α variáns megoldások.

1a. $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos kx\,dx,\ k \geq 0,\quad b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin kx\,dx,\ k \geq 1$ 2

$f(x) \sim \frac{a_0}{\cancel{2}} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$ 2

1b. $f(x) = x^2$ páros $\Rightarrow b_k = 0,\ \forall k$ 2

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{2\pi}\ \frac{x^3}{3}\Big|_{-\pi}^{\pi} = \frac{\pi^2}{3}$ 2

$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos kx\,dx = -\frac{1}{k\pi}\int_{-\pi}^{\pi} 2x \sin kx\,dx =$ 2

$= \frac{2}{\pi k^2}\ x\cos kx\Big|_{-\pi}^{\pi} = \frac{4}{k^2}(-1)^k$ 2

1c. $f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}(-1)^n \cos nx$ 2, $f(0) = 0 \Rightarrow -\frac{\pi^2}{12}$ 2

2a. $u = xy,\ v = y/x \Rightarrow y = \sqrt{uv},\ x = \sqrt{\frac{u}{v}}$ 2

$J = \begin{pmatrix} \frac{1}{2} u^{-1/2} v^{-1/2}, & -\frac{1}{2} u^{1/2} v^{-3/2} \\ \frac{1}{2} u^{-1/2} v^{1/2} & \frac{1}{2} u^{1/2} v^{-1/2} \end{pmatrix}$ 4, $\det J = \frac{1}{2v}$ 4

2b. $S = \iint_V 1\,dx\,dy$ 2 $= \int_2^3\int_3^4 \frac{1}{2v}$ 4 $du\,dv = \int_2^3 \frac{1}{2v}\,dv = \frac{1}{2}\ln\frac{3}{2}$ 4

3a $f \in C[x,y] \cap D(x,y)$ 2 $\Rightarrow \exists c \in (x,y) : f(x) - f(y) = f'(c)(x-y)$ 3

3b. Ha $f'(c) = 0,\ \forall c \Rightarrow f(x) = f(y),\ \forall x,y \in [a,b]$ 4

4a $\lim |a_n|^{1/n} = \begin{cases} < 1 & \text{konv.} \\ > 1 & \text{div} \\ = 1 & ? \end{cases}$ 4

4b1. Hányados kr. $\frac{(n+1)!}{2^{n+1}+1} \cdot \frac{2^n+1}{n!}$ 3 $= \frac{(n+1)(1+2^{-n})}{2+2^{-n}}$ 4 $\to \infty$ div 2

4.b.2 előjelváltó 1 ⊕ $\frac{n^2}{n^3+1} \to 0$ 2 ⊕ $\left(\frac{x^2}{x^3+1}\right)' < 0$ 2 mon. csökken

⊕ Leibnitz tétel 2 konv. felt.

4.b.3. gyökkr. 3 $(n^{1/n} - 1)^2$ 2 $\to 0$ 2 konv.

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α 2 olal

5. homogén: $xy' - y = 0$ $\quad \frac{dy}{y} = \frac{dx}{x}$ 2, $y = cx$ 2

$y = c(x)x$ $\quad x(c'(x)x + c(x)) - c(x)x = x^3 + 1$

$c'(x) = x + \frac{1}{x^2}$, 3

$c(x) = \frac{x^2}{2} - \frac{1}{x} + c$ 3, $y = \frac{x^3}{2} - 1 + cx$, $5 = 3 + 2c$, $c = 1$ 2

6a. gyök kr. $|x+1| \cdot 3 \cdot \sqrt[n]{\frac{1+(2/3)^n}{n}} \to 3|x+1|$, $R = 1/3$ 3

$(-4/3, -2/3)$ $\quad -2/3$: $\sum \frac{1+(2/3)^n}{n}$ = div + konv. = div. 2

$-4/3$: $\quad \sum (-1)^n \frac{1+(2/3)^n}{n}$ konv. $\quad KT = [-4/3, 2/3)$ 2

6b hányados kr. $\frac{(n+1)!}{(n+1)^2} |x-5|^{n+1} \frac{n^2}{n!|x-5|^n} = \frac{n^2}{n+1} |x-5| \to \infty$, $x \neq 5$

$R = 0$ 3

$KT = \{5\}$ 2

6c. gyök kr. $\left(1 + \frac{1}{n}\right)^n |x-1| \to e|x-1|$, $R = 1/e$ 3

2

β variáns

1. UA

2a. UA $\quad$ 2b. $S = \int_2^4 \int_2^5 \frac{1}{2v} \, du\, dv = \frac{3}{2} \int_2^4 \frac{1}{v} \, dv = \frac{3}{2} \ln 2$

3. UA

4. UA

5. homogén UA: $y = c(x)x$ $\quad 2+2=$ 4, $c'(x) = 1 + \frac{1}{x^2}$ 3

$c(x) = x - \frac{1}{x} + c$ 3 $\quad y = x^2 - 1 + cx$, $5 = 3 + 2c$, $c = 1$ 2

6a gyök kr. $5|x+1| \sqrt[n]{\frac{(3/5)^n + 1}{n}} \to 5|x+1|$, $R = \frac{1}{5}$ 3

$(-6/5, -4/5)$, $-4/5$: div. 2 $\quad -6/5$ konv. $\quad KT = [-6/5, -4/5)$ 2

6b. $R = 0$ 3, $KT = \{-10\}$ 2 $\quad$ 6c. $R = 1/e$

$2+3=5$