2017.10.19.

Rendszerelmélet

1. ZH

Elméleti összefoglaló

$$\delta[k]=\begin{cases}0, & k\in\mathbb{Z}^-,\\ 1, & k=0,\\ 0, & k\in\mathbb{Z}^+.\end{cases} \qquad \varepsilon[k]=\begin{cases}0, & k\in\mathbb{Z}^-,\\ 1, & k\in\mathbb{N}.\end{cases}$$

$$\varepsilon[k]=\sum_{i=0}^{\infty}\delta[k-i] \qquad \delta[k]=\varepsilon[k]-\varepsilon[k-1]$$

$$\delta(t,T)=\frac{\varepsilon(t)-\varepsilon(t-T)}{T} \qquad \varepsilon(t)=\begin{cases}0, & t\in\mathbb{R}^-\\ 1, & t\in\mathbb{R}^+\end{cases}$$

$$\int_{-\infty}^{\infty}\delta(t,T)\,dt=1 \qquad \varepsilon'(t)=\delta(t)$$

DI: $u[k]=\delta[k] \Rightarrow y[k]=h[k]$ (impulzusválasz)

Konvolúció: $y[k]=h[k]*u[k]=\sum_{i=-\infty}^{\infty}h[k-i]u[i]=\sum_{i=-\infty}^{\infty}h[i]u[k-i]$

GV stabilis rendszer $\Leftrightarrow \sum_{i=-\infty}^{\infty}|h[k]|<\infty$

$u[k]=\varepsilon[k] \Rightarrow y[k]=g[k]$ (ugrásválasz)

$h[k]=g[k]-g[k-1] \qquad g[k]=\sum_{i=-\infty}^{k}h[i]$

FI: $u(t)=\delta(t) \Rightarrow y(t)=h(t)$ (impulzusválasz)

Konvolúció: $y(t)=h(t)*u(t)=\int_{-\infty}^{\infty}h(t-\tau)u(\tau)\,d\tau=\int_{-\infty}^{\infty}h(\tau)u(t-\tau)\,d\tau$

GV stabilis rendszer $\Leftrightarrow \int_{-\infty}^{\infty}|h(t)|\,dt<\infty$

$u(t)=\varepsilon(t) \Rightarrow y(t)=g(t)$ (ugrásválasz)

$h(t)=g'(t) \qquad g(t)=\int_{-\infty}^{\infty}h(\tau)\varepsilon(t-\tau)\,d\tau=\int_{-\infty}^{t}h(\tau)\,d\tau$

Aszimptotikus stabilitás:

DI: $|\lambda_i|<1$

FI: $Re\{\lambda_i\}<0$

ASZ $\Rightarrow$ GV

Karakterisztikus egyenlet $n=2$ esetén: $\lambda^2+a_1\lambda+a_2=0=F(\lambda)$

DI: Jury-kritérium: $(n=2)$

$$1+a_1+a_2>0$$
$$1-a_1+a_2>0$$
$$|a_2|<1$$

FI: Routh-Hurwitz-kritérium: $(n=2)$

$$a_1>0$$
$$a_2>0$$

$$\underline{x}'=\underline{\underline{A}}\,\underline{x}+\underline{B}u$$

$$y=\underline{C}^T\underline{x}+Du$$

$$\underline{x}[k]=\underline{\underline{A}}^k\underline{x}[0]+\sum_{i=0}^{k-1}\underline{\underline{A}}^{k-1-i}\underline{B}u[i]$$

$$\underline{x}(t)=e^{\underline{\underline{A}}t}\underline{x}(-0)+\int_{-0}^{t}e^{\underline{\underline{A}}(t-\tau)}\underline{B}u(\tau)d\tau$$

$$y[k]=\begin{cases}\underline{C}^T\underline{x}[0]+Du[0] & k=0\\ \underline{C}^T\underline{\underline{A}}^k\underline{x}[0]+\underline{C}^T\sum_{i=0}^{k-1}\underline{\underline{A}}^{k-1-i}\underline{B}u[i]+Du[k] & k>0\end{cases}$$

$$y(t)=\underline{C}^T\cdot e^{\underline{\underline{A}}t}\underline{x}(-0)+\underline{C}^T\int_{-0}^{t}e^{\underline{\underline{A}}(t-\tau)}\underline{B}u(\tau)d\tau+Du(t) \qquad t>0$$

$$h[k]=D\delta[k]+\varepsilon[k-1]\underline{C}^T\underline{\underline{A}}^{k-1}\underline{B}$$

$$h(t)=D\delta(t)+\varepsilon(t)\underline{C}^Te^{\underline{\underline{A}}t}\underline{B}$$

$$\underline{\underline{L}}_1=\frac{\underline{\underline{A}}-\lambda_2\underline{\underline{E}}}{\lambda_1-\lambda_2} \qquad \underline{\underline{L}}_2=\underline{\underline{E}}-\underline{\underline{L}}_1$$

$$\underline{\underline{A}}^k=\sum_{i=1}^{N}\lambda_i^k\underline{\underline{L}}_i$$

$$e^{\underline{\underline{A}}t}=\sum_{i=1}^{N}e^{\lambda_i t}\underline{\underline{L}}_i$$

$$\underline{\underline{M}}_o=\begin{bmatrix}\underline{C}^T\\ \underline{C}^T\cdot\underline{\underline{A}}\end{bmatrix} \qquad \underline{\underline{M}}_c=[\underline{B}\ \ \underline{\underline{A}}\underline{B}]$$

$\det(\underline{\underline{M}}_o)\neq0 \Rightarrow$ megfigyelhető

$\det(\underline{\underline{M}}_c)\neq0 \Rightarrow$ irányítható

$$\underline{K}^T=[0\ 1]\cdot\underline{\underline{M}}_c^{-1}\cdot\varphi_c(\underline{\underline{A}})$$

$$\underline{\underline{M}}_c^{-1}=\frac{\mathrm{adj}(\underline{\underline{M}}_c)}{\det(\underline{\underline{M}}_c)}=\begin{bmatrix}M_{22} & -M_{12}\\ -M_{21} & M_{11}\end{bmatrix}\cdot\frac{1}{\det(\underline{\underline{M}}_c)}$$

DI: $x[k] = X\cos(\vartheta k + \varphi)$ FI: $x(t) = X\cos(\omega t + \varphi)$

$\overline{X}$ fazor (komplex amplitúdó): $\overline{X} = Xe^{j\varphi}$

DI: $x[k] = \overline{X}e^{j\vartheta k}$ FI: $x(t) = \overline{X}e^{j\omega t}$

$$\overline{H} = \frac{\overline{Y}}{\overline{U}}$$

DI: $H(e^{j\vartheta})$ FI: $H(j\omega)$

DI: $e^{j\vartheta}\underline{\overline{X}} = \underline{\underline{A}}\,\underline{\overline{X}} + \underline{B}\,\overline{U}$ $\overline{Y} = \underline{C}^T\underline{\overline{X}} + D\overline{U}$

FI: $j\omega\underline{\overline{X}} = \underline{\underline{A}}\,\underline{\overline{X}} + \underline{B}\,\overline{U}$ $\overline{Y} = \underline{C}^T\underline{\overline{X}} + D\overline{U}$

DI: $H(e^{j\vartheta}) = \underline{C}^T\left[e^{j\vartheta}\underline{\underline{E}} - \underline{\underline{A}}\right]^{-1}\underline{B} + D$

FI: $H(j\omega) = \underline{C}^T\left[j\omega\underline{\underline{E}} - \underline{\underline{A}}\right]^{-1}\underline{B} + D$