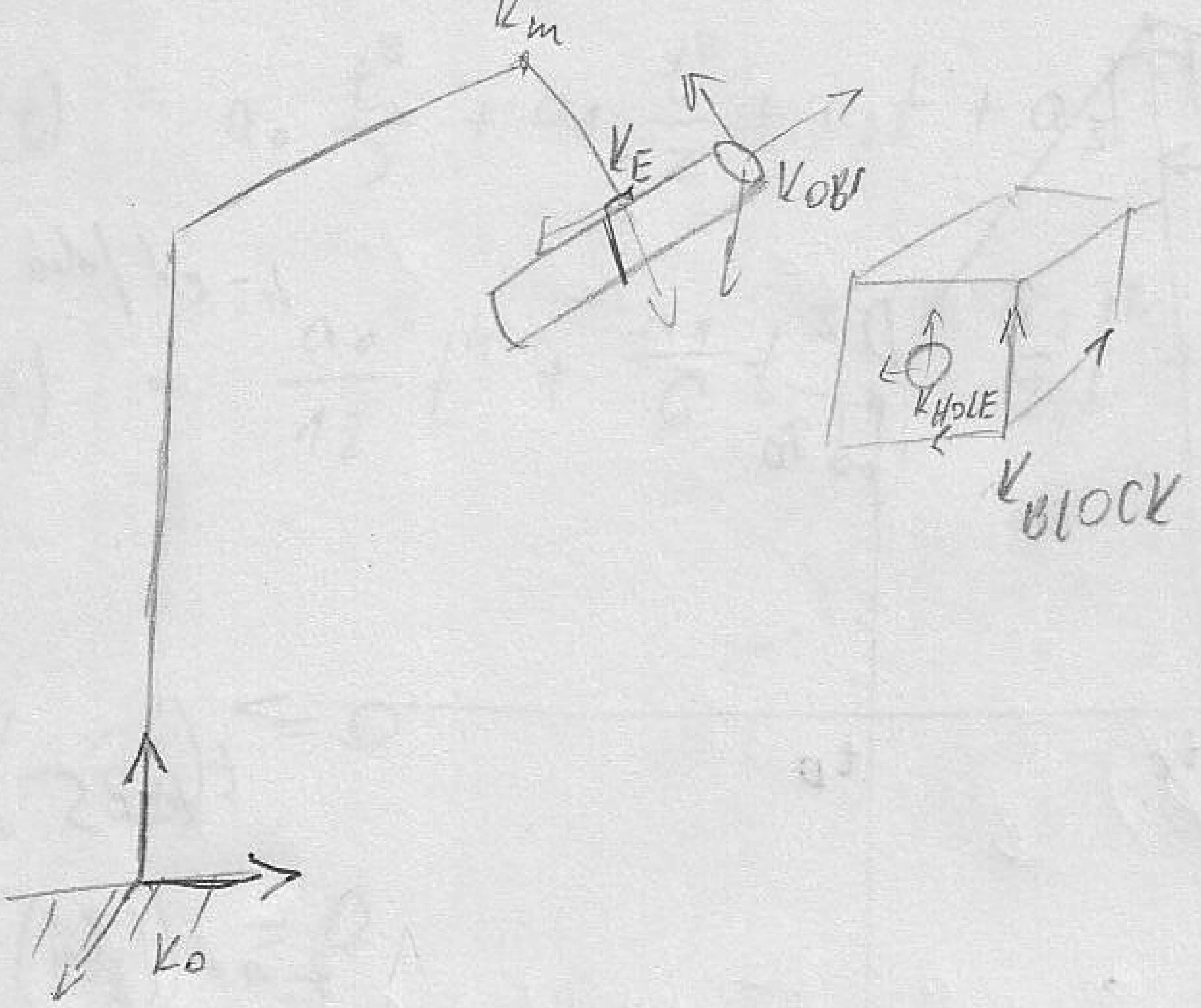
Pályatervezés (csütörtök)



$$\begin{tikzcd}
K_B \arrow[r, "\overline{\overline{T}}_{B,0}"] \arrow[dr, "\overline{\overline{T}}_{B,CONV}"] \arrow[dd, "\overline{\overline{T}}_{B,CAM}"'] & K_0 \arrow[r, "\overline{\overline{T}}_{0,m}(\overline{q})"] & K_m \arrow[r, "\overline{\overline{T}}_{m,E}"] & K_E \\
 & \overline{\overline{T}}_{CONV} \arrow[r, "\overline{\overline{T}}_{CONV,OBJ}"] & K_{OBJ} \arrow[ur] & \\
K_{CAM} \arrow[urr] & & &
\end{tikzcd}$$

$$\bar{\bar{T}}_{0,m} = \underbrace{\bar{\bar{T}}_{B,0}^{-1}\,\bar{\bar{T}}_{B,CONV}\left[\bar{\bar{T}}_{B,CONV}^{-1}\,\bar{\bar{T}}_{B,CAN}\,\bar{\bar{T}}_{CAN,OBJ}\right]}_{\bar{\bar{COORD}}}\,\underbrace{\bar{\bar{T}}_{OBJ,E}}_{\bar{\bar{POS}}}\,\underbrace{\bar{\bar{T}}_{m,E}^{-1}}_{\bar{\bar{TOOL}}^{-1}}$$

poz és orientáció



$$\begin{tikzcd}
K_B \arrow[r, "\bar{\bar{T}}_{BO}"] \arrow[dr, "\bar{\bar{T}}_{B,BLOCK}"'] & K_0 \arrow[r, "\bar{\bar{T}}_{om}(q)"] & K_m \arrow[r, "\bar{\bar{T}}_{m,E}"] & K_E & K_{OBJ} \arrow[l, "\bar{\bar{T}}_{OBJ,E}"'] \\
 & K_{BLOCK} \arrow[rr, "\bar{\bar{T}}_{BLOCK,HOLE}"] & & K_{HOLE} \arrow[ur, "\bar{\bar{T}}_{HOLE,OBJ}"'] &
\end{tikzcd}$$

$$\bar{\bar{T}}_{om} = \underbrace{\bar{\bar{T}}_{BO}^{-1}\,\bar{\bar{T}}_{B,BLOCK}\,\bar{\bar{T}}_{BLOCK,HOLE}}_{\bar{\bar{COORD}}}\,\underbrace{\bar{\bar{T}}_{HOLE,OBJ}}_{\bar{\bar{POS}}}\,\underbrace{\bar{\bar{T}}_{OBJ,E}\,\bar{\bar{T}}_{m,E}^{-1}}_{\left(\bar{\bar{T}}_{m,E}\,\bar{\bar{T}}_{OBJ,E}^{-1}\right)^{-1}}$$

(POSE)

1 tagot megspóroltunk

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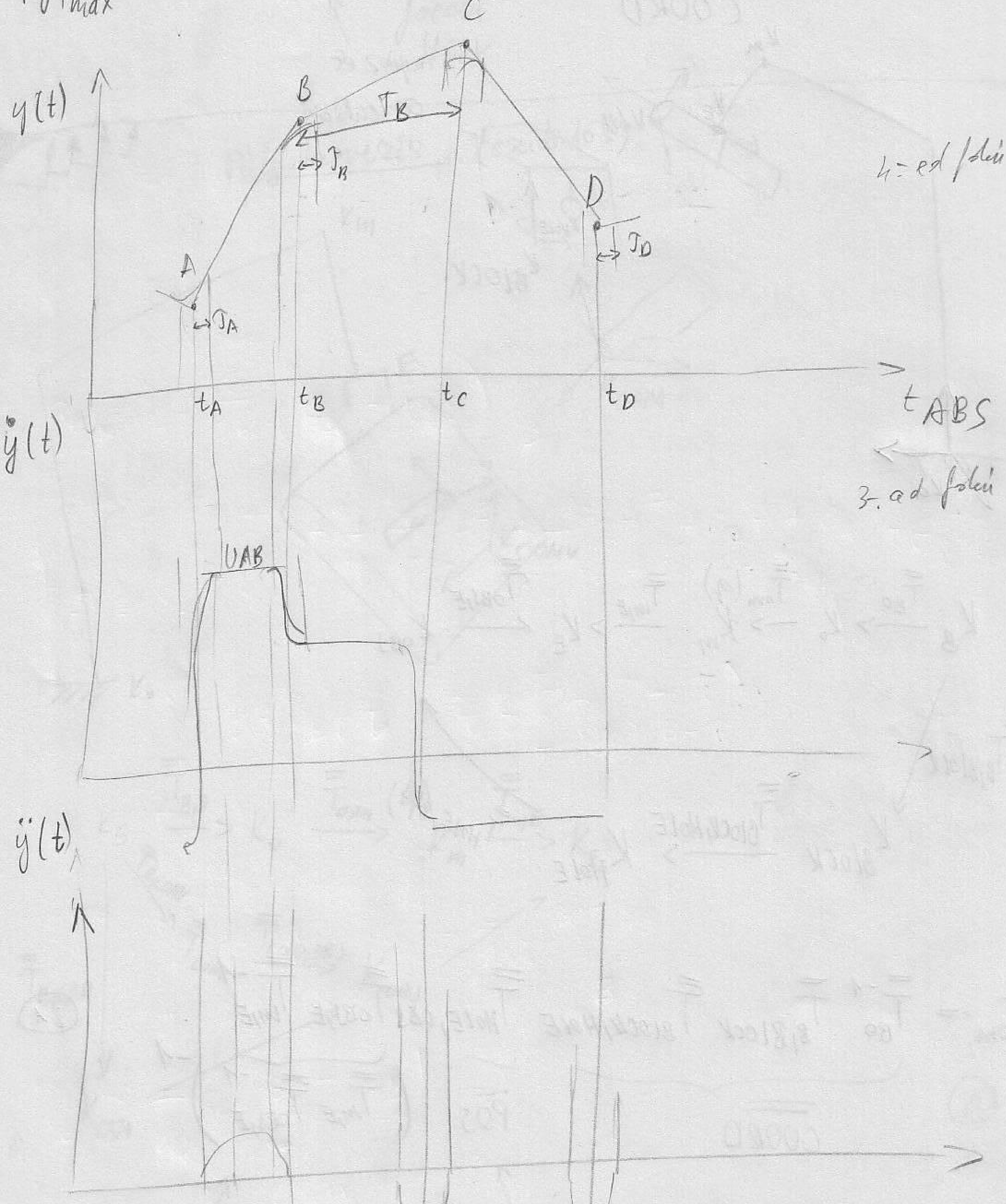
Séma: $\{y_i\} = \{\ldots, A, B, C, D, \ldots\}$,

$\{t_i\} = \{\ldots, t_A, t_B, \ldots\}$, abszolút idő

(majd áttérünk relatív idő)

$|\ddot{y}|_{max}$ adott (gyorsulás)



Ha szakaszosan konstans → jerk i nem jó szakadjuk

$t = t_{abs} - t_B$ relatív idő

$$\ddot{y}(t) = a_0 t^2 + a_1 t + a_2$$

$$\dot{y}(t) = a_0 \frac{t^3}{3} + a_1 \frac{t^2}{2} + a_2 t + a_3$$

$$y(t) = \frac{a_0}{12} t^4 + \frac{a_1}{6} t^3 + \frac{a_2}{2} t^2 + a_3 t + a_4$$

$$\ddot{y}(-\tau_B) = 0$$

$$\ddot{y}(\tau_B) = 0$$

$$\dot{y}(-\tau_B) = v_{AB} = \frac{B - B'}{\tau_B}$$

$$\dot{y}(\tau_B) = v_{BC} = \frac{C - B}{T_B}$$

$$y(\tau_B) = B + v_{BC} \tau_B$$

(23)

$$a_0 = -\frac{3}{4}\,\frac{U_{BC} - U_{AB}}{\tau_B^3}$$

$$a_1 = 0$$

$$a_2 = \frac{3}{4}\,\frac{U_{BC} - U_{AB}}{\tau_B}$$

$$a_3 = \frac{U_{AB} + U_{BC}}{2}$$

$$a_4 = B + \frac{3}{16}\left(U_{BC} - U_{AB}\right)\tau_B$$

$$y(t) = \begin{cases} \frac{a_0}{12}t^4 + \frac{a_1}{6}t^3 + \frac{a_2}{2}t^2 + a_3 t + a_4, & t \in [-\tau_B, \tau_B) \\ B + U_{BC}\,t, & t \in [\tau_B, T_B - \tau_1) \end{cases}$$

$$|\ddot{y}|_{max} \geq |\ddot{y}(0)| = |a_2| = \frac{3}{4}\,\frac{|U_{BC} - U_{AB}|}{\tau_B}$$

$$\tau_B \geq \frac{3}{4}\,\frac{|U_{BC} - U_{AB}|}{|\ddot{y}|_{max}}$$

Eltérés: $-\frac{3}{16}\left(v_{BC} - v_{AB}\right)\tau_B$

$T_B \geq \tau_B + \tau_C$

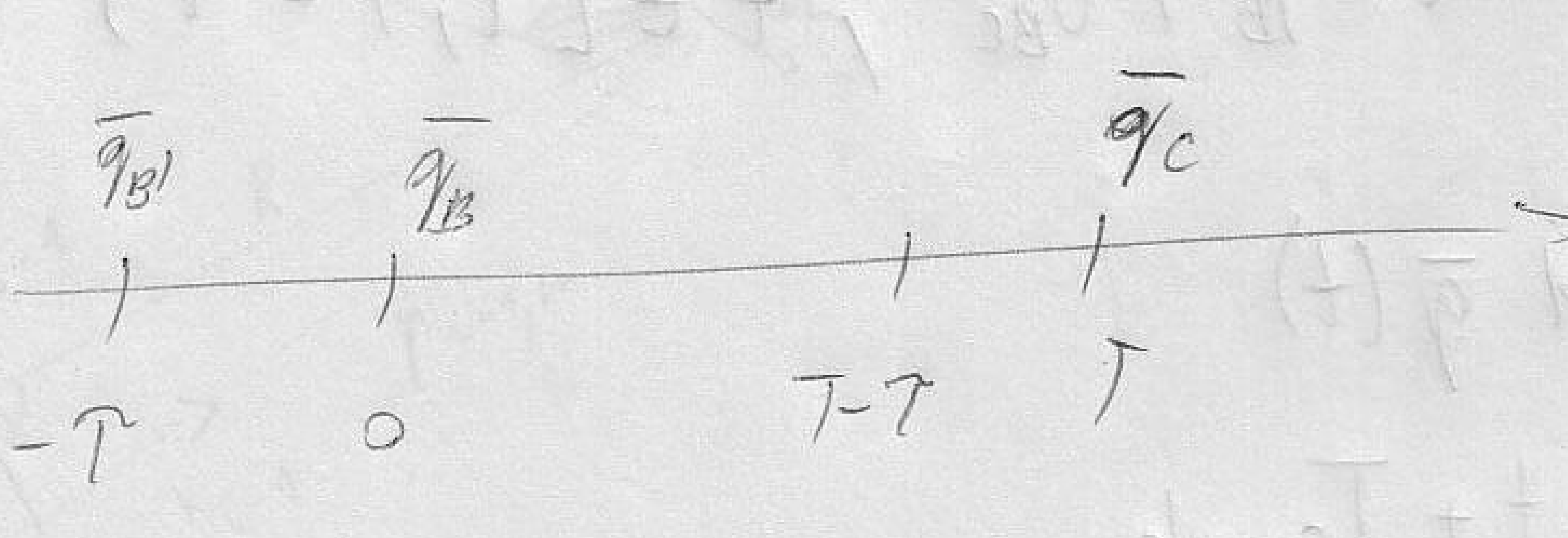
Ha a ref. pontokon át is akarunk menni, megduplázzuk őket

Pályatervezés csuklóváltozókban

$\bar{q} = (q_1, \dots, q_n)^T$

τ legyen konstans

$$\tau = \max_i \left\{ \frac{3}{2} \frac{|\dot{q}_i|_{max}}{|\ddot{q}_i|_{max}} \right\},$$



$\bar{q}_{B'} := \bar{q}(T-\tau)$

$\bar{q}_B := \bar{q}_C$

$$q_C := \text{solve}\{\overline{\overline{COORD}} * \overline{\overline{POS}} * \overline{\overline{TOOL}}^{-1}\}_D$$

$$T_i := \frac{|q_{Ci} - q_{Bi}|}{|\dot{q}_i|_{max}}, \forall i \longrightarrow \max\{\max_i\{T_i\}, 2\tau\} =: T$$

$$\bar{v}_{AB} = \frac{\bar{q}_B - \bar{q}_{B'}}{\tau}, \bar{v}_{BC} = \frac{\bar{q}_C - \bar{q}_B}{T}$$

$$\longrightarrow \bar{a}_0, \dots, \bar{a}_4$$

$$\bar{q}(t) = \begin{cases} \frac{\bar{a}_0}{12} t^4 + \frac{\bar{a}_1}{6} t^3 + \frac{\bar{a}_2}{2} t^2 + \bar{a}_3 t + \bar{a}_4 \\ \bar{q}_B + \bar{v}_{BC}, \quad t \in [\tau, T-\tau) \end{cases}$$

send $\bar{q}(t)$

$t := t + T_{sample}$

i $t < T - \tau$

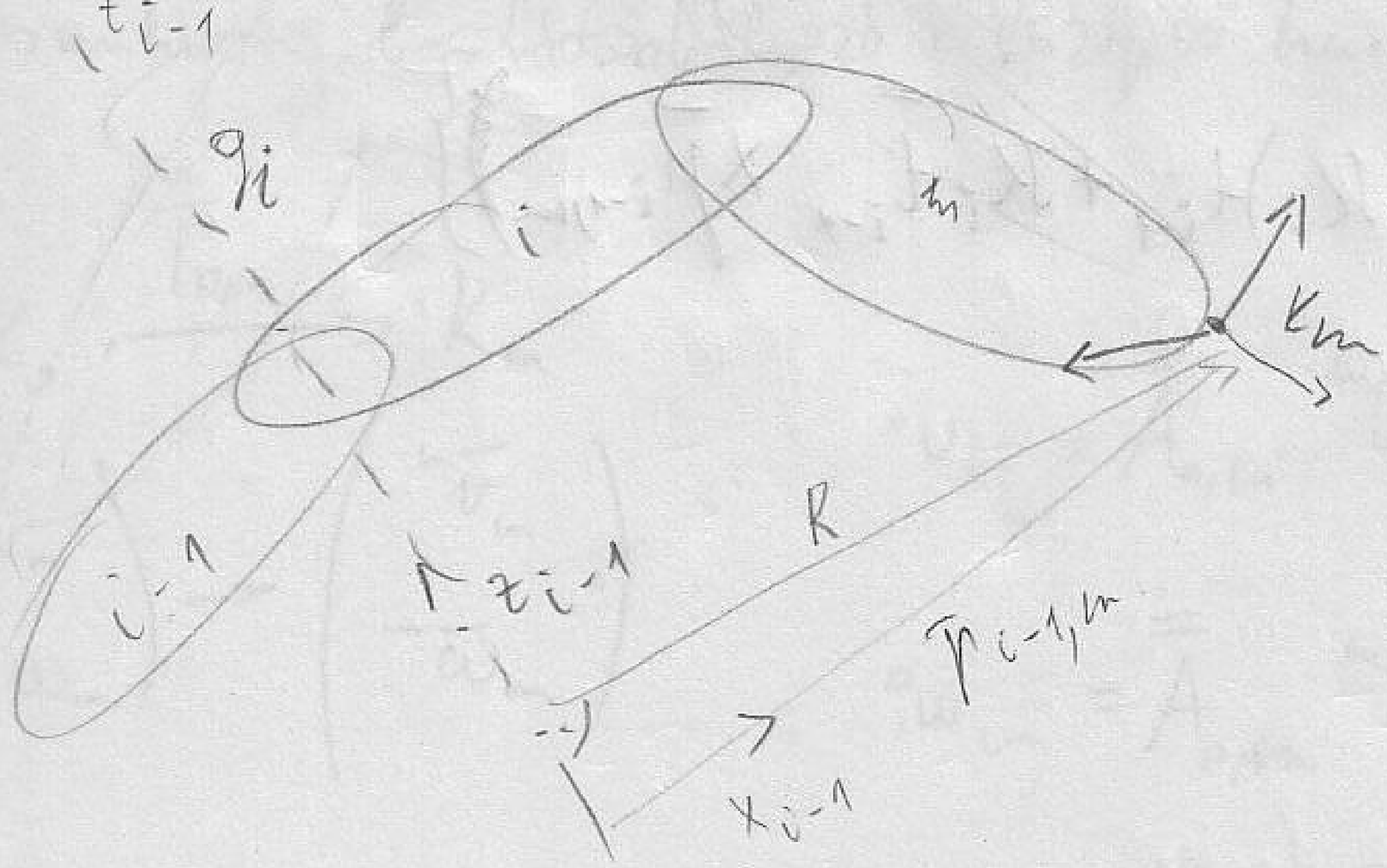
$\downarrow n$

α

Köv óra

$$T_{om}(\bar{q}) = \overline{\overline{COORD}} * \overline{\overline{POS}} * \overline{\overline{TOOL}}^{-1}$$

$$\overline{\overline{T}}_{om}(t) = \begin{bmatrix} \bar{A}(t) & r(t) \\ 0^T & 1 \end{bmatrix} \longrightarrow \text{solve inv geom}$$



$\boxed{R}$ $K_i = 1$ | $\boxed{T}$ $K_i = 0$ | parciális sebesség

parciális szögsebesség

$$\bar{d}_{i-1} = (1-K_i)\bar{t}_{i-1} + K_i \bar{t}_{i-1} \times \bar{p}_{i-1,m}$$

$$K_i \bar{t}_{i-1}$$

$$K_{i-1} \xrightarrow{\bar{\bar{T}}_{i-1,m}} K_m$$

$$^m t_{i-1} = \bar{\bar{A}}_{i-1,m} \{K_i \bar{t}_{i-1}\}$$

$$^m d_{i-1} = \bar{\bar{A}}^T_{i-1,m} \{(1-K_i) t_{i-1} + K_i \bar{t}_{i-1} \times \bar{p}_{i-1,m}\}$$

$$^m v_m = \sum_{i=1}^{m} {}^m d_{i-1} \dot{q}_i$$

$$^m \omega_m = \sum_{i=1}^{m} {}^m \bar{t}_{i-1} \dot{q}_i$$

$$\begin{pmatrix} {}^{m}v_m \\ {}^{m}\omega_m \end{pmatrix} = \underbrace{\begin{bmatrix} {}^{m}d_0 & {}^{m}d_1 & \dots & {}^{m}d_{m-1} \\ {}^{m}t_0 & \dots & \dots & {}^{m}t_{m-1} \end{bmatrix}}_{{}^{m}\bar{\bar{J}}(\bar{q})} \dot{q} = {}^{m}\bar{\bar{J}}_m(\bar{q})\dot{q}$$

pszeudoinverz

$$\dot{\bar{q}} = {}^{m}\bar{\bar{J}}_m^{\#} \begin{pmatrix} {}^{m}v_m \\ {}^{m}\omega_m \end{pmatrix}$$

Háromhurkos kaszkádszabályozó: középső hurok sebesség

$$K_0 \xrightarrow{\bar{\bar{T}}_{0,m}} K_m$$

$$\begin{pmatrix} {}^{0}v_m \\ {}^{0}\omega_m \end{pmatrix} \to \begin{pmatrix} {}^{m}\bar{v}_m \\ {}^{m}\bar{\omega}_m \end{pmatrix} \qquad {}^{0}v_m = \bar{\bar{A}}_{0,m}\, {}^{m}\bar{v}_m \qquad {}^{0}\omega_m = \bar{\bar{A}}_{0,m}\, {}^{m}\omega_m$$

$$\begin{pmatrix} {}^{0}v_m \\ {}^{0}\omega_m \end{pmatrix} = \begin{bmatrix} \bar{\bar{A}}_{0,m} & \bar{\bar{0}} \\ \bar{\bar{0}} & \bar{\bar{A}}_{0,m} \end{bmatrix} \left\{ {}^{m}\bar{\bar{J}}(\bar{q})\dot{\bar{q}} \right.$$

$$\bar{\bar{T}}_{i-1,m} = \begin{bmatrix} \bar{\bar{A}}_{i-1,m} & \bar{r}_{i-1,m} \\ \bar{0}^T & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \bar{\bar{A}} & \bar{r} \\ \bar{0}^T & 1 \end{bmatrix}$$

$$\bar{\bar{A}}^T \bar{t}_{i-1} = \begin{bmatrix} \bar{l}^T \\ \bar{m}^T \\ \bar{n}^T \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} l_z \\ m_z \\ n_z \end{pmatrix}$$

Denavit - Hartenberg
konvenció: Z irányban mozgatunk

$$\bar{\bar{A}}\ \bar{t}_{i-1} \times \bar{p} = \begin{bmatrix} \bar{l}^T \\ \bar{m}^T \\ \bar{n}^T \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{bmatrix} \bar{l}^T \\ \bar{m}^T \\ \bar{n}^T \end{bmatrix} \cdot$$

$$\begin{pmatrix} -p_y \\ p_x \\ 0 \end{pmatrix} = \begin{pmatrix} -l_x p_y + l_y p_x \\ -m_x p_y + m_y p_x \\ -n_x p_y + n_y p_x \end{pmatrix}$$

$n > 6$ redundáns robot $m = DOF$

$$\begin{pmatrix} \overline{U}_m \\ \overline{\omega}_m \end{pmatrix} = \overline{\overline{J}}(\overline{q})\dot{\overline{q}} \quad , \quad \|\dot{\overline{q}}\|^2 \to \min.$$

<u>Séma</u>:

$\overline{\overline{A}}_{n \times m}$, $n < m$, $\overline{x} \in \mathbb{R}^m$, $y \in \mathbb{R}^n$

$\overline{\overline{A}}\,\overline{x} = \overline{y} \quad \|x\|^2 \to \min$

Lagrange multiplikátor szabály: (Kuhn-Tucker Karusch még erősebb):

$$F = \langle \overline{x}, \overline{x} \rangle + \langle \lambda, \overline{\overline{A}}\overline{x} - \overline{y} \rangle$$

$$F'_x = \overline{0}$$

$\|x\|^2 = \langle \overline{x}, \overline{x} \rangle$ konvex funkcionál

$\overline{\overline{A}}\overline{x} - \overline{y} = \overline{0}$ lineáris korlátozás

$$F = \langle \bar{x}, \bar{x} \rangle + \langle \bar{\lambda}, \bar{\bar{A}} \bar{x} \rangle - \langle \bar{\lambda}, \bar{y} \rangle =$$

$$= \langle \bar{x}, \bar{x} \rangle + \langle \bar{\bar{A}}^T \lambda, \bar{x} \rangle - \langle \bar{\lambda}, \bar{y} \rangle$$

$$f'_{\bar{x}} = 2\bar{x} + \bar{\bar{A}}^T \bar{\lambda} = \bar{0} \Rightarrow \bar{x} = -\frac{1}{2} \bar{\bar{A}}^T \bar{\lambda}$$

$$\bar{\bar{A}} \left(-\frac{1}{2} \bar{\bar{A}}^T \bar{\lambda} \right) = \bar{y} \Rightarrow \bar{\lambda} = -2 \left(\bar{\bar{A}} \bar{\bar{A}}^T \right)^{-1} \bar{y}$$

$$\bar{x} = -\frac{1}{2} \bar{\bar{A}}^T (-2) \left(\bar{\bar{A}} \bar{\bar{A}}^T \right)^{-1} \bar{y} \Rightarrow \bar{x} = \bar{\bar{A}}^T \left(\bar{\bar{A}} \bar{\bar{A}}^T \right)^{-1} \bar{y}$$

$$\dot{q} = \bar{\bar{J}}^T \left(\bar{\bar{J}}_m \bar{\bar{J}}_m^T \right)^{-1} \begin{pmatrix} v_m \\ \omega_m \end{pmatrix}$$

$m<6$ hiányzó szabadságfok $\quad m = DOF$

$$\begin{pmatrix}\bar{v}_m \\ \bar{\omega}_m\end{pmatrix} = \bar{\bar{J}}_m(\bar{q})\,\dot{\bar{q}}$$

① Csak annyi egyenletet tekintünk, amennyi a manipulálható változók száma $\quad \dim \dot{\bar{q}} = m$

SCARA RRTR

Táblázat

1. v_x
2. v_y
3. v_z
6. ω_z

② Általános esetben (nincs fenti technológiai interpretáció)

$$\bar{y} = \bar{\bar{A}}\bar{x}, \quad \bar{y} \in \mathbb{R}^n, \quad \bar{x} \in \mathbb{R}^m, \quad m<n$$

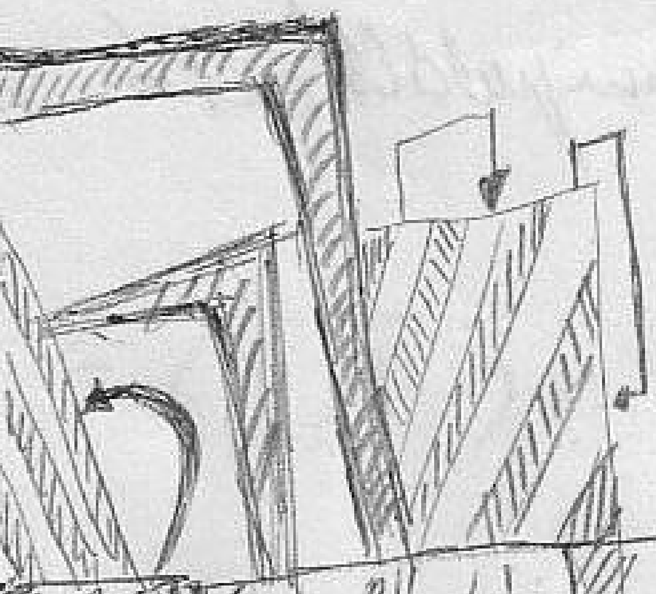
$$\|\bar{\bar{A}}\bar{x} - \bar{y}\|^2 \to \min \qquad (LS \text{ opt.} = \text{kompromisszum})$$

$$F = \langle \bar{\bar{A}}\bar{x} - \bar{y}, \bar{\bar{A}}\bar{x} - \bar{y} \rangle = \langle \bar{\bar{A}}\bar{x}, \bar{\bar{A}}\bar{x} \rangle - 2\langle \bar{\bar{A}}\bar{x}, \bar{y} \rangle +$$

$$+ \langle \bar{y}, \bar{y} \rangle \overset{\#}{=} \langle \bar{\bar{A}}^T\bar{\bar{A}}\bar{x}, \bar{x} \rangle - 2\langle \bar{\bar{A}}^T\bar{y}, \bar{x} \rangle + \langle \bar{y}, \bar{y} \rangle$$

$$F'_{\bar{x}} = \bar{0} = \bar{\bar{A}}^T \bar{\bar{A}} \bar{x} - \bar{\bar{A}}^T \bar{y} \Rightarrow \bar{x} = \underbrace{\left(\bar{\bar{A}}^T \bar{\bar{A}}\right)^{-1} \bar{\bar{A}}^T}_{\bar{\bar{A}}^{\#}} \bar{y}$$

$\bar{\bar{A}}^{\#}$ LS



K. előadás (gyakorlat jellegű cucc)

R^6 típusú robot, utolsó három Euler

$\underbrace{X, Y, Z}, \underbrace{O, A, T}_{\text{Euler szögek}}$ ⟵ robot programozási nyelvben így hívják

GO,

GO STRAIGHT LINE

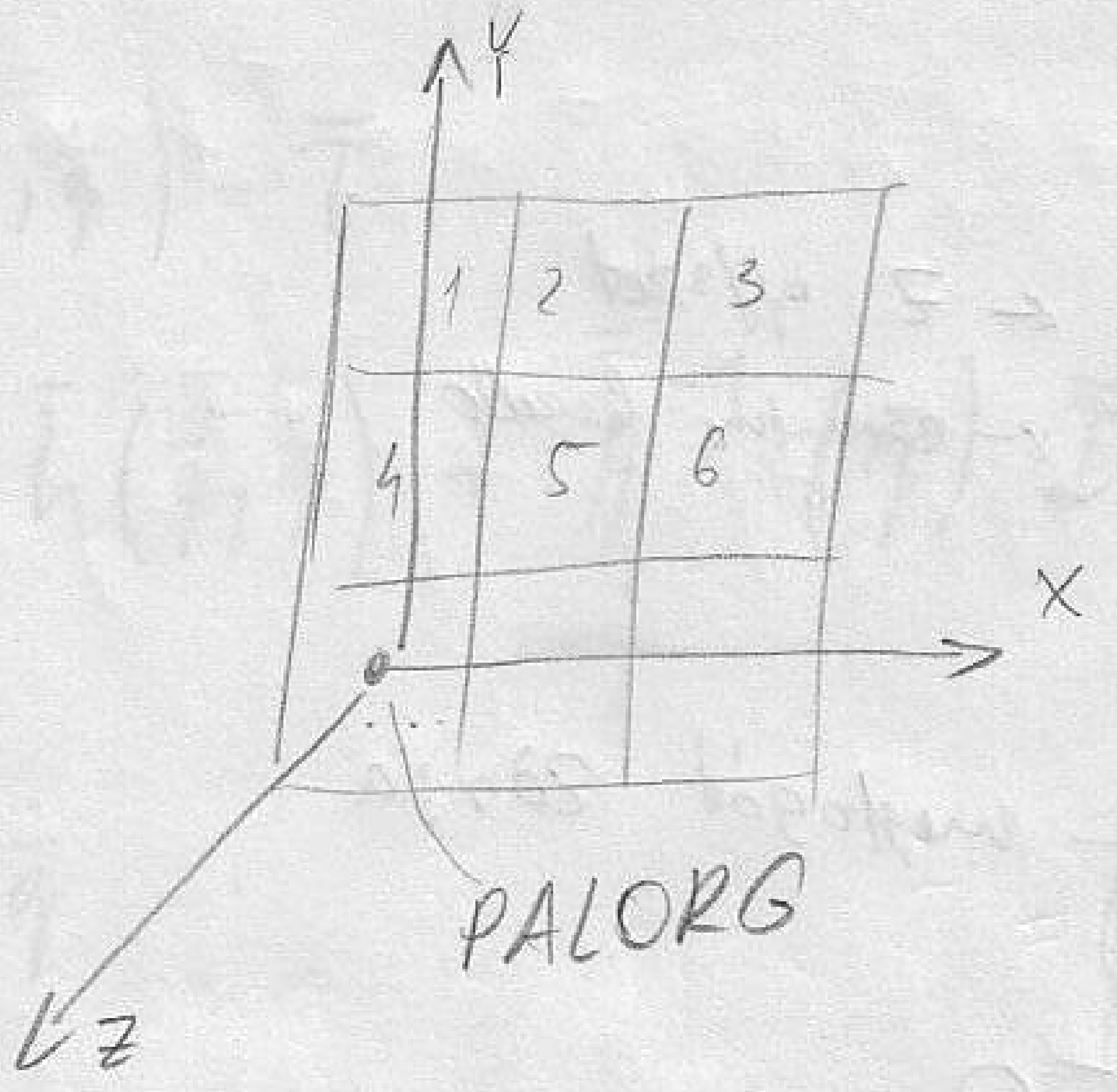
NEAR ⟵ kis távolság cél koord Z tengelye mentén

NEAR STRAIGHT LINE

GOCON

PICKUP POZÍCIÓ (megszólal a végállás kapcsoló)

CONRDY



robot programnyelvben gyakran csak globális változókat használnak

LOCATE – ~~hozzárendelé~~ értékadás 3D pontok esetén

MNEMONIC, OPERANDUS

```
(* - - - - - -
(* FRAME PROGRAM
(* - - - - - -
   LOCATE   PALLET=PALORG
   SET      GOCON = 3
   SET      CONRDY = 2
   SET      PX = 1
   SET      PY = 1
10 OUT      GOCON
   CALL     ONEOBJ
   IF       PX=3 THEN JUMP 20
   SET      PX=PX+1
   SHIFT    PALLET=100,0,0
   JUMP     10
20 IF       PY=3 THEN JUMP 30
   SET      PX=1
   SET      PY=PY+1
   SHIFT    PALLET=200,100,0
   JUMP     10
   HALT
(* - - -
```

(* SUBROUTINE ONEOBS

```
GONEAR   PICKUP, 50      ← Z offszet
WAITIN   CONRDY          ← esemény bevelés
G~~OSTRAIGHT LI~~NE
GOS      PICKUP
CLOSE                    ← megfogót zárja
GOSNEAR  PICKUP, 50
GONEAR   PALLET, 50
GOS      PALLET
OPEN
GOSNEAR  PALLET, 50
RETURN
```

(* END ONEOBS

#name q̄

name $(x, y, z, o, a, t)^T$

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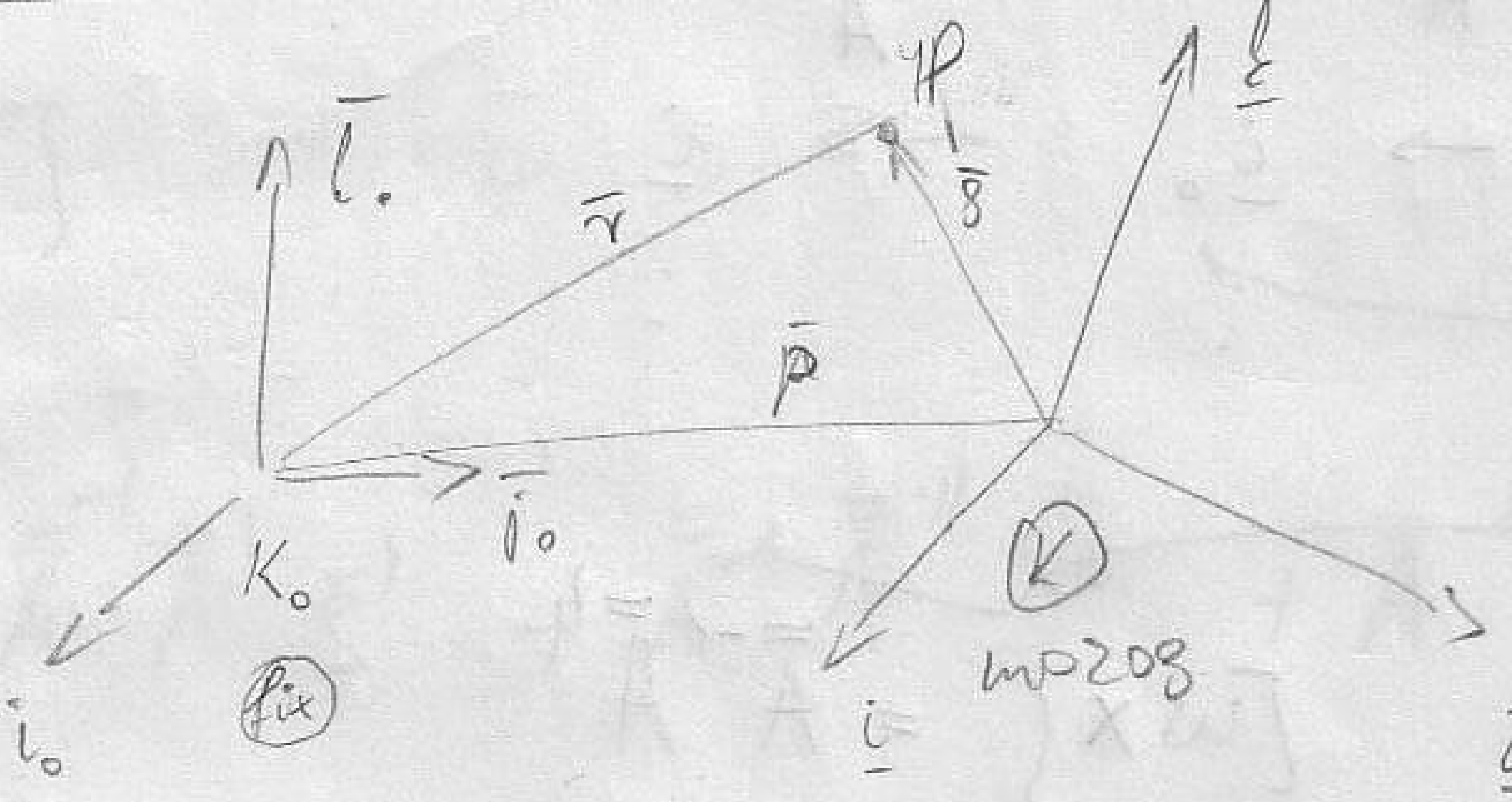
$$\bar{\bar{H}}(\bar q)\ddot{\bar q} + \bar h(\bar q, \dot{\bar q}) = \bar \tau$$

$$\ddot{\bar q} = -\bar{\bar H}^{-1}(\bar q)\,\bar h(\bar q, \dot{\bar q}) + \bar{\bar H}^{-1}(\bar q)\,\bar \tau$$

$$\left.\begin{aligned}
{}^i\omega_i &= \bar{\bar\Gamma}_i\,\dot{\bar q}\\
{}^i\bar\varepsilon_i &= \bar{\bar\Gamma}_i\,\ddot{\bar q} + \bar\Phi_i\\
{}^i\bar v_i &= \bar{\bar\Omega}_i\,\dot{\bar q}\\
{}^i\bar a_i &= \bar{\bar\Omega}_i\,\ddot{\bar q} + \bar\Theta_i
\end{aligned}\right\}$$

kinematika
nincs benne
erő / tömeg
tehetetlenségi nyomaték!



A Föld felszínét inerciarendszernek tekintjük.

$$\bar r = \bar s + \bar p$$

$$r'_x \bar{i}_0 + r_y \bar{j}_0 + r_z \bar{k}_0 = \rho'_x \bar{i} + \rho'_y \bar{j} + \rho'_z \bar{k} + \rho_x \bar{i}' + \rho_y \bar{j}' +$$

$$+ \rho_x \bar{i}_0 + \rho_y \bar{j}_0 + \rho'_z \bar{k}_0$$

$$\cancel{\underline{i} + d\underline{i} = \left(\bar{\bar{1}} + d\varphi[\bar{t}\right.}$$

$$\underline{i} + d\underline{i} = \left(\bar{\bar{1}} + d\varphi[t\times]\right)\underline{i} = \underline{i} + d\varphi \bar{t} \times \bar{i}$$

$$\underline{i}' = \frac{\underline{i} + d\underline{i} - \underline{i}}{dt} = \underbrace{\frac{d\varphi\, t}{dt}}_{\bar{\omega}} \qquad \times \bar{i} = \bar{\omega} \times \bar{i}$$

$$\underbrace{i_0 \xrightarrow{} \underline{i} \xrightarrow{\bar{\bar{A}}'} \underline{i}_0}_{[{}^{\circ}\bar{\omega}\times]} \xrightarrow{\bar{\bar{A}}^{-1}} \underline{i}$$

$$[\omega\times] = \bar{\bar{A}}^{-1} \bar{\bar{A}}'$$

$$r' = \boxed{\bar{s}' + \bar{\omega} \times \bar{s}} + \bar{p}$$

Koord függő alak

$$\bar{r} = \bar{\bar{A}}\bar{s} + \bar{p}$$

$$\bar{r}' = \bar{\bar{A}}\bar{s}' + \bar{\bar{A}}' s + \bar{p}$$

$$r'' = \bar{\bar{A}}\bar{s}'' + \bar{\bar{A}}' \bar{s}' + \bar{\bar{A}}'\bar{s} + \bar{p}''$$

$$\bar{\bar{A}}^{-1}\bar{r}' = s' + \underbrace{\bar{\bar{A}}^{-1}\bar{\bar{A}}'}_{[\bar{\omega}\times]}\bar{s} + \bar{\bar{A}}^{-1}\bar{p}'$$

$$\bar{\bar{A}}^{-1}\bar{r}'' = \bar{s}' + 2\bar{\bar{A}}^{-1}\bar{\bar{A}}'\bar{s}' + \bar{\bar{A}}^{-1}\bar{\bar{A}}''\bar{s}^{t} + \bar{\bar{A}}^{-1}\bar{p}''$$

$$\bar{\bar{A}}^{-1}\bar{\bar{A}} = \bar{\bar{I}} \longrightarrow (\bar{\bar{A}}^{-1})'\bar{\bar{A}} + \bar{\bar{A}}^{-1}\bar{\bar{A}}' = \bar{\bar{0}}$$

$$\boxed{\left(\bar{\bar{A}}^{-1}\right)' = -\bar{\bar{A}}^{-1}\bar{\bar{A}}'\bar{\bar{A}}^{-1}}\ !$$

$$[\bar{\omega}\times] = \bar{\bar{A}}^{-1}\bar{\bar{A}}' \longrightarrow [\bar{\varepsilon}\times] =$$

$$= -\underbrace{\bar{\bar{A}}^{-1}\bar{\bar{A}}'}_{[\bar{\omega}\times]}\underbrace{\bar{\bar{A}}^{-1}\bar{\bar{A}}'}_{[\bar{\omega}\times]} + \bar{\bar{A}}^{-1}\bar{\bar{A}}''$$

$$\bar{\bar{A}}^{-1}\bar{r}'' = \bar{s}'' + 2\bar{\omega}\times\bar{s}' + \bar{\varepsilon}\times\bar{s} + \bar{\omega}\times(\bar{\omega}\times\bar{s}) + \bar{\bar{A}}^{-1}\bar{p}''$$

K kerel origójának sebessége: $\bar{\bar{A}}^{-1}\bar{p}' = \bar{v}_K$

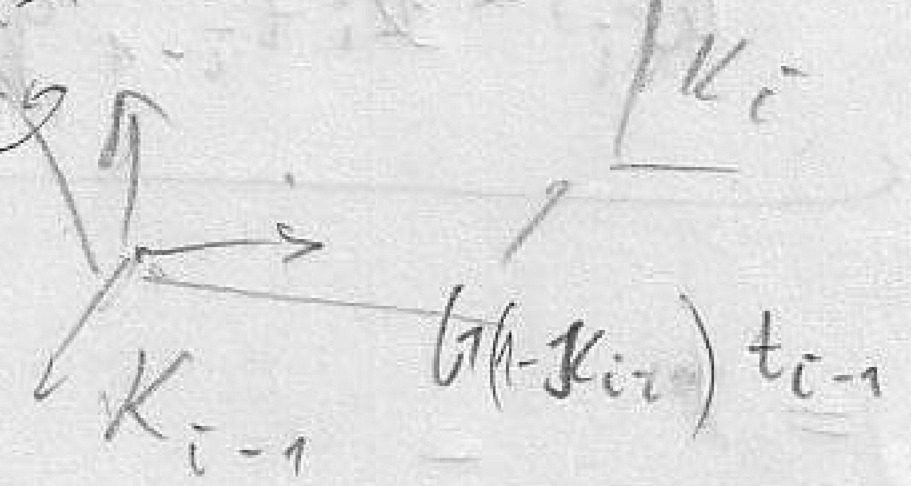
gyorsulása: $\bar{\bar{A}}^{-1}\bar{p}'' = a_K$

szögseb $\bar{\omega} = \bar{\omega}_K$

szöggyors $\bar{\varepsilon} = \bar{\varepsilon}_K$

$$\bar{v}_\rho = \bar{v}_K + \bar{\omega}_K \times \bar{\rho} + \bar{\rho}'$$

$$\bar{a}_\rho = \bar{a}_K + \bar{\varepsilon}_K \times \bar{\rho} + \bar{\omega}_K \times \left(\bar{\omega}_K \times \bar{\rho}\right) + \bar{\rho}'' + 2\bar{\omega}_K \times \bar{\rho}'$$



$$\bar{\rho} := \bar{r}_{i-1,i}$$

$${}^{i}\bar{\omega}_i = \bar{A}^{-1}_{i-1,i}\; {}^{i-1}\bar{\omega}_i$$

$$\vdots$$

$${}^{i}\bar{a}_i = \bar{A}^{-1}_{i-1,i}\; {}^{i-1}\bar{a}_{i-1}$$

$$\bar{\rho} := \bar{r}_{i-1,i}$$

$$\bar{\rho}' = (1-\kappa_i)\, t_{i-1}\, \dot{q}_i + \kappa_i \bar{t}_{i-1}\, \dot{q}_i \times \bar{r}_{i-1,i} :=$$

$$:= d_{i-1}\, \dot{q}_i$$

$${}^{i-1}\bar{\omega}_i = {}^{i-1}\bar{\omega}_{i-1} + \kappa_i \bar{t}_{i-1}\, \dot{q}_i$$

$$\rho'' = d_{i-1}\, \ddot{q}_i + \kappa_i \bar{t}_{i-1}\, \dot{q}_i \times d_{i-1}\, \dot{q}_i$$

$${}^{i-1}\bar{\varepsilon}_i = {}^{i-1}\bar{\varepsilon}_{i-1} + \kappa_i \bar{t}_{i-1}\, \ddot{q} + {}^{i-1}\bar{\omega}_i \times \kappa_i\, t_{i-1}\, \dot{q}_i$$

$${}^{i-1}a_i = {}^{i-1}\bar{a}_{i-1} + {}^{i-1}\bar{\varepsilon}_{i-1} \times \bar{r}_{i-1,i} + {}^{i-1}\bar{\omega}_{i-1} \times \left({}^{i-1}\bar{\omega}_{i-1} \times \bar{r}_{i-1,i}\right)$$

$\bar{d}_{i-1}\ddot{q}_i + \varkappa_i \bar{t}_{i-1}\dot{q}_i \times d_{i-1}\dot{q}_i$

$2\,{}^{i-1}\bar{\omega}_{i-1} \times \bar{d}_{i-1}\dot{q}_i$

${}^{i-1}\bar{\omega}_i - \varkappa_i \bar{t}_{i-1} q_i$

$\bar{\bar{A}}_{i-1,i}\,,\ \bar{\Gamma}_{i-1,i}$

${}^{i}\bar{d}_{i-1} = \bar{\bar{A}}^{-1}_{i-1,i}\left\{(1-\varkappa_i)\bar{t}_{i-1} + \varkappa_i \bar{t}_{i-1} \times \bar{\Gamma}_{i-1,i}\right.$

${}^{i}\bar{t}_{i-1} = \bar{\bar{A}}^{-1}_{i-1,i}\left\{\varkappa_i \bar{t}_{i-1}\right\}$

$\bar{\bar{\Gamma}}_i = \left[\bar{\bar{A}}^{-1}_{i-1,i}\Gamma_{i-1} \,\middle|\, {}^{i}\bar{t}_{i-1}\right],\quad {}^{i}\bar{\omega}_i = \bar{\bar{\Gamma}}_i \dot{q}$

$\phi_i = \bar{\bar{A}}^{-1}_{i-1,i}\phi_{i-1} + {}^{i}\bar{\omega}_i \times \times\, {}^{i}\bar{t}_{i-1}\dot{q}_i$

$$\bar{\Omega}_i = \left[\bar{\bar{A}}^{-1}_{i-1,i} \left\{ \bar{\bar{\Gamma}}_{i-1} - [\bar{p}_{i-1,i} \times] \bar{\bar{\Gamma}}_{i-1} \right\} \Big| \, {}^{i}\bar{d}_{i-1} \right]$$

$$\bar{\Theta}_i = \bar{\bar{A}}^{-1}_{i-1,i} \left\{ \bar{\theta}_{i-1} + \bar{\phi}_{i-1} \times \bar{p}_{i-1,i} + {}^{i-1}\bar{\omega}_{i-1} \times \right.$$

$$\left. \times \left({}^{i-1}\omega_{i-1} \times \bar{p}_{i-1,i} \right) \right\} +$$

$$+ 2\, {}^{i}\bar{\omega}_i \times {}^{i}d_{i-1} \, \dot{q}_i \times {}^{i}\bar{d}_{i-1} \, \dot{q}_i$$

---

edu.iit.bme.hu $\qquad$ $\overset{F = ma}{\checkmark}$ = forgómozgások

Autonóm robotok és járművek $\qquad$ Newton - Euler egyenletek

VIMHA 358

A-148 o.

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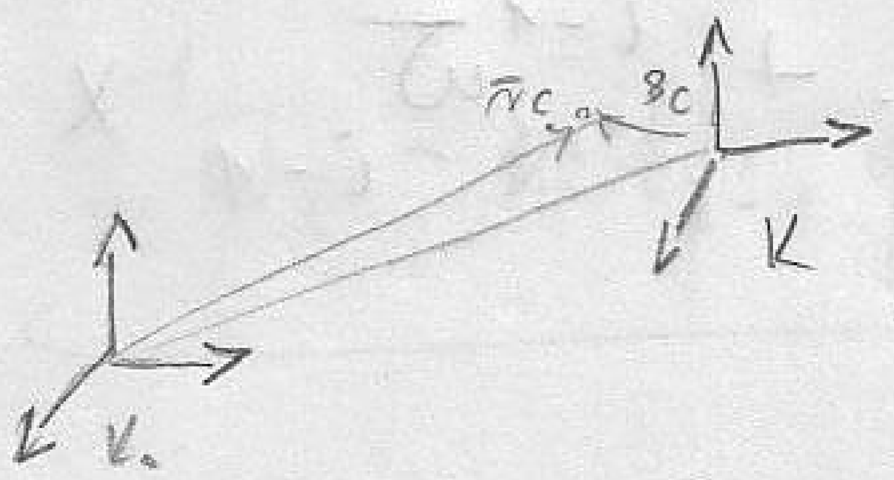
$$\bar{F}_{total} = m\ddot{\bar{r}}_c$$

$$\bar{\bar{I}}\,\dot{\bar{\omega}} + \bar{\omega} \times \left(\bar{I}_c \bar{\omega} \right) = \bar{M}_{total}$$ nyomaték



Lagrange egyenletek

$L = K - P$ ← potenciális

← kinetikus

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{q}_i} - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} = \tau_i$$

Appel egyenlet

$$\frac{\partial G}{\partial \ddot{q}_i} + \frac{\partial P}{\partial q_i} = \tau_i$$

$K = \frac{1}{2} \operatorname{dim} v_\rho^2$

$K = \frac{1}{2} \int\limits_S v_\rho^2 \, dm$

$\bar{v}_\rho = \bar{v} + \bar{\omega} \times \bar{\rho}$

$\bar{v}_\rho^2 = \langle \bar{v} + \bar{\omega} \times \bar{\rho}, \bar{v} + \bar{\omega} \times \rho \rangle = \langle \bar{v}, \bar{v} \rangle + 2 \langle \bar{v}, \bar{\omega} \times \rho \rangle +$

$\langle \rho, \bar{v} \times \bar{\omega} \rangle$

$+ \langle \bar{\omega} \times \bar{\rho}, \bar{\omega} \times \bar{\rho} \rangle$

$\langle \bar{\rho} \times \bar{\omega}, \bar{\rho} \times \bar{\omega} \rangle$

$\langle [\rho \times] \bar{\omega}, [\rho \times] \bar{\omega}$

$\langle [\bar{\rho} \times][\bar{\rho} \omega] \bar{\omega}, \bar{\omega} \rangle$

$m = \int\limits_S dm \qquad m \bar{\rho}_c = \int \bar{\rho} \, dm \quad \Rightarrow \bar{\rho}_c = \frac{\int \bar{\rho} \, dm}{m}$

$$[\bar{\rho}\times]^T[\bar{\rho}\times] = \begin{bmatrix} 0 & \rho_z & -\rho_y \\ -\rho_z & 0 & \rho_x \\ \rho_y & -\rho_x & 0 \end{bmatrix} \begin{bmatrix} 0 & \rho_z & -\rho_y \\ \vdots & & \\ & & \end{bmatrix} =$$

$$= \begin{matrix} \rho_y^2+\rho_z^2 & -\rho_x\rho_y & -\rho_x\rho_z \\ * & \rho_x^2+\rho_z^2 & -\rho_y\rho_z \\ * & \times & \rho_x^2+\rho_y^2 \end{matrix}$$

$$\bar{\bar{K}} = \int [\rho\times]^T[\rho\times]\,dm = \begin{bmatrix} K_x & -K_{xy} & K_{xz} \\ * & K_y & K_{yz} \\ * & * & K_z \end{bmatrix}$$

$$K_z = \int_S \left(\rho_x^2+\rho_y^2\right) dm$$

$$K_{xy} = \int_S \rho_x\rho_y\, dm \qquad \text{stb.}$$

$$K = \frac{1}{2}\langle \bar{v}, \bar{v} \rangle m + \langle m\bar{s}_c, \bar{v} \times \bar{\omega} \rangle + \frac{1}{2}\langle \bar{\bar{K}}\bar{\omega}, \bar{\omega} \rangle$$

$$K = \frac{1}{2}\langle \bar{v}_c, \bar{v}_c \rangle m + \frac{1}{2}\langle \bar{\bar{K}}_c \bar{\omega}, \bar{\omega} \rangle$$

$$\bar{v}_c = \bar{v} + \bar{\omega} \times \bar{s}_c = \bar{v} - [s_c \times]\bar{\omega} =$$

$$= \bar{\bar{\Omega}}\dot{q} - [\bar{s}_c \times]\bar{\bar{\Gamma}}\dot{q} = \underbrace{\left(\bar{\bar{\Omega}} - [s_c \times]\bar{\bar{\Gamma}}\right)}_{\bar{\bar{\Omega}}_c}\dot{q} =$$

$$= \bar{\bar{\Omega}}_c \dot{q}$$

$$K = \frac{1}{2}\langle \bar{\bar{\Omega}}_c \dot{q}, \bar{\bar{\Omega}}_c \dot{q} \rangle m + \frac{1}{2}\langle K_c \bar{\bar{\Gamma}}\dot{q}, \bar{\bar{\Gamma}}\dot{q} \rangle$$

$$K = \frac{1}{2}\left\langle \left(\bar{\bar{\Omega}}_c^T \bar{\bar{\Omega}}_c m + \bar{\bar{\Gamma}}^T \bar{\bar{K}}_c \bar{\bar{\Gamma}}\right)\dot{q}, \dot{q} \right\rangle$$

$$K = \frac{1}{2}\left\langle \underbrace{\sum_{s=1}^{m}\left(\bar{\bar{\Omega}}_{cs}^T \bar{\bar{\Omega}}_{cs} m_s + \bar{\bar{\Gamma}}_s^T \bar{\bar{K}}_{cs} \bar{\bar{\Gamma}}_s\right)}_{\bar{\bar{H}}}\dot{q}, \dot{q} \right\rangle =$$

$$= \frac{1}{2}\langle \bar{\bar{H}}(q)\dot{q}, \dot{q} \rangle$$

dlt inercia mátrix

$$\bar{\bar{H}} = \left[D_{jl}\right]^{tt}_{n \times n}$$

$$K = \frac{1}{2} \sum_j \sum_l D_{jl}(\bar{q})\, \dot{q}_j \dot{q}_l$$

$G = \frac{1}{2} m a^2$ "gyorsulás energia"

Gibbs fv

$$G = \frac{1}{2} \int_{[S]} \bar{a}_S^{\,2}\, dm$$

S - összes pont halmaza
S futó pont

$$\bar{a}_S = \langle \bar{a} + \bar{\varepsilon} \times \bar{s} + \bar{\omega} \times (\bar{\omega} \times \bar{s}),\ \bar{a} + \bar{\varepsilon} \times \bar{s} + \bar{\omega} \times (\bar{\omega} \times \bar{s}) \rangle$$

$$= \langle \bar{a}, \bar{a} \rangle + \langle [\bar{s}\times]^T [s\times] \bar{\varepsilon}, \bar{\varepsilon} \rangle +$$

$$+ 2 \langle \bar{s}, -\bar{\varepsilon} \times \bar{a} + \bar{\omega} \times (\bar{\omega} \times \bar{s}) \rangle - 2 \langle ([\bar{s}\times]^T [s\times] \omega) \times \bar{\omega}, \bar{\varepsilon} \rangle +$$

$+ \ldots$

$$G = \frac{1}{2}\langle \bar{a}, \bar{a}\rangle m + \langle m \bar{\varrho}_c, \bar{a} \times \bar{\varepsilon} + \bar{\omega} \times (\omega \times a)\rangle +$$

$$+ \frac{1}{2}\langle \bar{\bar{K}}\bar{\varepsilon} - 2(\bar{\bar{K}}\bar{\omega}) \times \omega, \varepsilon \rangle$$

$$\bar{a}_c = \bar{a} + \bar{\varepsilon} \times \bar{\varrho}_c + \bar{\omega} \times (\omega \times \varrho_c)$$

Amikor a tk-ba helyezzük keretet, azt a DH kerettel párhuzamosra vesszük

$$\bar{a}_c = \bar{\bar{\Omega}}_c \ddot{\bar{q}} + \bar{\Theta}_c$$

$$\bar{\bar{\Omega}}_c = \bar{\bar{\Omega}} - [\bar{\varrho}_c \times]\bar{\bar{\Gamma}}$$

$$\bar{\Theta}_c = \bar{\Theta} + \bar{\Phi} \times \bar{\varrho}_c + \bar{\omega} \times (\bar{\omega} \times \bar{\varrho}_c)$$

Appell-egyenlet

$$\frac{\partial G}{\partial \ddot{q}_i} + \frac{\partial P}{\partial q_i} = \tau_i$$

$$\frac{\partial G}{\partial \ddot{q}_i} = \frac{1}{2}\left\langle \frac{\partial \bar{a}_c}{\partial \ddot{q}_i}, \bar{a}_c \right\rangle m + \frac{1}{2}\left\langle \bar{\bar{K}}_c \frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i}, \bar{\varepsilon} \right\rangle + \frac{1}{2}\,\Sigma$$

$$\left\langle \bar{K}_c \varepsilon - 2(\bar{K}_c \omega) \times \bar{\omega}, \frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i} \right\rangle$$

$$= \left\langle \frac{\partial \bar{a}_c}{\partial \ddot{q}_i}, \bar{a}_c \right\rangle m + \left\langle \bar{\bar{K}}_c \bar{\varepsilon} - (\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega}, \frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i} \right\rangle$$

$$\bar{a}_c = \bar{\bar{\Omega}}_c \ddot{\bar{q}} + \bar{\Theta}_c$$

$\frac{\partial \bar{a}_c}{\partial \ddot{q}_i}$ az $\bar{\bar{\Omega}}_c$ mátrix i-edik oszlopa

$$\Downarrow$$

$$\bar{\Omega}_{c,i}$$

$\frac{\partial \bar{\varepsilon}}{\partial \ddot{q}_i}$ a $\bar{\bar{\Gamma}}$ mátrix i-edik oszlopa $\Rightarrow \bar{\Gamma}_i$

$$\frac{\partial G}{\partial \ddot{q}_i} = \bar{\Omega}_{c,i} \left(\bar{\bar{\Omega}}_c \ddot{\bar{q}} + \bar{\Theta}_c \right) m + \bar{\Gamma}_i^T \left\{ \bar{\bar{K}}_c \left(\bar{\bar{\Gamma}} \ddot{\bar{q}} + \bar{\Phi} \right) - (\bar{\bar{K}}_c \bar{\omega}) \times \bar{\omega} \right\}$$

$$\frac{\partial G}{\partial \ddot{q}_i} + \frac{\partial P}{\partial q_i} = \tau_i \longleftrightarrow \frac{\partial G}{\partial \ddot{\bar{q}}} + \frac{\partial P}{\partial \bar{q}} = \bar{\tau}$$

$$\left(\bar{\bar{\Omega}}_c^T \bar{\bar{\Omega}}_c m + \bar{\bar{\Gamma}}^T \bar{\bar{K}}_c \bar{\bar{\Gamma}} \right) \ddot{\bar{q}} + \ldots$$

$$+ \bar{\bar{\Omega}}_c^T \Theta_c m + \bar{\bar{\Gamma}}^T \{ \bar{\bar{K}}_c \phi - (\bar{\bar{K}} \bar{\omega}) \times \bar{\omega} \}$$

$$\underbrace{\sum_{s=1}^{m} \left(\bar{\bar{\Omega}}_{cs}^T \bar{\bar{\Omega}}_{cs} m_s + \bar{\bar{\Gamma}}_s^T \bar{\bar{K}}_{cs} \bar{\bar{\Gamma}}_s \right)}_{\bar{\bar{H}}(\bar{q})} \ddot{\bar{q}} +$$

$$\underbrace{+ \sum_{s=1}^{m} \left\{ \bar{\bar{\Omega}}_{cs}^T \bar{\Theta}_{cs} m_s + \bar{\bar{\Gamma}}^T \left[\bar{\bar{K}}_{cs} \bar{\phi}_s - \left(\bar{\bar{K}}_{cs} \bar{\omega}_s \right) \times \bar{\omega}_s \right] \right\}}_{\bar{h}_{cc}(\bar{q}, \dot{\bar{q}})} +$$

$$+ \underbrace{\frac{\partial P}{\partial \bar{q}}}_{\bar{h}_g(\bar{q})} = \bar{\tau}$$

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Robot dinamikus modellje

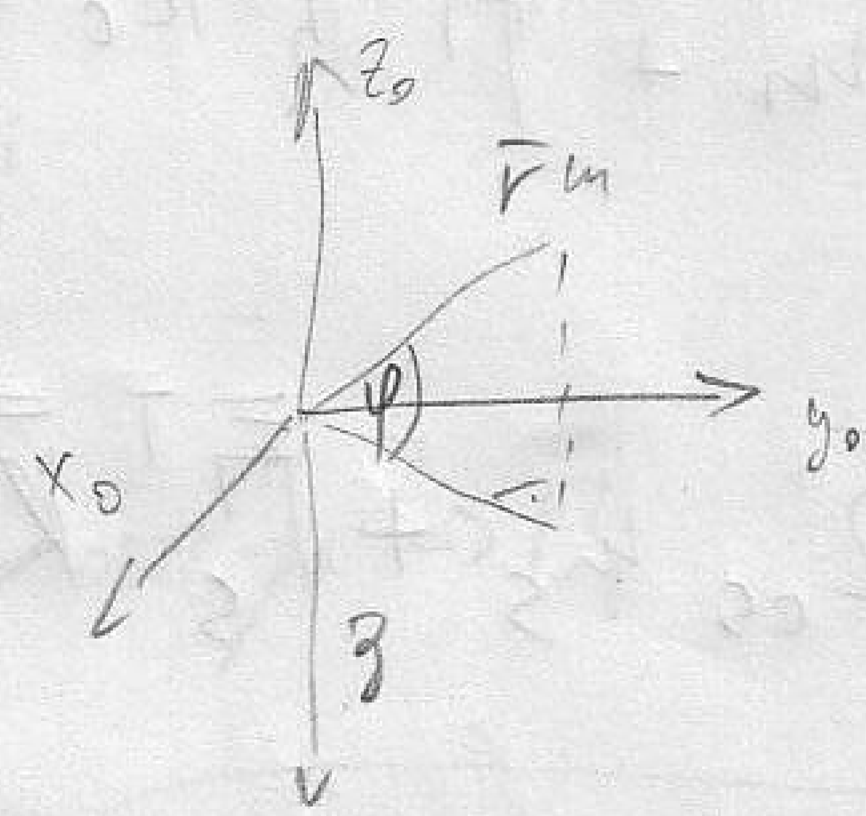
$$\bar{\bar{H}}(\bar{q})\ddot{\bar{q}} + \bar{h}(\bar{q},\dot{\bar{q}}) = \bar{\tau}$$

$$\bar{h} = \bar{h}_{cc} + \bar{h}_{g}$$

$$P = mgh$$

$$h = |\bar{r}| \sin\varphi$$

$$\langle \bar{g}, \bar{r} \rangle =$$



$$= |\bar{g}|\,|\bar{r}| \underbrace{\cos(90^\circ + \varphi)}_{-\sin\varphi}$$

$$P = -\langle \bar{g}, \bar{r} \rangle\, m$$

$$P = \sum_{s=1}^{m} m_s \left[\bar{g}^T \; 0\right] T_{0s}(\bar{q}) \begin{pmatrix} \bar{s}_{cs} \\ 1 \end{pmatrix}$$

Newton - Euler, Appel levez kell

~~Newton - Appel levezetése~~ nem kell,