pzh 2 / α variáns

1. $|a_n|^{\frac{1}{n}} \to \frac{4}{5}|x+3|$ (2), $R=\frac{5}{4}$ (2), $KT = [-17/4, -\frac{7}{4})$. (4)

2a. $t=x-2$ $\quad \frac{1}{-4x-2} \overset{(2)}{=} \frac{1}{-4t-10} = -\frac{1}{10}\cdot\frac{1}{1+\frac{4}{10}t}$ (2)

$= -\frac{1}{10}\sum\left(-\frac{4}{10}\right)^n (x-2)^n$ (2) $\quad R=\frac{10}{4}$ (1)

2b. $g(x)=\frac{e^{5x}+e^{x}}{2}$ (2) $\quad e^{5x}=\sum\frac{(5x)^n}{n!}$

$e^{5(x-2)}=\sum\frac{5^n(x-2)^n}{n!}$, $e^{5x}=e^{10}\sum\frac{5^n(x-2)^n}{n!}$ (2)

$e^{x} = e^{2}\sum\frac{(x-2)^n}{n!}$ (2) $\quad R=\infty$ (1)

3. a) nem $\quad y=x \Rightarrow f(x,y)=\frac{1}{6}$, $f(0,0)=0$

b) $f_x = \frac{2xy^2(4x^4+2y^4)-x^2y^2\cdot16x^3}{(4x^4+2y^4)^2}$, $(x,y)\neq(0,0)$, $f_x(0,0)=$

$f_y = \frac{2x^2y(\dots)-x^2y^2\cdot 8y^3}{(4x^4+2y^4)^2}$, $(x,y)\neq(0,0)$, $f_y(0,0)=$

c) $(x,y)\neq(0,0)$

d) nem létezik: $f(0,0)=0$, $f(tv)=c\neq0$, $t\neq0$

4. $f_x = 4x^3-2x-2y=0$ (5) $\Rightarrow x^3=y^3$, $x=y$, $4x^3-4x=0$ (2)

$f_y=4y^3-2x-2y=0$

(3) $(0,0)$, $(1,1)$, $(-1,-1)$, $\quad f_{xx}=12x^2-2$, $f_{yy}=12y^2-2$ (5)

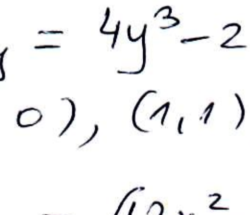
$f_{xy}=-2$

$D=(12x^2-2)(12y^2-2)-4$, $D(1,1)>0$, $f_{xx}(1,1)>$

(5) $(1,1)$ – min, $(-1,-1)$ – min, $D(0,0)=0$

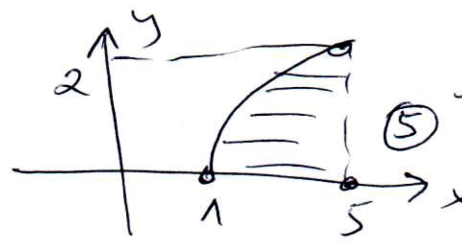
$f(x,x)=2x^4-4x^2<0$ (x kicsi), $f(x,-x)=2x^4>$

$(0,0)$ – nyeregpont. (5)

5.  $1-x\le y\le\sqrt{1-x^2}$ (6)

$\iint = \int_0^1\int_{1-x}^{\sqrt{1-x^2}} xy\,dy\,dx = \int_0^1 x\cdot\frac{1}{2}\left((1-x^2)-(1-x)^2\right)dx$ (6)

$=\int_0^1 (x^2-x^3)\,dx = \frac{1}{3}-\frac{1}{4}=\frac{1}{12}$ (6)

6. 

$$\iint = \int_1^5 \int_0^{\sqrt{x-1}} y e^{(x-1)^2} dx \quad (5)$$

$$= \frac{1}{2}\int_1^5 e^{(x-1)^2}(x-1)\,dx$$

$$= \frac{1}{4} e^{(x-1)^2}\Big|_1^5 = \frac{e^{16}-1}{4} \quad (5)$$

IMSC. $x^2+y^2+z^2 \to \min$, $x^2 = z^2+1$

$f(y,z) = y^2+2z^2+1 \to \min$ $\quad f_y = 2y$, $f_z = 4z$

$y=0$, $z=0$

$(\pm 1, 0, 0)$ – kritikus pontok, $f(y,z) \geq 1 \Rightarrow \boxed{(\pm 1, 0, 0)}$

pzh 2/β variáns

1. $$\left|\frac{a_{n+1}}{a_n}\right| = |x| \frac{(2n+3)^{n+1}}{(n+1)!}\cdot\frac{n!}{(2n+1)^n} = |x|\frac{2n+3}{n+1}\left(1+\frac{2}{2n+1}\right)^n \to |x| 2 \quad (4)$$

$$R = 1/2e \quad (4)$$

2a. $t = x+1$, $f = \frac{1}{2t+3}$ (2) $= \frac{1}{3}\cdot\frac{1}{\frac{2}{3}t+1}$ (2) $= \frac{1}{3}\sum\left(-\frac{2}{3}\right)^n (x+1)^n$ (2)

$R = 3/2$ (1)

2b. $g(x) = \frac{e^{5x}+e^x}{2}$ (2)

$e^{5(x-4)} = \sum \frac{5^n (x-4)^n}{n!}$, $e^{5x} = e^{20}\sum\frac{5^n(x-4)^n}{n!}$ (2)

$e^x = e^4 \sum \frac{(x-4)^n}{n!}$ (2) $R = \infty$ (1)

3. lásd α variáns

4. $f_x = 3x^2+3y = 0$ (5)

$f_y = 3y^2+3x = 0$

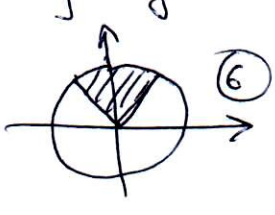
$y = -x^2$, $x^4+x = 0$

$(0,0)$ nyereg, $(-1,-1)$ min (5)

$D = 36xy - 9$ (5)

$D(0,0) < 0$ nyereg (5)

$D(-1,-1) > 0$, $f_{xx} = 6x < 0$ (5)

5.  (6)

$$\int_0^{1/\sqrt{2}}\int_x^{\sqrt{1-x^2}} xy\,dy\,dx = \frac{1}{2}\int_0^{1/\sqrt{2}}(x-2x^3) = 1/16 \quad (6$$

$$\int_{-1/\sqrt{2}} \ldots = -1/16.\ \text{Tehát } \underline{0} \quad (6$$