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A VARIÁNS (RÉSZLETES)

1, T.: Ha a_n konvergens, akkor korlátos. (3)

$\boxed{10}$ B.: $\exists A = \lim_{n\to\infty} a_n \in \mathbb{R}. \Rightarrow \varepsilon = 1,\ \exists N(1):\ \forall n \geq N:$

$|a_n - A| < 1 \Rightarrow |a_n| < |A| + 1$

(7) Legyen $K := \max\{|a_1|, |a_2|, \ldots |a_{N-1}|, |A| + 1\}$

$K \in \mathbb{R}$, és $\forall n$-re $|a_n| \leq |K|$ ✓

2, $\boxed{6}$ T_1: Ha f diff.-ható x_0-ban, és f-nek lokális szélsőértéke van x_0-ban, akkor $f'(x_0) = 0$. (Lokális szélsőért. szükséges feltétele)

(3)

(3) T_2: Ha f kétszer diff.-ható x_0-ban, és $f'(x_0) = 0$, $f''(x_0) \neq 0$, akkor f-nek lok. szélsőértéke van x_0-ban. $f''(x_0) < 0$ esetén lok. max., $f''(x_0) > 0$ esetén lok. min.

egyik elég

(3) T_2': Ha f diff.-ható x_0 egy környezetében, $f'(x_0) = 0$, és f' előjelet vált x_0-ban, akkor f-nek lok. szélsőértéke van x_0-ban. Ha f' pozitívból negatívba vált $\Rightarrow$ lok. max, Ha f' negatívból pozitívba vált, $\Rightarrow$ lok. min.

4, $\boxed{10}$ $f(x) = \dfrac{x}{\sqrt{1+x^4}}$;

$$f'(x) = \frac{\sqrt{1+x^4} - x\dfrac{4x^3}{2\sqrt{1+x^4}}}{1+x^4} = \frac{1-x^4}{(1+x^4)^{3/2}} = \underbrace{\frac{1+x^2}{(1+x^4)^{3/2}}}_{>0}(1+x)(1-x)$$ (3) (3)

x	$x < -1$	-1	$-1 < x < +1$	$+1$	$1 < x$
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	↘	lok. min	↗	lok. max	↘

(4)

3, $\boxed{10}$
$$\frac{2x^2+x+2}{(x-1)(3+2x^2)} = \frac{A}{x-1} + \frac{Bx+C}{3+2x^2} \quad (4)$$

$$2x^2+x+2 = 2Ax^2 + 3A + Bx^2 + Cx - Bx - C$$

$$\left.\begin{array}{ll} x^2: & 2A+B=2 \\ x: & C-B=1 \\ 1: & 3A-C=2 \end{array}\right\}$$

Összeadva őket: $5A = 5 \Rightarrow A = 1$; $B = 2-2A = 0$; $C = 1+B = 1$ (3)

$$\int \frac{2x^2+x+2}{(x-1)(3+2x^2)}\,dx = \int \frac{1}{x-1}\,dx + \int \frac{1}{3+2x^2}\,dx =$$

$$= \ln|x-1| + \frac{1}{3}\int \frac{1}{1+\left(\sqrt{\frac{2}{3}}x\right)^2}\,dx = \ln|x-1| + \underbrace{\frac{1}{3}\sqrt{\frac{3}{2}}}_{\frac{1}{\sqrt{6}}}\,\mathrm{arctg}\left(\sqrt{\frac{2}{3}}x\right) + C$$

(1) (2)

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4, $\boxed{12}$ $y' + 2xy = 2xe^{-x^2}$ elsőrendű, inhomogén lineáris.

(H): $y' = \frac{dy}{dx} = -2xy \Rightarrow \int \frac{dy}{y} = \int -2x\,dx$

$\Rightarrow \ln|y| = -x^2 + C \Rightarrow y_{H,ált}(x) = K\,e^{-x^2}$; $K \in \mathbb{R}$ (5)

Variálás:

$\boxed{7}$

$y_{I,p}(x) = K(x)\,e^{-x^2}$ (2) ; Beírva:

$$K'(x)\,\cancel{e^{-x^2}} - \cancel{2xK(x)e^{-x^2}} + \cancel{2xK(x)e^{-x^2}} = 2x\,\cancel{e^{-x^2}}$$

$$K'(x) = 2x \Rightarrow K(x) = x^2$$

$$y_{I,p} = x^2 e^{-x^2} \quad (3)$$

$$y_{I,ált} = y_{I,p} + y_{H,ált} = (K+x^2)\,e^{-x^2}, \quad K \in \mathbb{R}. \quad (2)$$

5, $\boxed{9}$

$f(x) = \operatorname{arc\,tg} x$

$$f'(x) = \frac{1}{1+x^2} \overset{(2)}{=} \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n} \overset{(2)}{,} \quad \text{ha } |q|<1, \text{ azaz } |x|<1.$$

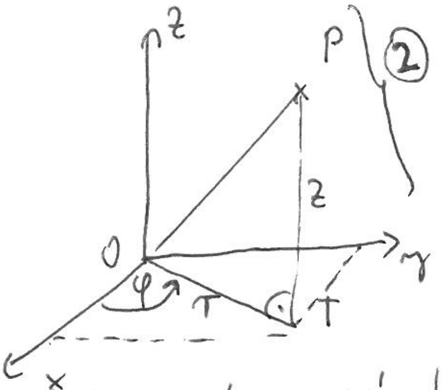
$\uparrow$ $q = -x^2$

$$f(x) = f(0) + \int_0^x f'(t)\,dt = 0 + \sum_{n=0}^{\infty} (-1)^n \int_0^x t^{2n}\,dt =$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \cdot x^{2n+1} \overset{(4)}{=} x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots \quad (1)$$

$\boxed{R=1}$ (marad)

6, a, $\boxed{8}$,



(2)

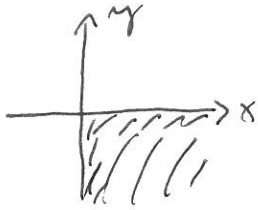
$$\left.\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ z &= z \end{aligned}\right\} (2)$$

(4)

$$J = \begin{vmatrix} x'_r & x'_\varphi & x'_z \\ y'_r & y'_\varphi & y'_z \\ z'_r & z'_\varphi & z'_z \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\sin\varphi & 0 \\ \sin\varphi & r\cos\varphi & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} =$$

$$= r(\cos^2\varphi + \sin^2\varphi) = \underline{\underline{r}}$$

b, $\boxed{10}$



$$V = \begin{cases} x^2 + y^2 \le 4 \\ x \ge 0,\ y \le 0 \\ 0 \le z \le x \end{cases} \iff \begin{aligned} &0 \le r \le 2 \\ &-\tfrac{\pi}{2} \le \varphi \le 0 \\ &0 \le z \le r\cos\varphi \end{aligned}$$

$$I = \int_{r=0}^{2} \int_{\varphi=-\frac{\pi}{2}}^{0} \int_{z=0}^{r\cos\varphi} \underbrace{r\cos\varphi}_{x}\, \underbrace{r\sin\varphi}_{y}\, r \, dz\, d\varphi\, dr \overset{(5)}{=}$$

$$= \int_{r=0}^{2} r^3 \int_{\varphi=-\frac{\pi}{2}}^{0} \cos\varphi \sin\varphi \cdot \underbrace{\big[z\big]_0^{r\cos\varphi}}_{r\cos\varphi} d\varphi\, dr = \left(\int_{r=0}^{2} r^4\,dr\right)\left(\int_{\varphi=-\pi/2}^{0} \sin\varphi\cos^2\varphi\, d\varphi\right) =$$

$$= \left[\frac{r^5}{5}\right]_0^2 \cdot \left[-\frac{\cos^3\varphi}{3}\right]_{-\pi/2}^{0} = \frac{32}{5} \cdot \left(\frac{-1}{3}\right) = \underline{\underline{-\frac{32}{15}}} \quad (5)$$

(*)

7, $\boxed{13}$ $f(x) = |\sin x|$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |\sin x|\, dx = \frac{2}{\pi}\int_0^{\pi} \sin x\, dx = \frac{2}{\pi}\Big[-\cos x\Big]_0^{\pi} = \underline{\underline{\frac{4}{\pi}}}$$ ① ③

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{|\sin x| \cos x}_{\text{páros}}\, dx = \frac{2}{\pi}\int_0^{\pi} \sin x \cos x\, dx = \frac{1}{\pi}\int_0^{\pi} \sin(2x)\, dx = \underline{\underline{0}}$$ ① ④

$$b_5 = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{|\sin x| \sin(5x)}_{\text{páratlan}}\, dx = 0$$ ① ③

8, (*) $\boxed{12}$

$$\mathcal{F}[f(x+h)](\omega) = \int_{-\infty}^{\infty} e^{-i\omega x} f(\underbrace{x+h}_{t})\, dx = \int_{-\infty}^{\infty} e^{-i\omega(t-h)} f(t)\, dt =$$

$$= e^{i\omega h}\, \mathcal{F}[f](\omega)$$

Csak kimondás: 4p. Bizonyítás: 8p.

B VARIÁNS (TÖMÖR)

1, Lásd A. 2, Lásd A,

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3, $\int \frac{5x^2+x+3}{(x+1)(2+5x^2)}dx = \int \frac{1}{x+1}dx + \int \frac{1}{2+5x^2}dx =$

$= \ln|x+1| + \frac{1}{2}\sqrt{\frac{2}{5}}\,\mathrm{arc\,tg}\left(\sqrt{\frac{5}{2}}x\right) + C$

—∘—

4, $y_{H,\text{ált}} = K e^{x^2}$; $K \in \mathbb{R}$

$y_{I,p} = x^2 e^{x^2}$; $y_{I,\text{ált}} = (K + x^2)e^{x^2}$; $K \in \mathbb{R}$.

—∘—

5, $f'(x) = \frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n \cdot x^n$, ha $|x| < 1$.

$\ln(1+x) = \ln 1 + \int_0^x f'(t)\,dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1}$; $R = 1$

—∘—

6, a, Lásd A.

b, $\int_{r=0}^{3} \int_{\varphi=0}^{\pi/2} \int_{z=0}^{r\sin\varphi} r\cos\varphi\, r\sin\varphi\, r \, dz\, d\varphi\, dr =$

$= \left(\int_{r=0}^{3} r^4\,dr\right) \cdot \left(\int_{\varphi=0}^{\pi/2} \cos\varphi \sin^2\varphi\,d\varphi\right) = \left[\frac{r^5}{5}\right]_0^3 \cdot \left[\frac{\sin^3\varphi}{3}\right]_0^{\pi/2} = \frac{3^5}{5} \cdot \frac{+1}{3}$

—∘—

7, $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x|\,dx = \frac{4}{\pi}$

$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{|\cos x|\cos x}_{\text{páros}}\,dx = \frac{1}{\pi}\left(\int_0^{\pi/2} \cos^2 x\,dx - \int_{\pi/2}^{\pi} \cos^2 x\,dx\right) = 0$

$b_3 = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{|\cos x|\sin(3x)}_{\text{páratlan}}\,dx = 0$

—∘—

8, Lásd A.