INFO AN1. PPZH1 | —1— | 2018. 12. 12.

1, a, $z = r\,e^{i\varphi}$ n-edik gyökei: $\sqrt[n]{r}\cdot e^{i\left(\frac{\varphi}{n}+\frac{2k\pi}{n}\right)}$ (4), $k=0,1,\dots,n-1$ (1)

$\boxed{5}$

b, $1+i = \sqrt{2}\cdot e^{i\frac{\pi}{4}}$ (3); $(1+i)^8 = 16\cdot e^{2\pi i} = 16 = 16\,e^{i\,0}$ (3)

$\boxed{12}$

$z_k = \sqrt[3]{16}\cdot e^{i\frac{2k\pi}{3}}$ (6); $z_0 = 2\sqrt[3]{2}$; $z_1 = e^{i\frac{2\pi}{3}} = (-1+\sqrt{3}\,i)\sqrt[3]{2}$

$(k=0,1,2)$ $z_2 = e^{i\frac{4\pi}{3}} = (-1-\sqrt{3}\,i)\sqrt[3]{2}$

2, a, $\lim_{n\to\infty} a_n = A$, ha $\forall \varepsilon>0$ esetén $\exists N(\varepsilon)\in\mathbb{N}$, melyre

$\boxed{5}$ $|a_n - A| < \varepsilon$, ha $n > N(\varepsilon)$

b, $\left|\frac{2n}{n+3} - 2\right| = \frac{|2n-2(n+3)|}{n+3}$ (2) $= \frac{|-6|}{n+3}$ (3) $< \frac{+6}{n} < \varepsilon$, ha $n > \frac{6}{\varepsilon}$

$\boxed{10}$

$N(\varepsilon) \geq \frac{6}{\varepsilon}$ (5)

3, a, $a_n = \frac{n^2-2n+3-(n^2+n-2)}{\sqrt{n^2-2n+3}+\sqrt{n^2+n-2}}$ (3) $= \frac{-3n+5}{\sqrt{n^2-2n+3}+\sqrt{n^2+n-2}}$ (2) $=$

$\boxed{10}$

$= \frac{-3+\overbrace{5/n}^{\to 0}}{\sqrt{1-\underbrace{2/n+3/n^2}_{\downarrow\, 0}}+\sqrt{1+\underbrace{1/n-2/n^2}_{\downarrow\, 0}}}$ (3) $\longrightarrow \frac{-3}{1+1} = -\frac{3}{2}$ (2)

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b, $b_n = \frac{3^n+n^2}{4^n-n} = \frac{\overbrace{(3/4)^n}^{\to 0} + \overbrace{n^2/4^n}^{\to 0}}{1-\underbrace{n/4^n}_{\downarrow\, 0}} \longrightarrow \frac{0}{1} = 0$

$\boxed{8}$

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c, Rendőr elvvel: (2)

$\boxed{10}$ $3 = \sqrt[n]{3^n} \leq c_n = \sqrt[n]{3^n+2^n} \underset{\text{ha } n\geq 1}{\leq} \sqrt[n]{3^n+3^n} = \underbrace{\sqrt[n]{2}\cdot 3}_{\downarrow\, 3}$ (4) (4)

Tehát $\lim_{n\to\infty} c_n = 3$

4, Teljes indukcióval igazoljuk, hogy $2 \leq a_n \leq 5$

20 α, $2 \leq a_1 = 3 \leq 5$ ✓

β, T.f.h. $2 \leq a_n \leq 5$

γ, $5 = \frac{10}{2} \geq \frac{10}{a_n} \geq \frac{10}{5} = 2$

$2 = 7 - 5 \leq 7 - \frac{10}{a_n} = a_{n+1} \leq 7 - 2 = 5$ ✓ 7

Teljes indukcióval igazoljuk, hogy a_n monoton nő:

α, $a_1 = 3 < a_2 = 7 - \frac{10}{3} = \frac{11}{3} = 3 + \frac{2}{3}$

7 β, T.f.h. $0 < a_n < a_{n+1}$

γ, $\frac{10}{a_n} > \frac{10}{a_{n+1}} \Rightarrow 7 - \frac{10}{a_n} = a_{n+1} < 7 - \frac{10}{a_{n+1}} = a_{n+2}$

Mivel a_n felülről korlátos és monoton nő, ezért konvergens 2

$A = 7 - \frac{10}{A} \Rightarrow A^2 - 7A + 10 = (A-2)(A-5) = 0$ 2

$A_1 = 2$ ↯ ; $\underline{\underline{A_2 = 5 \text{ a határérték}}}$ 2

5, a, Ha n páros: $a_{2k} = \left(1 + \frac{1}{2k}\right)^{2k} \longrightarrow e$ 3

10 Ha n páratlan: $a_{2k+1} = \left(1 + \frac{-1}{2k+1}\right)^{2k+1} \longrightarrow e^{-1} = \frac{1}{e}$ 3

Tehát $S = \{e, \frac{1}{e}\}$ 1 ; $\underline{\lim}\, a_n = \frac{1}{e}$ 1 , $\overline{\lim}\, a_n = e$ 1 , $\nexists \lim a_n$ 1

10 b, Az előzőhöz hasonló gondolatmenettel a $c_n = b_n^n = \left(1 + \frac{(-1)^n}{n^2}\right)^{n^2}$

sorozat torlódási pontjai: e, és $\frac{1}{e}$ 5 $\Rightarrow c_n$ korlátos $\exists K$ és ε:

$\Rightarrow \quad 0 < \varepsilon < b_n^n < K \quad / \sqrt[n]{\ }$

$$\sqrt[n]{\varepsilon} < b_n < \sqrt[n]{K}$$

$$\sqrt[n]{\varepsilon} \to 1, \quad \sqrt[n]{K} \to 1$$

Tehát $\underline{\underline{\lim_{n \to \infty} b_n = 1.}}$ 5