Kalkulus vizsga 4

2023. jan. 26.

1) Gauss elim:
20p)

$$\left(\begin{array}{ccc|c} 1 & 5 & -6 & 17 \\ -2 & 2 & 0 & 8 \\ 10 & 2 & 4 & 10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 5 & -6 & 17 \\ 0 & 12 & -12 & 42 \\ 0 & -48 & 64 & -160 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 5 & -6 & 17 \\ 0 & 12 & -12 & 42 \\ 0 & 0 & 16 & 8 \end{array}\right)$$

$II + 2I$ ③ $\qquad III + 4 \cdot II$ ③

$III - 10I$ ③

$$\rightarrow \begin{cases} x + 5y - 6z = 17 \Rightarrow x + 20 - 3 = 17 \Rightarrow \underline{x = 0} \\ 12y - 12z = 42 \Rightarrow 12y - 6 = 42 \rightarrow 12y = 48 \rightarrow \underline{y = 4} \\ 16z = 8 \Rightarrow \underline{z = 1/2} \end{cases}$$

① ④

$\det A = \underbrace{1 \cdot 12 \cdot 16}_{} = 192$ (1,5)

g. elim. után főátlóbeli elemek szorzata (1,5)

g. elim. után nem nulla sorok száma (1,5)

$r(A) = 3$ → g. elim. után nem nulla sorok száma

(1,5)

2) (15p)

$$\lim \frac{\overset{dom}{\boxed{3^n}} \cdot 3^{-1} - 2 \cdot 2^n - 5 \cdot n^4}{1 - 100n^2 - \boxed{7^n} + \sqrt{n}} =$$

(1,5) (1,5)

Erdősorrend: ④
$\sqrt{n} << n^2 << n^4 << 2^n$
$<< 3^n << 7^n$

$$\lim \frac{3^n}{7^n} ① \cdot \frac{\overbrace{3^{-1} - 2\left(\frac{2}{3}\right)^n - 5 \cdot \frac{n^4}{3^n}}^{②}}{\frac{1}{7^n} - 100 \cdot \frac{n^2}{7^n} - 1 + \frac{\sqrt{n}}{7^n}} =$$

dom

$$\lim \underset{\Downarrow \atop 0}{\left(\frac{3}{7}\right)^n} \cdot \frac{3^{-1} - 2\overset{\to 0}{\left(\frac{2}{3}\right)^n} - 5 \overset{\to 0}{\frac{n^4}{3^n}}}{\underbrace{\underset{\downarrow \atop 0}{\frac{1}{7^n}} - 100 \underset{\downarrow \atop 0}{\frac{n^2}{7^n}} - 1 + \underset{\downarrow \atop 0}{\frac{\sqrt{n}}{7^n}}}_{②}} = 0 \cdot \frac{1/3}{-1} = \underline{\underline{0}}$$

① ① ①

3.)
(15p) $\lim\limits_{x\to 0} \dfrac{x^2-5x}{6x-2x^2}\Big|_0 = \lim\limits_0 \dfrac{\cancel{x}(x-5)}{\cancel{x}\cdot(6-2x)}$ ⑤ ⊜

behely: $\frac{0}{0}$ ⑤

L'H → ez is max pont
vagy
gyöktényezős alak

$= \lim\limits_0 \dfrac{x-5}{6-2x}\Big|_{\text{behely}} = \underline{\underline{\dfrac{-5}{6}}}$ ⑤ ✓.

4.) $f(x) = \dfrac{x^3-3x}{x-1}$ $D_f = \mathbb{R} \setminus \{1\}$ ① $x_0 = 2$
(15p)

$$f'(x) = \frac{(3x^2-3)\cdot(x-1)-(x^3-3x)\cdot 1}{(x-1)^2} =$$

$$= \frac{3x^3 \cancel{-3x} - 3x^2 + 3 - x^3 \cancel{+3x}}{(x-1)^2} =$$

$$= \frac{2x^3-3x^2+3}{(x-1)^2}$$ ⑥

Érintő egyenlete: $y = f'(x_0)\cdot(x-x_0)+f(x_0)$ ②

$f'(2) = \dfrac{2\cdot 8-3\cdot 4+3}{1^2} = 7$ ② $f(2) = \dfrac{8-3\cdot 2}{1} = 2$ ②

<u>e:</u> $y = 7\cdot(x-2)+2$

$\underline{y = 7x-12}$ ②

5) a) $\int e^{3x+1}\,dx = \frac{e^{3x+1}}{3} + c$ ②

(8p+12p)

lin. hely.

$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + c$ ②, ahol $\int f = F$.

$a = 3,\ f(x) = e^x \rightarrow F(x) = \int e^x = e^x$. ④

$b = 1$

N.-L. ②

b) $\int_0^{\frac{\pi}{2}} \sin x \cdot \cos^3 x\,dx \overset{1}{=} \left[-\frac{\cos^4 x}{4}\right]_{x=0}^{\frac{\pi}{2}} = -\frac{\overset{0}{\widetilde{\cos^4 \frac{\pi}{2}}}}{4} + \frac{\overset{1^4}{\widetilde{\cos^4 0}}}{4}$ ②

$= \underline{\underline{\frac{1}{4}}}$ ①

$\int \sin x \cdot \cos^3 x = -\int - \sin x \cdot \cos^3 x = -\frac{\cos^4 x}{4}$ ① ②

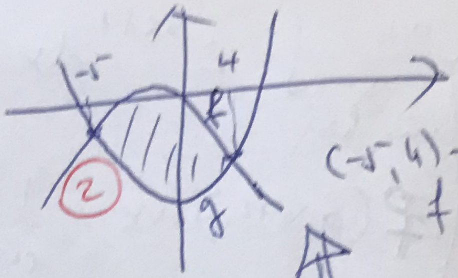
$\int f' \cdot f^3 = \frac{f^4}{4}$ ②

$f(x) = \cos x$ ②

$f'(x) = -\sin x$

6.)
(15p) $f(x) = -x^2 - x$

$g(x) = x^2 + x - 40$



(−5, 4)-n $f \geq g$ ①

Metszéspontok: $f(x) = g(x)$ ②

$$-x^2 - x = x^2 + x - 40$$

$$0 = 2x^2 + 2x - 40$$

$$0 = x^2 + x - 20$$

$$x_{1,2} = \frac{-1 \pm \sqrt{\overset{9}{1+80}}}{2} \begin{matrix} \nearrow 4 \\ \searrow -5 \end{matrix}$$ ②

Ter =

$$\int_{-5}^{4} f(x) - g(x)\,dx = \int_{-5}^{4} -x^2 - x - (x^2 + x - 40)\,dx =$$ ① ④

$$\int_{-5}^{4} -2x^2 - 2x + 40\,dx \overset{N-L}{=} \left[-2\frac{x^3}{3} - x^2 + 40x\right]_{x=-5}^{4} =$$ ① ① ②

$$= -2 \cdot \frac{64}{3} - 16 + 160 - \left(+2 \cdot \frac{125}{3} - 25 - 200\right) = \overset{243}{\cancel{495}}$$ ② ⑥

+) $z^2 - 10z + 34 = 0$
(15p)

$$z_{1,2} = \frac{10 \pm \sqrt{100 - 136}}{2} = \frac{10 \pm \sqrt{-36}}{2}$$ ④

$$\sqrt{-36} = \underbrace{\sqrt{-1}}_{\pm i} \cdot \underbrace{\sqrt{36}}_{6}$$ ⑥

$$z_{1,2} = \frac{10 \pm i \cdot 6}{2} = \underline{\underline{5 \pm 3i}}$$ ⑤