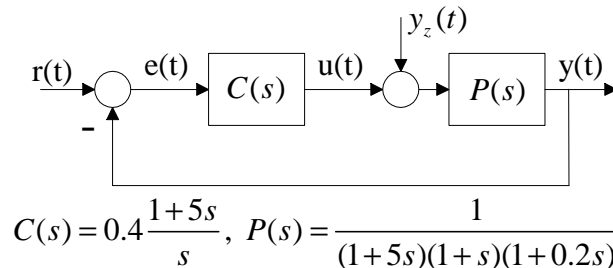


## Control theory Exam, December 8, 2008

Name: \_\_\_\_\_

Points: \_\_\_\_\_

1. Given the following control loop:



a./ Give the gain margin, phase margin and cut-off frequency of the system. Is the closed loop stable? **(4 points)**

In case of  $r(t) = 0$  and  $y_z(t) = 1(t)$ ,

b./ draw the  $y$  output signal of the system. **(4 points)**

c./ give the steady state value and settling time of the output signal. **(2 points)**

2. In a sampled data system the transfer function of the process is:

$$P(s) = \frac{0.4}{s^2 - 0.8s - 0.2} e^{-2s}. \quad \text{The sampling time is } T_s = 1.$$

a./ Give the  $G(z)$  discrete transfer function of the process in zero-pole form if zero order hold is applied **(4 points)**

b./ Is the  $G(z)$  discrete transfer function stable? Explain your answer. **(2 points)**

c./ Give the values of the step response of  $G(z)$  in first five sampling points. **(2 points)**

3. A continuous process is given by its state matrices:

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -8 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, c = [2 \quad 0], d = 0.$$

a./ Design a state variable feedback control system that the closed loop system is a second order oscillating element with damping factor of 0.7 and time constant of 0.2. **(5 points)**

b./ Calculate the  $k_c$  input gain value for unity static reference following. **(2 points)**

4. The transfer function of a continuous plant is  $P(s) = \frac{1}{(1+s)(1+10s)}$ .

In case of  $u(t) = \sin(0.5t)$  input signal the steady state output signal is  $y(t) = A \sin(0.5t + \varphi)$ .

Give the value of  $A$  and  $\varphi$ . **(5 points)**

## Control theory Exam, December 8, 2008, Solution

1.

a.

$s = \text{zpk}('s')$ ,  $C = 0.4 * (1 + 5 * s) / s$ ,  $P = 1 / ((1 + 5 * s) * (1 + s) * (1 + 0.2 * s))$ ,  $L = C * P$ ,  $L = \text{minreal}(L)$

`figure(1), margin(L)`

**pm=65°, wc=0.37rad/sec, Gm=23.5dB,**

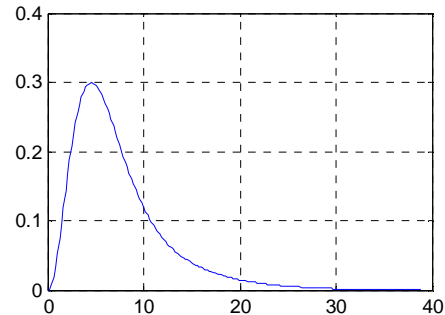
`gm=margin(L)`

**gm=15**

$Tz = P / (1 + L)$ ,

`figure(2), [y,t]=step(Tz), plot(t,y), grid`

**y\_vég=0, ts== 20 sec**



2.

a./

$s = \text{tf}('s')$ ,  $P = 0.4 / (s * s - 0.8 * s - 0.2)$ ,  $P = \text{zpk}(P)$ ,  $\%P = \text{tf}(P) \% P = 2 / ((s-1) * (5*s+1))$ ,  $P = \text{tf}(P)$

$Ts=1$ ,  $Td=2$ ,  $d = Td / Ts$ ,  $z = \text{zpk}('z', Ts)$ ,  $G1z = c2d(P, Ts)$ ,  $Gz = G1z / (z^d)$

**0.27065 (z+1.302)**

**z^2 (z-0.8187) (z-2.718)**

**b. Unstable, the absolute value of a pole is greater than 1.**

c.

`y=step(Gz,5)`

y= 0  
0  
0  
0.2706  
1.5802  
5.6099

3.

$a = [-1, 1; 0, -8]$ ,  $b = [1; 1]$ ,  $c = [2, 0]$ ,  $d = 0$

$T0 = 0.2$ ,  $kszi = 0.7$ ,  $\text{den} = [T0 * T0, 2 * T0 * kszi, 1]$ ,  $\text{pc} = \text{roots}(\text{den})$

**den = [0.0400 0.2800 1.0000],**

**pc =**

**-3.5000 + 3.5707i**

**-3.5000 - 3.5707i**

$k = \text{acker}(a, b, \text{pc})$ ,  $G = 1 / \text{dcgain}(a - b * k, b, c, d)$

**K = 2.3750 -4.3750**

**G = 1.3889**

4.

$s = \text{zpk}('s')$ ,  $P = 1 / ((1 + 1 * s) * (1 + 10 * s))$ ,

$w = 0.5$ ,  $[m, f] = \text{bode}(P, w)$

$A = m$ ,  $f_i = f$

**A=0.1754**

**f i=-105.2551**