

Képfrekvencia

03

Számítógépes látórendszerek

Dr. Szemenyei Márton

Adjunktus

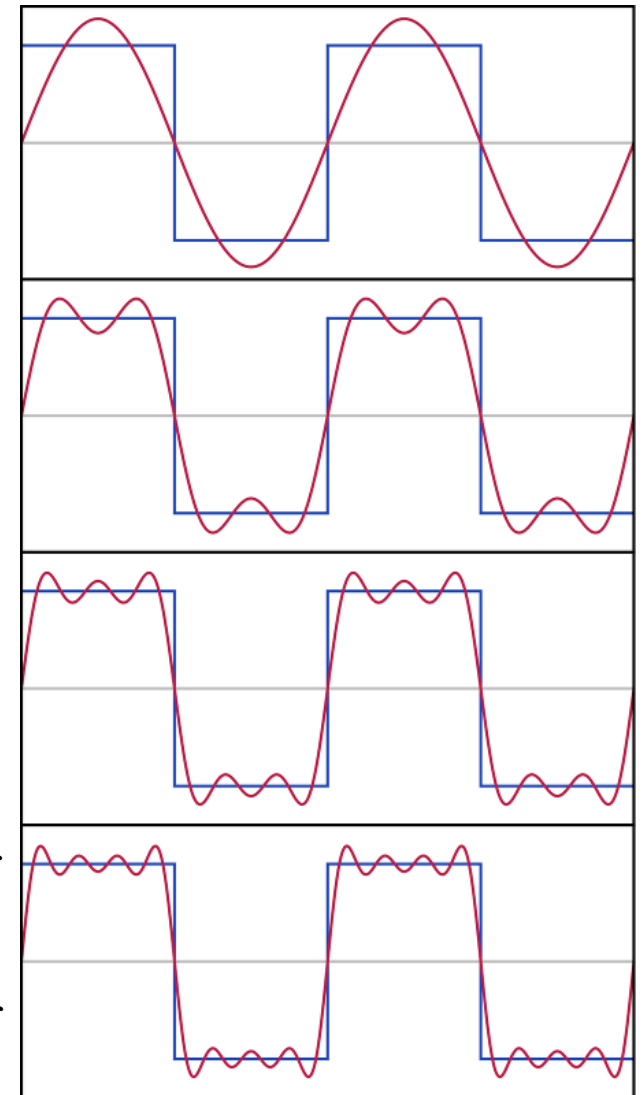
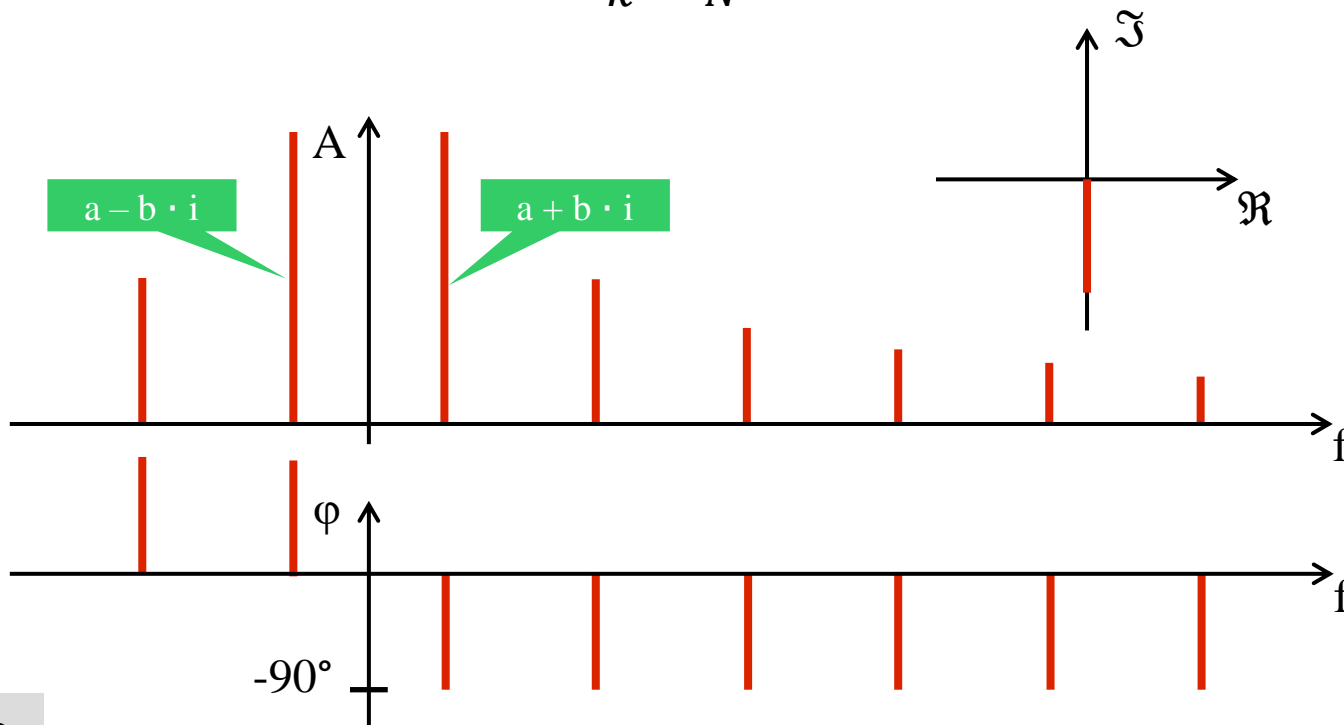
2022

Fourier tér



$$f(t) = a_0 + \sum_{k=1}^N a_k \cdot \sin(k\omega_0 \cdot t + \phi_k)$$

$$f(t) = \hat{f}_0 + \sum_{k=-N}^N \hat{f}_k \cdot e^{i \cdot k\omega_0 \cdot t}$$



Fourier transzformáció

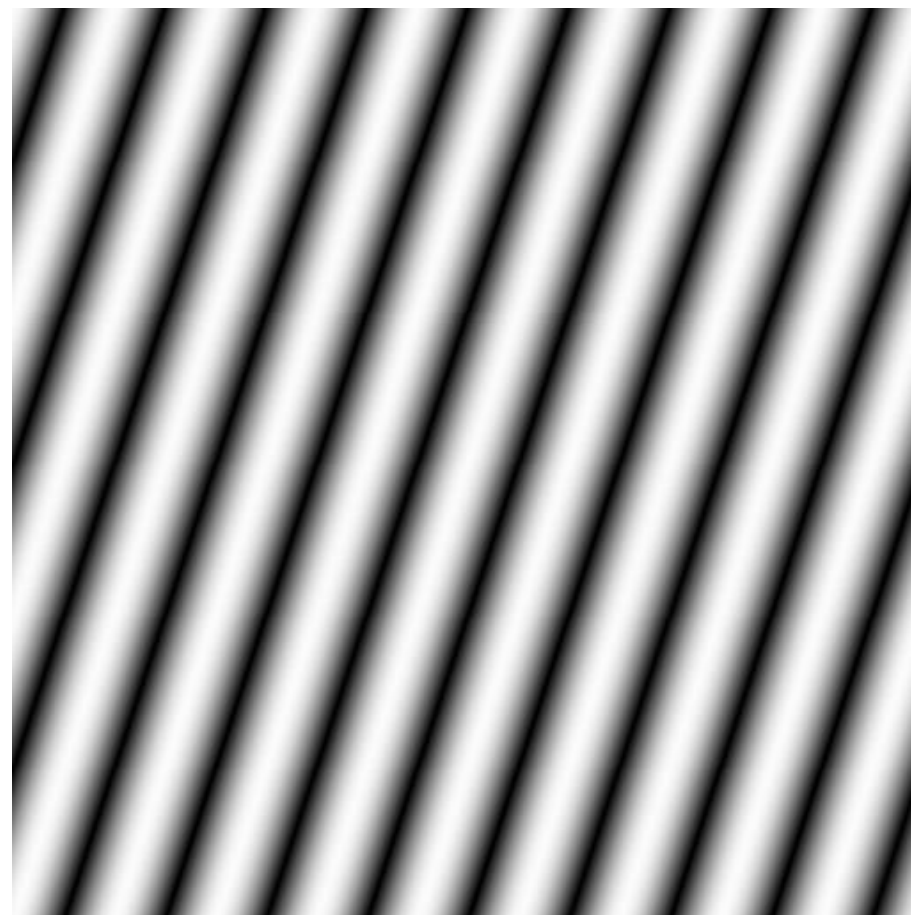


$$F(k, l) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) \cdot e^{-i\left(kx\frac{2\pi}{M} + ly\frac{2\pi}{N}\right)}$$

$$f(x, y) = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} F(k, l) \cdot e^{i\left(kx\frac{2\pi}{M} + ly\frac{2\pi}{N}\right)}$$

Függvény	Spektrum
Periodikus	Diszkrét
Diszkrét	Periodikus
Valós	Szimmetrikus
Szimmetrikus	Valós

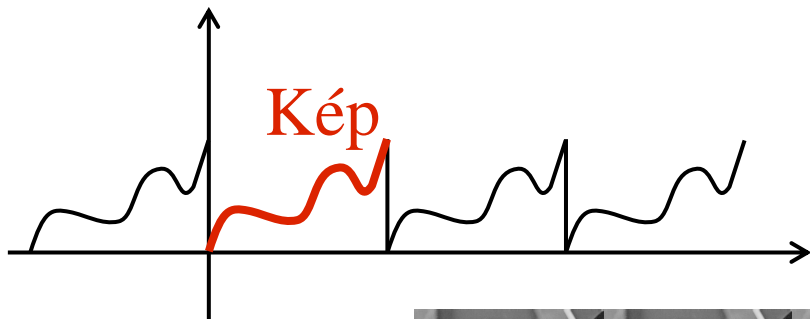
Képfrekvencia



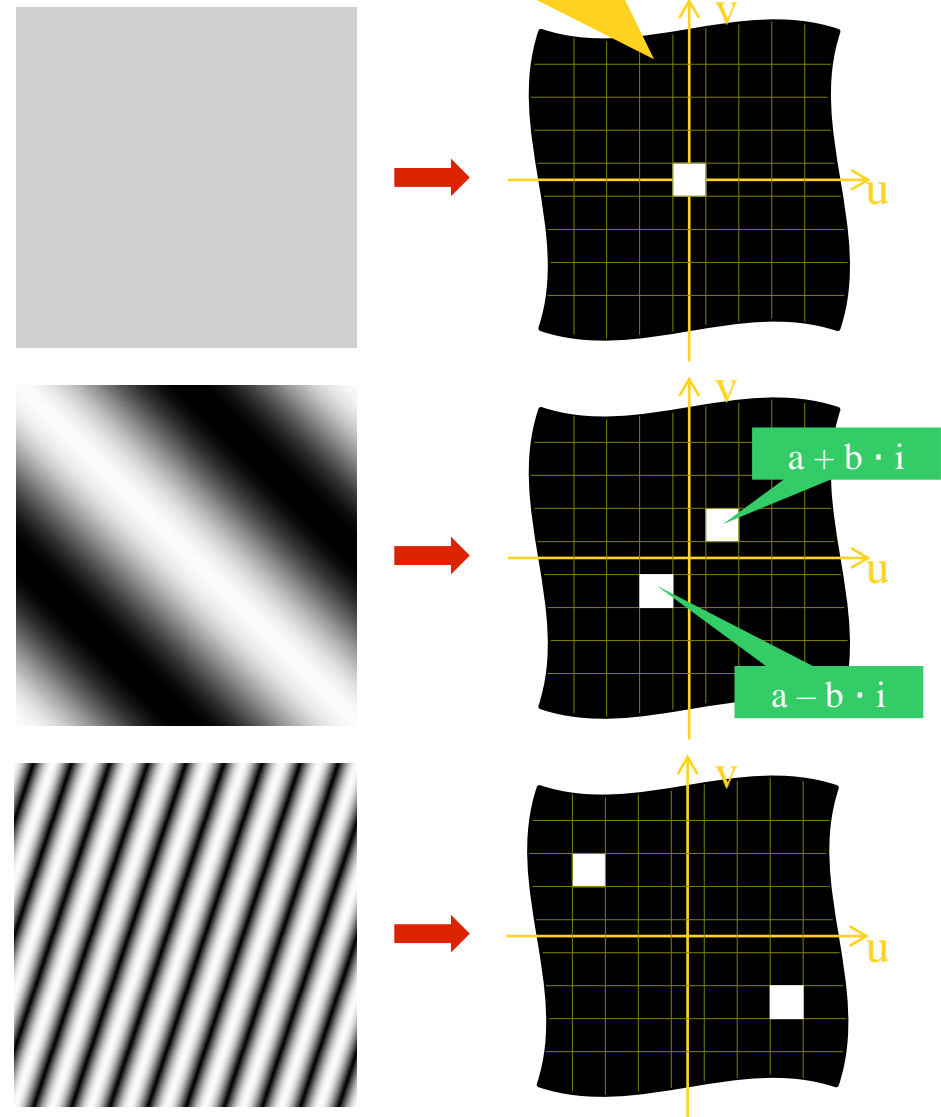
Fourier tér



$$f(x, y) = \sum_{m=-M}^M \sum_{n=-N}^N \hat{f}_{m,n} e^{imx} e^{iny}$$



Komplex frekvenciakép részlete

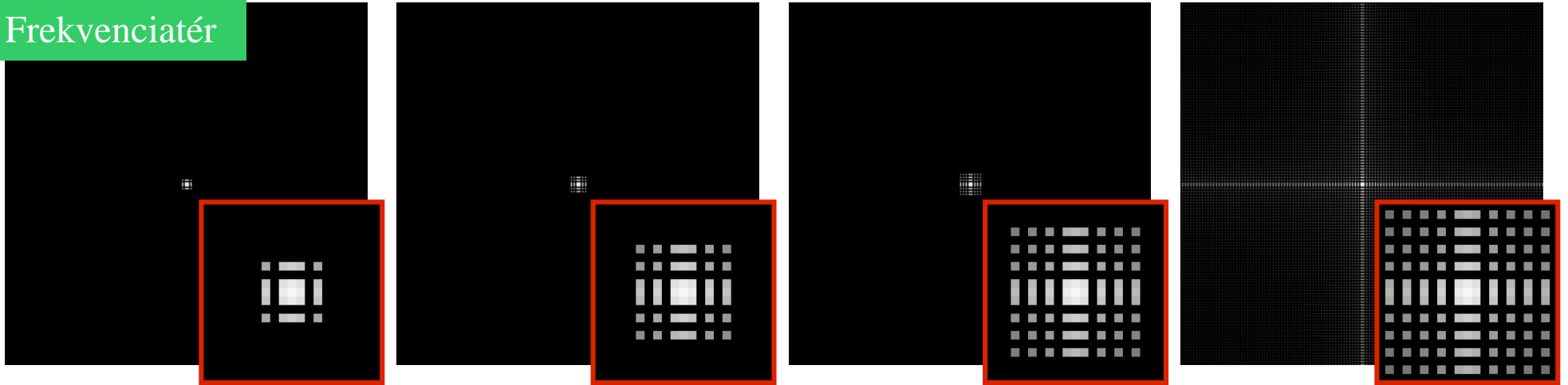


! Minden elem komplex!

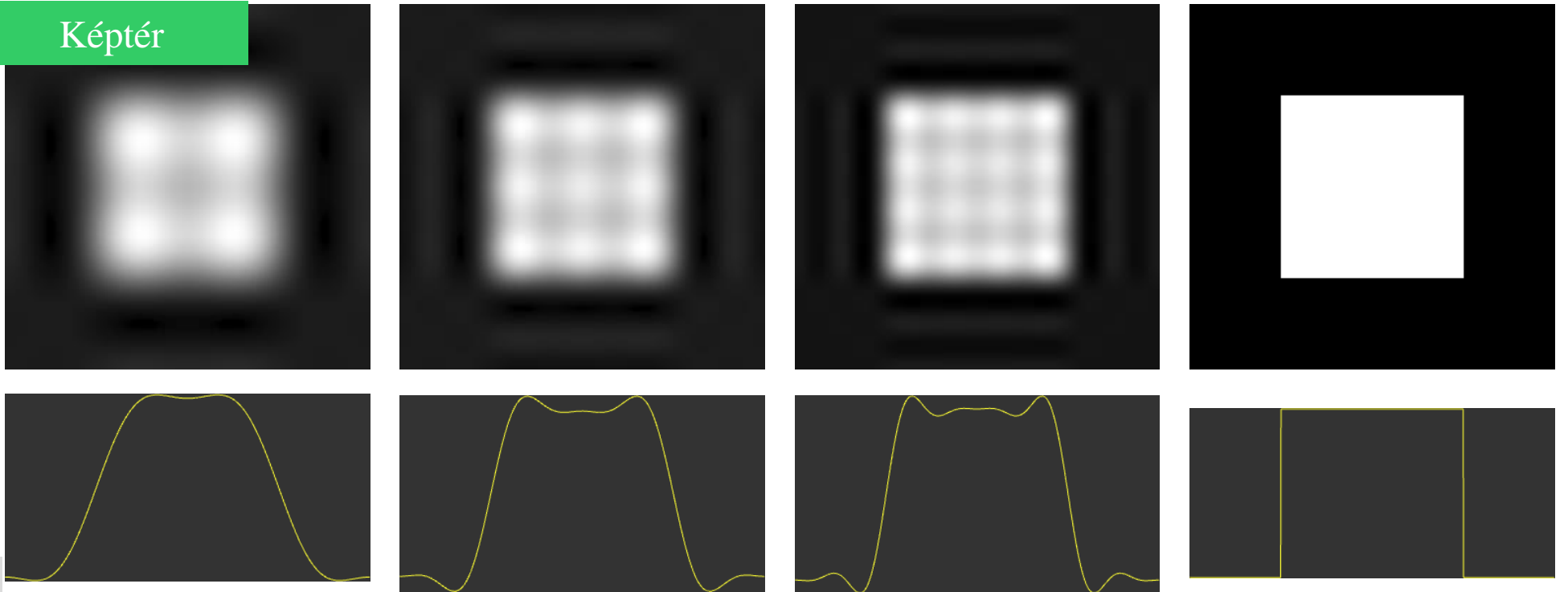


„Négyszögjel”

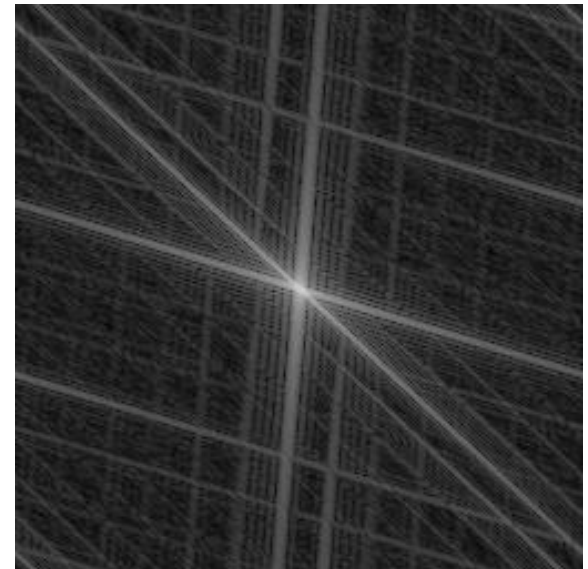
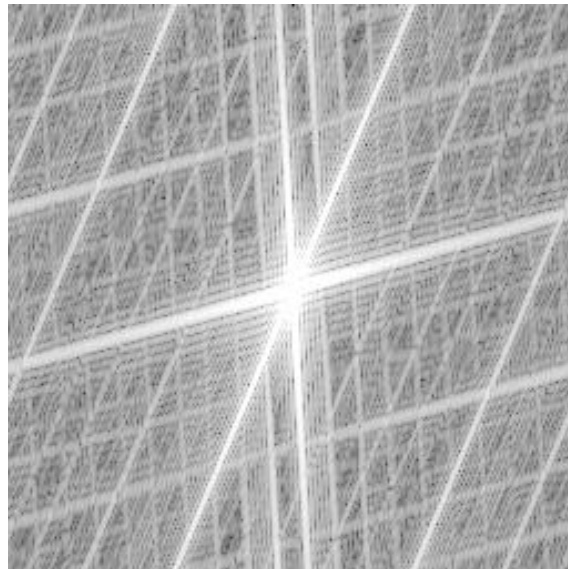
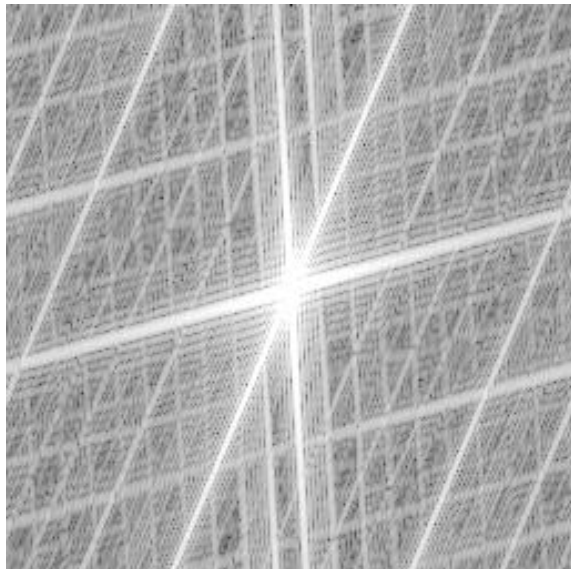
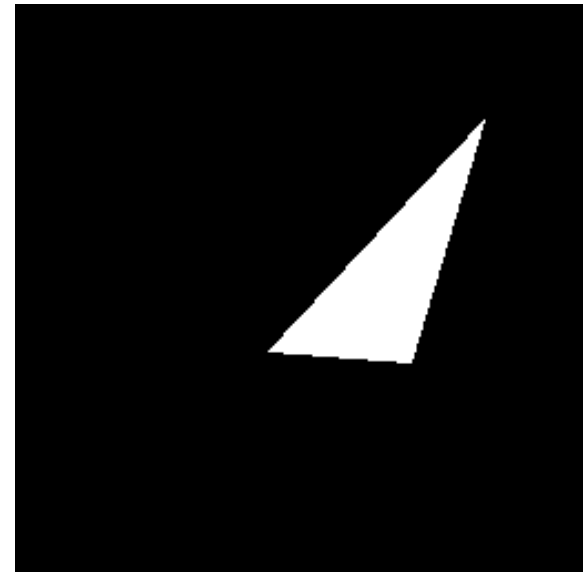
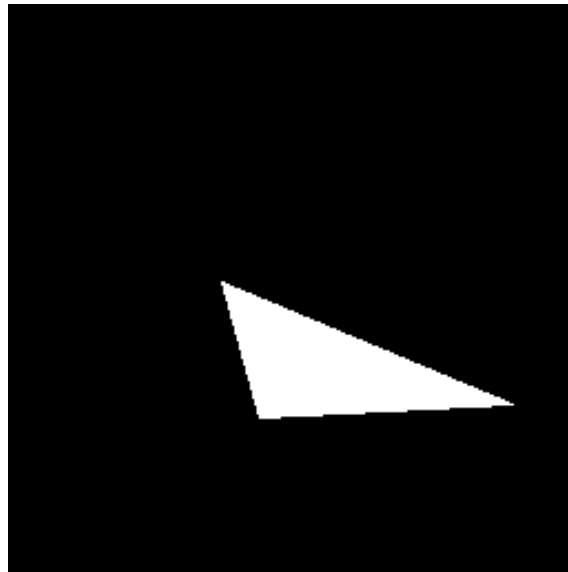
Frekvenciátér



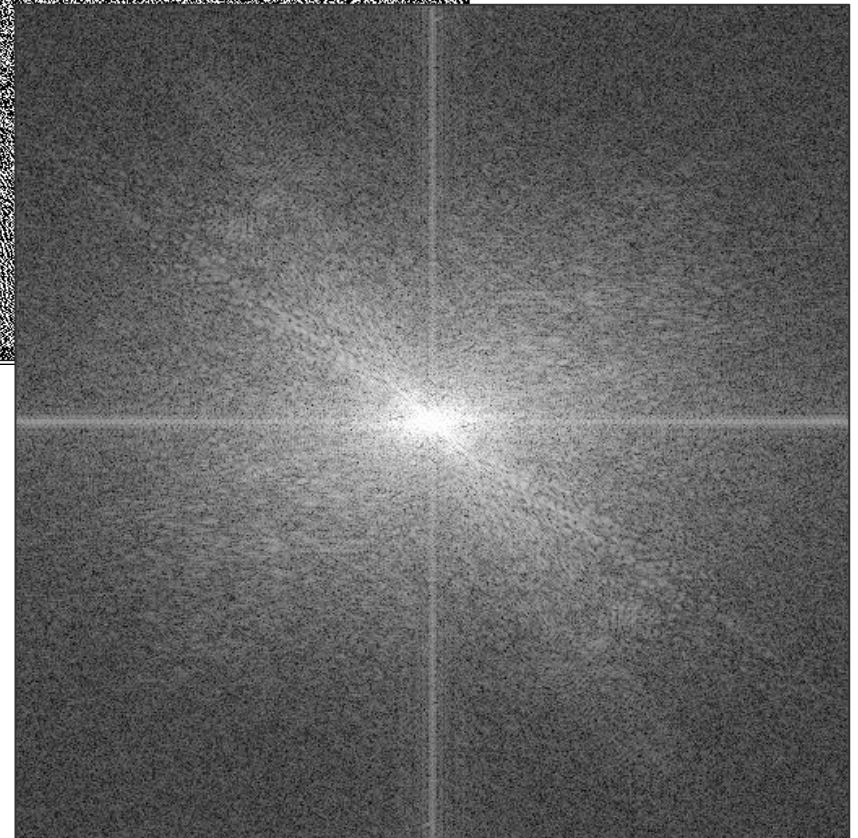
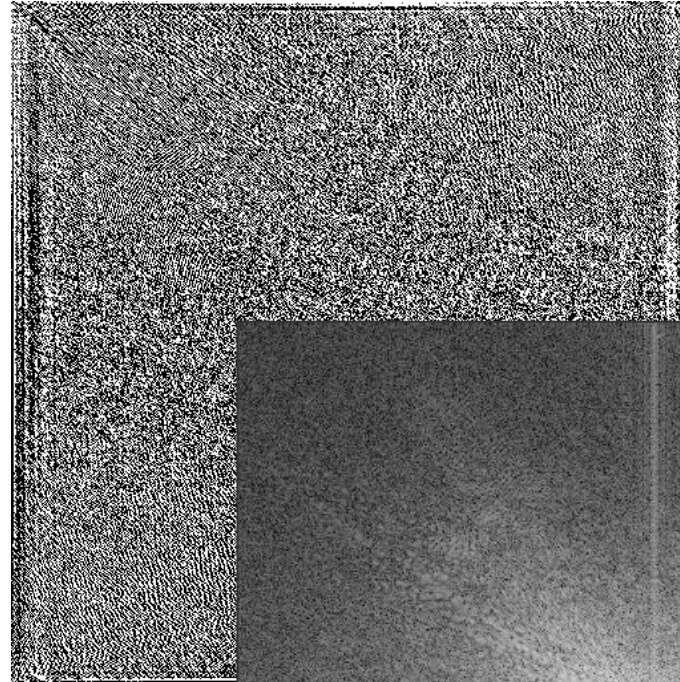
Képtér



Pozíció, orientáció



Fourier transzformáció

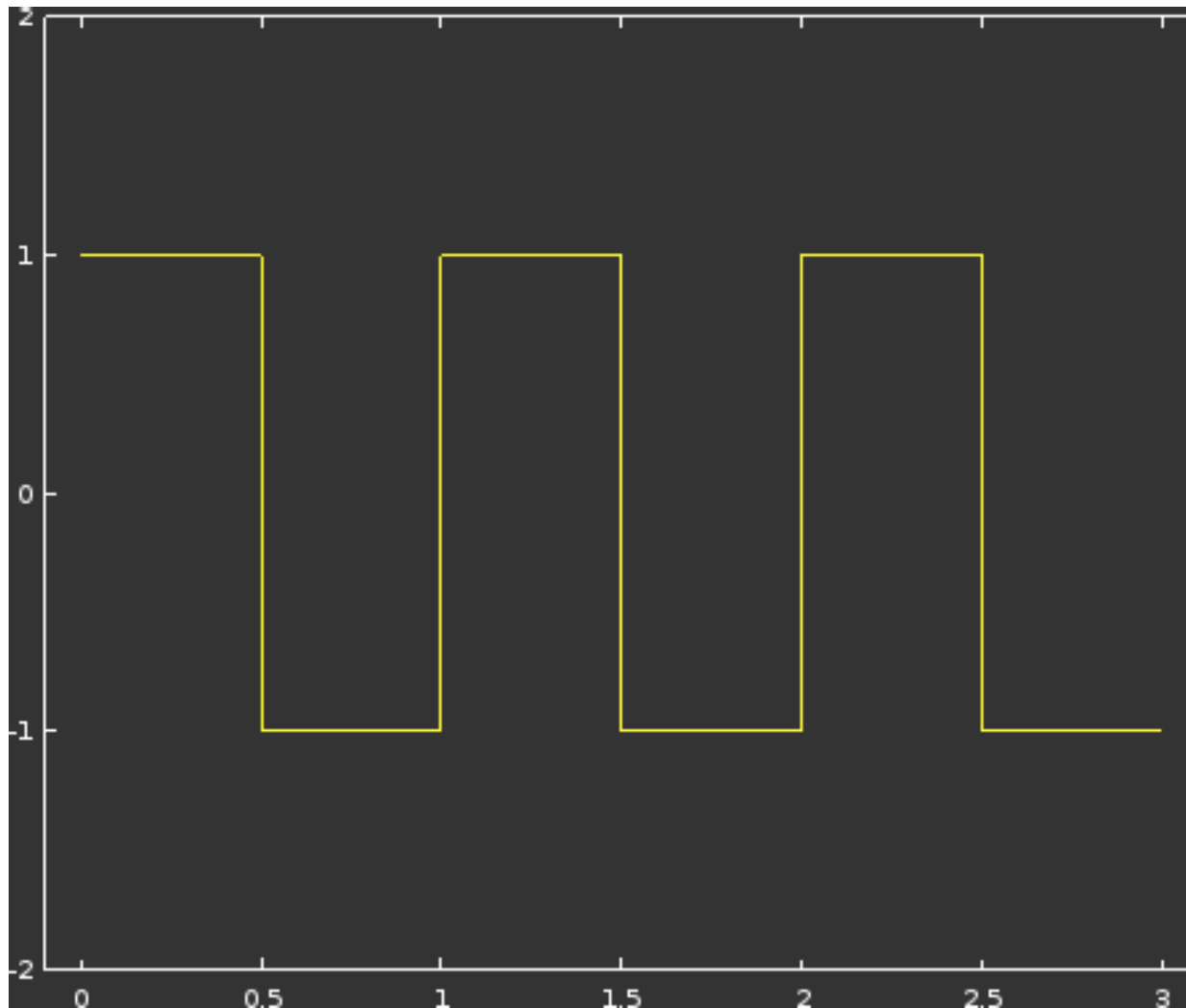


$$0.1 \cdot \log \left(1 + \sqrt{\Re^2(I_{uv}) + \Im^2(I_{uv})} \right)$$

Fázistorzítás



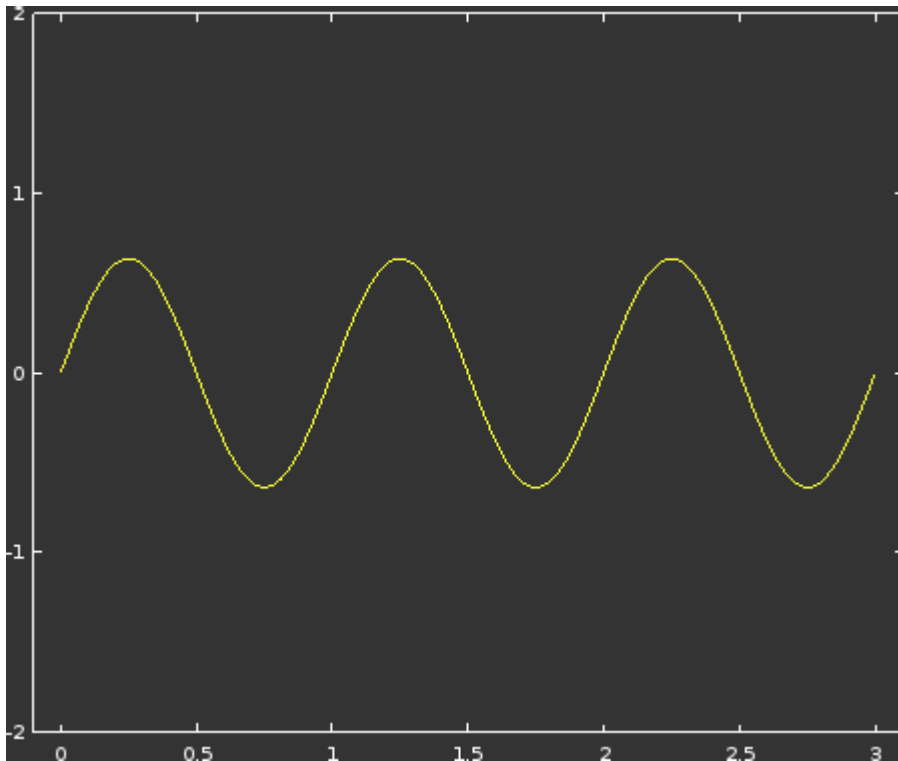
Kiinduló jel



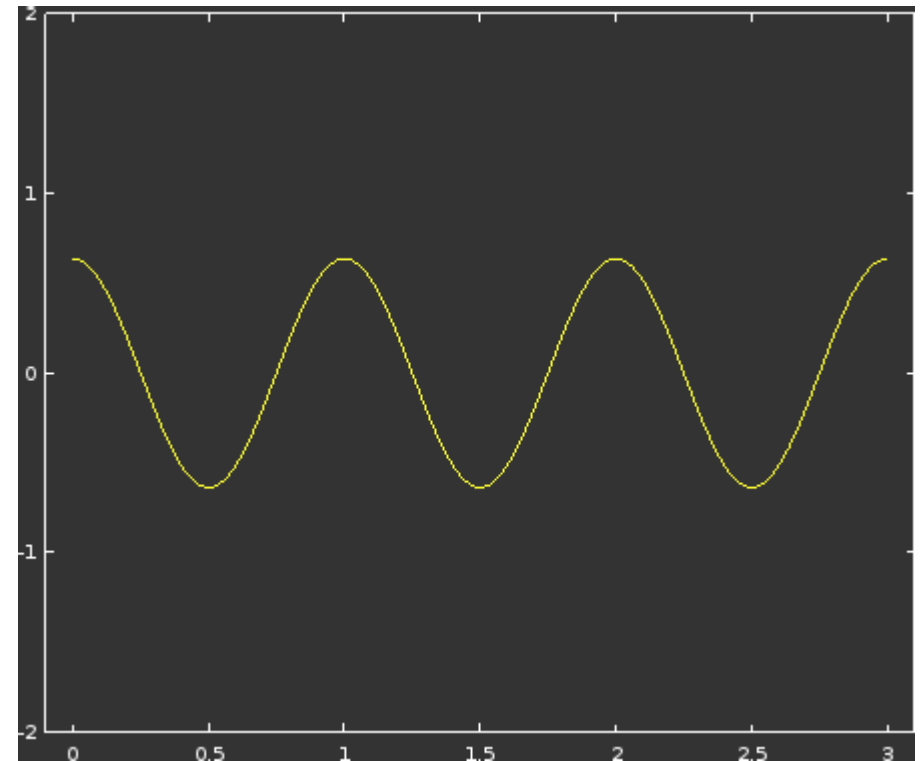
Fázistorzítás



1 felharmonikus



-90°



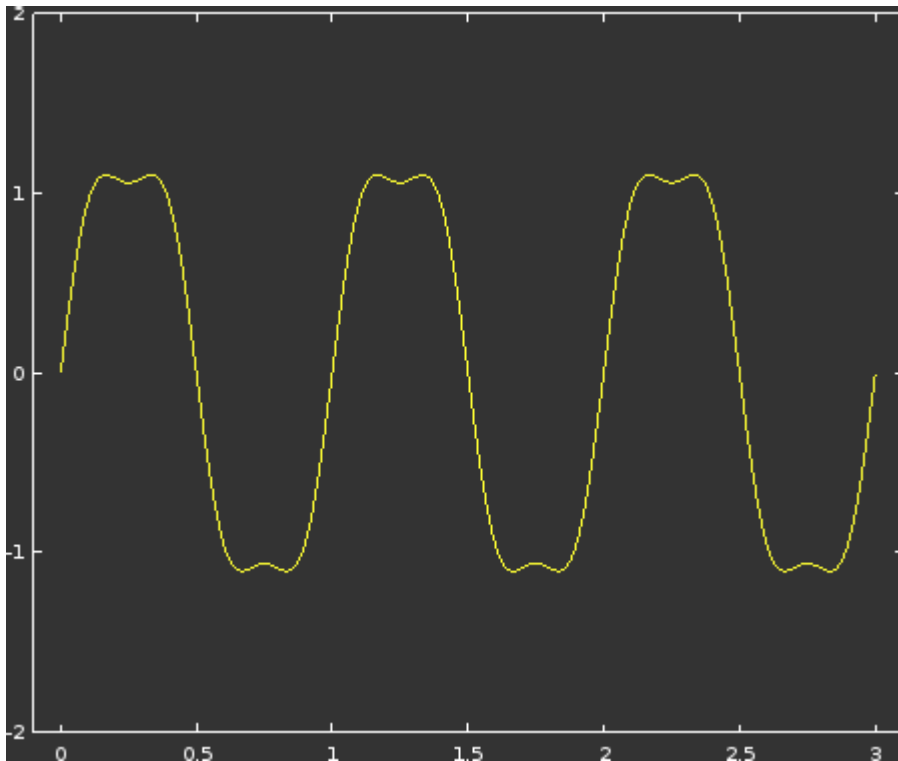
0°

fázis

Fázistorzítás

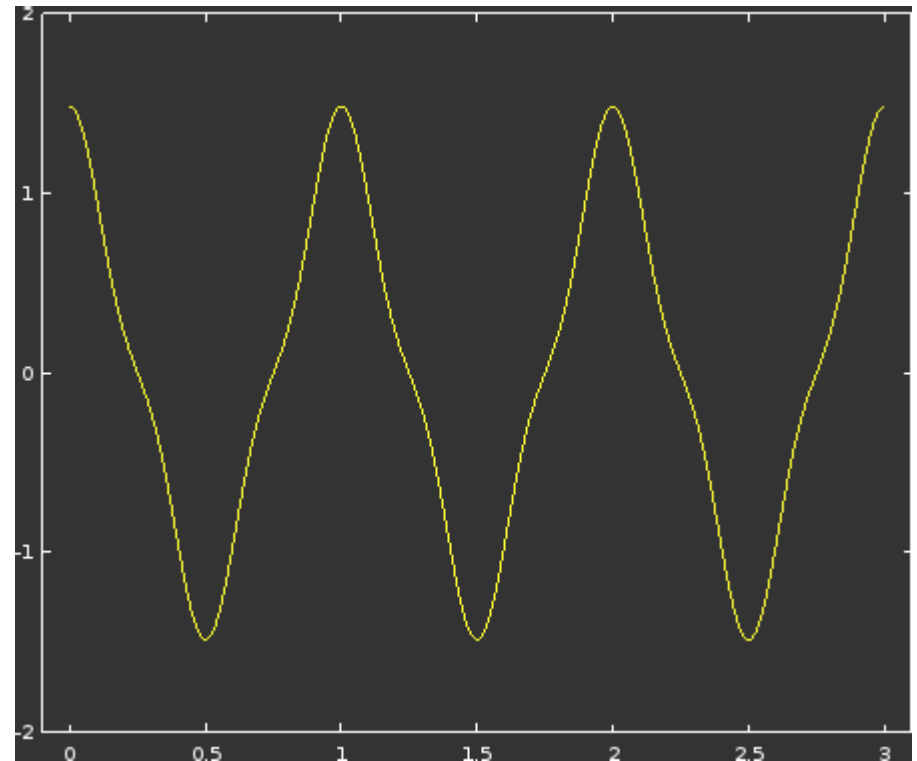


2 felharmonikus



-90°

fázis

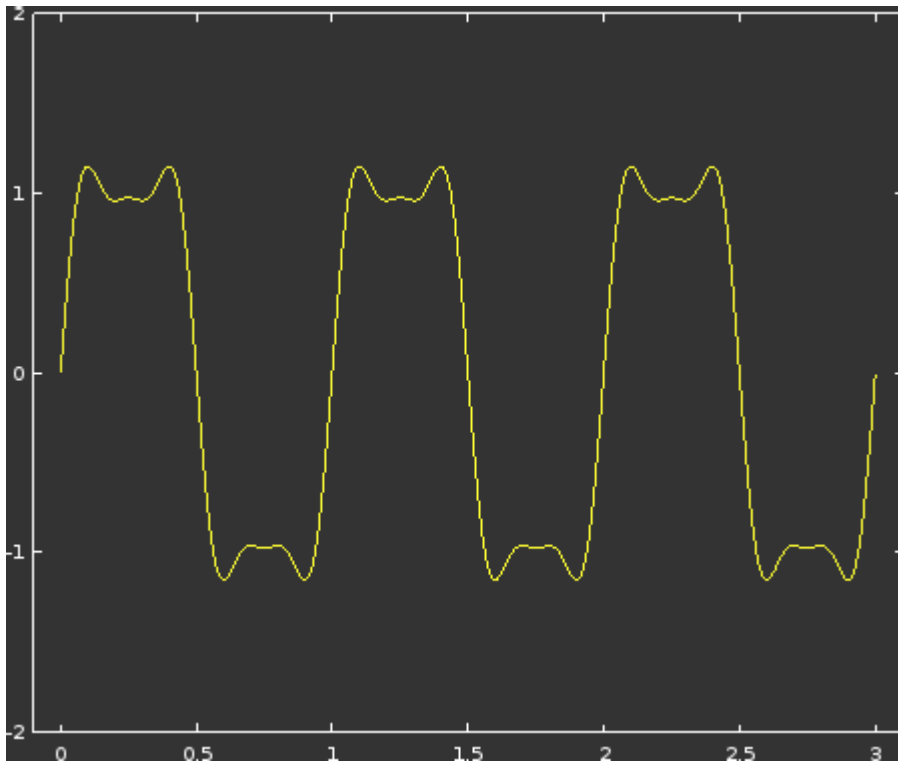


0°

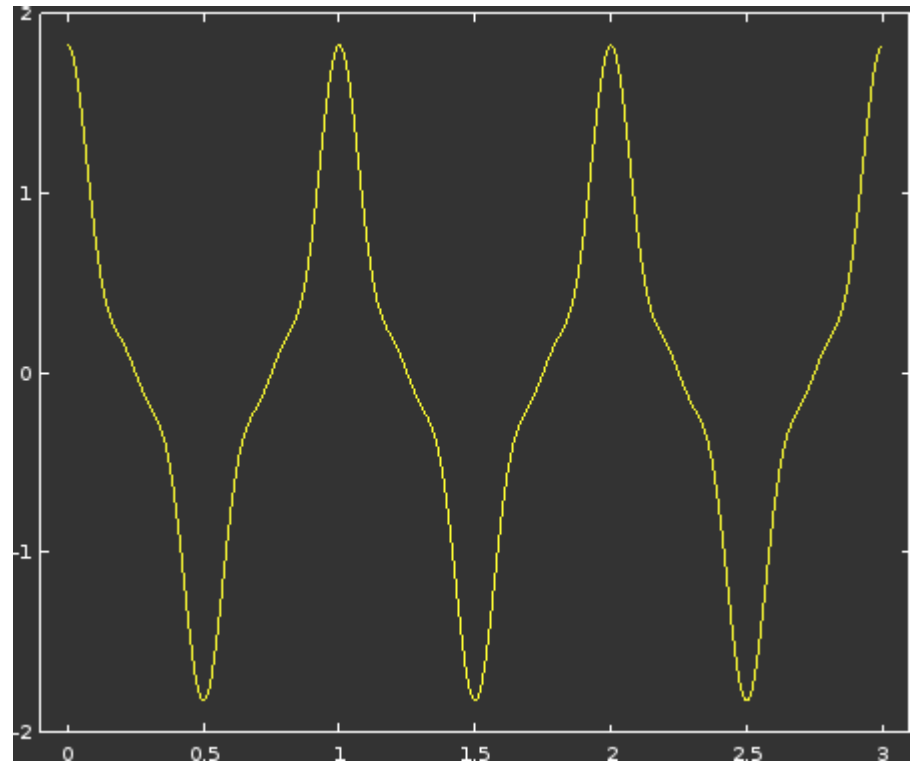
Fázistorzítás



3 felharmonikus



-90°



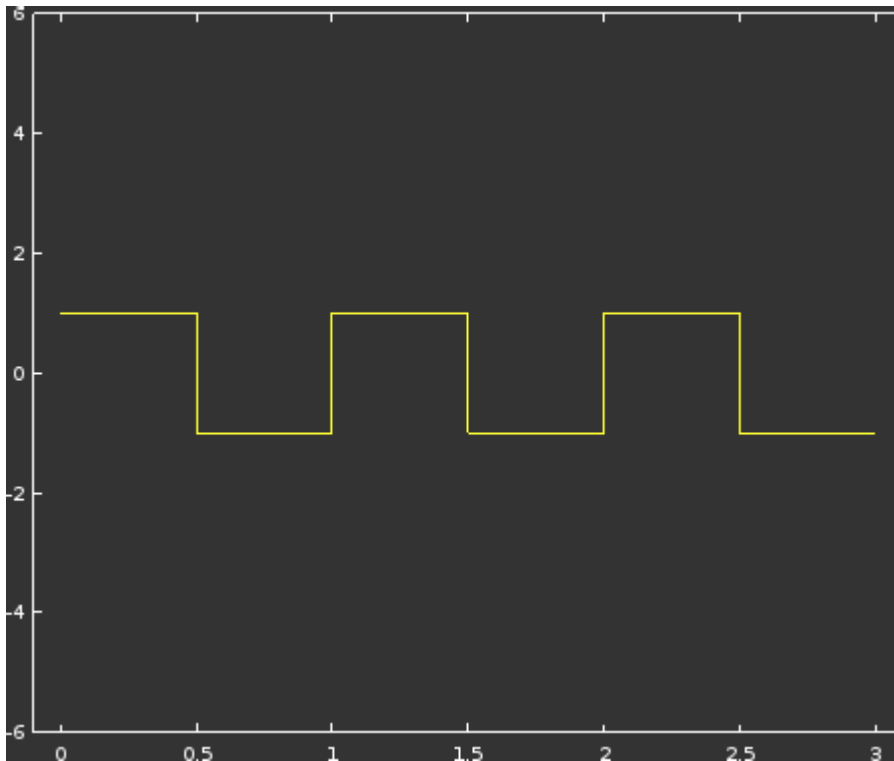
0°

fázis

Fázistorzítás

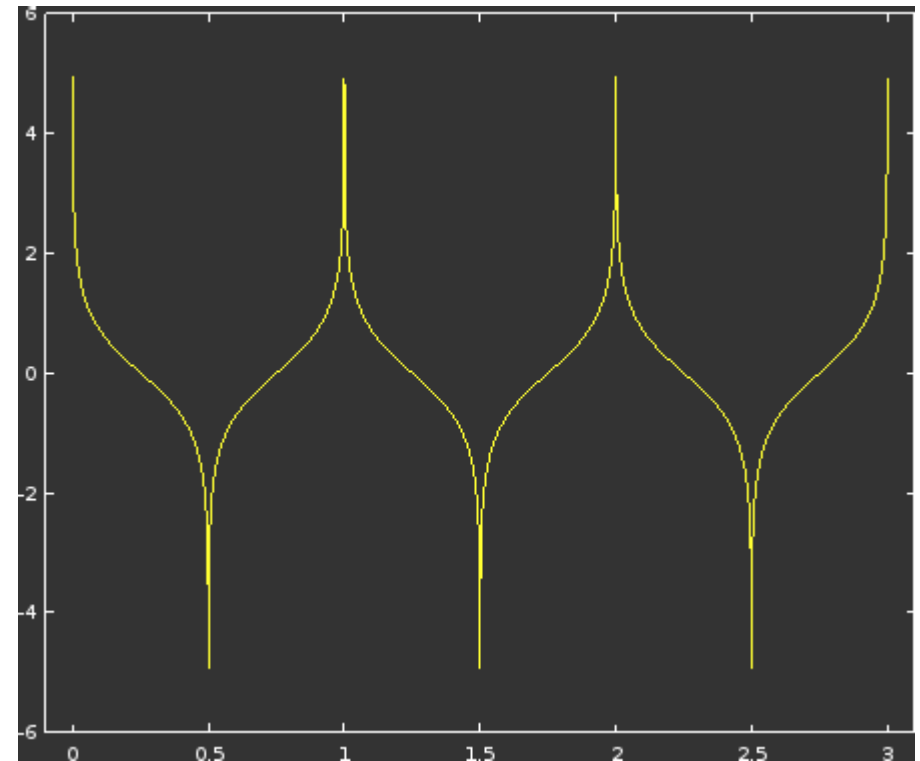


Összes felharmonikus



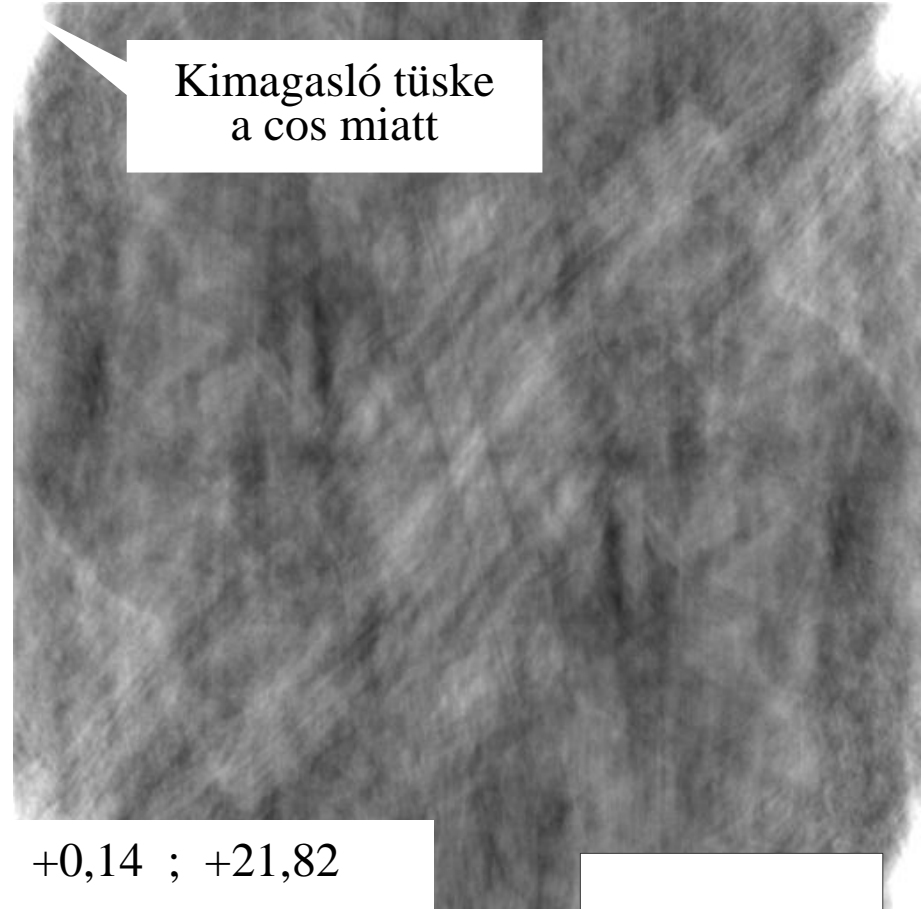
-90°

fázis



0°

Fázistorzítás



-1,08 ; +1,13

+0,14 ; +21,82

(Eredeti kép értéktartománya: 0 ; +1)

+90°

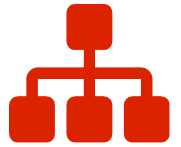
-90°



0°



FFT



1D FFT

Adatújrarendezés

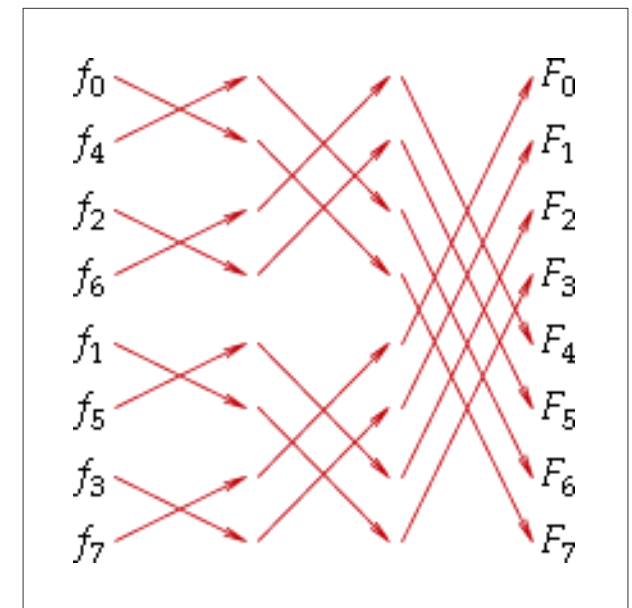
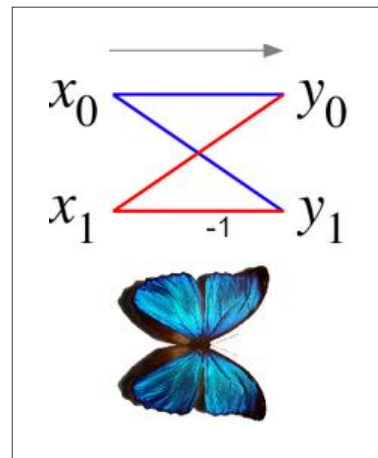
adat[x] \leftrightarrow **adat[ford(x)]** (pl. $\text{adat}[0100_{(2)}] \leftrightarrow \text{adat}[0010_{(2)}]$)

Pillangó műveletek

$\log_2 N$ alkalommal

$$y_0 = x_0 + x_1 \omega^k$$

$$y_1 = x_0 - x_1 \omega^k$$

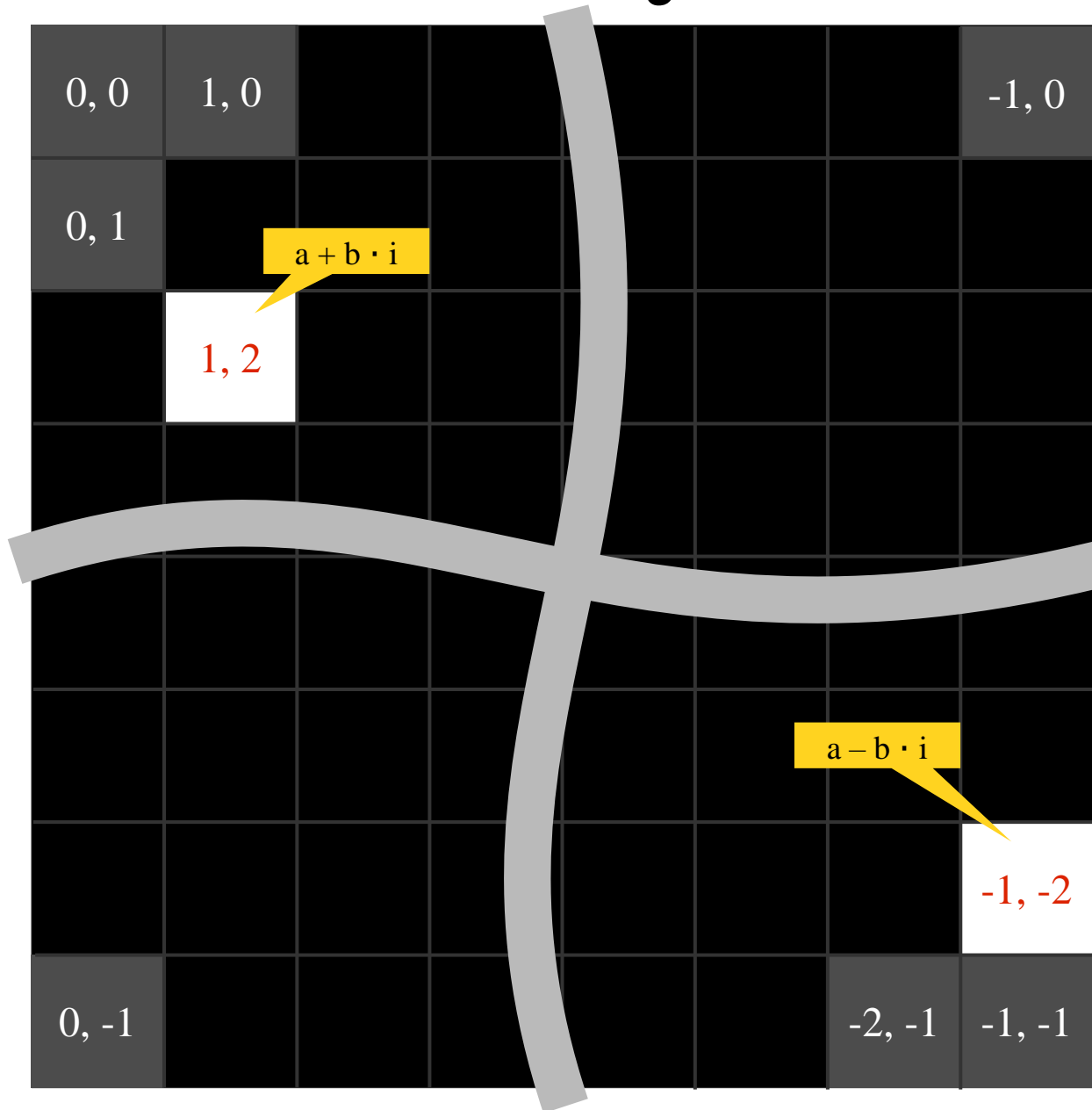


2D FFT

Először vízszintes, majd függőleges irányban...

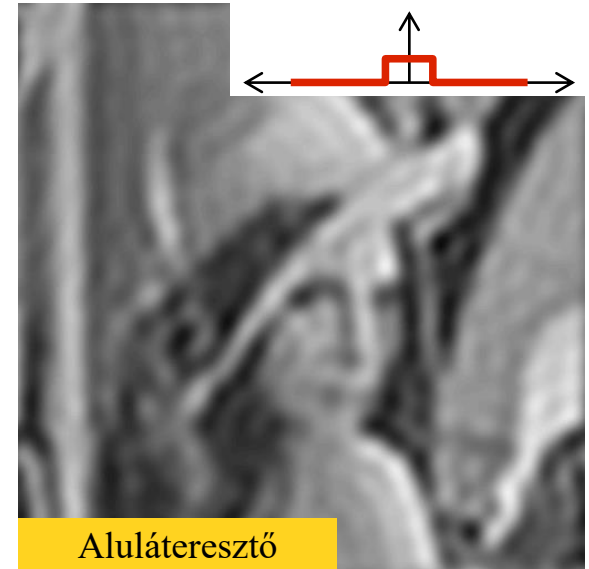
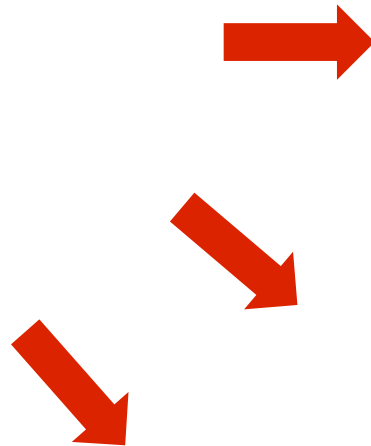


FFT eredménye



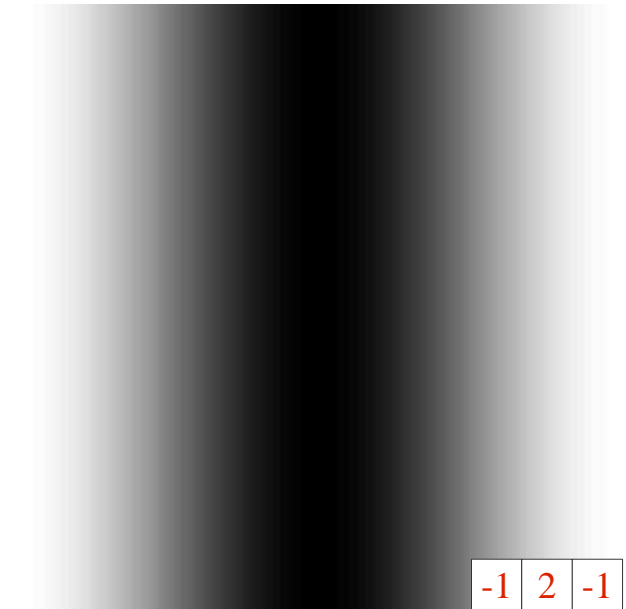
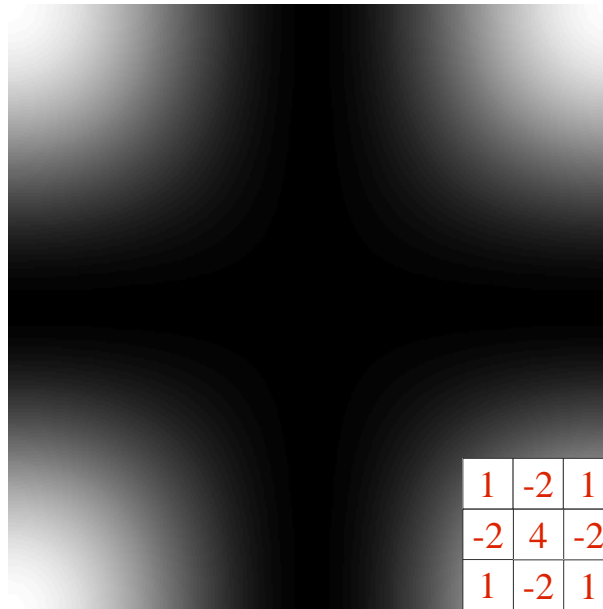
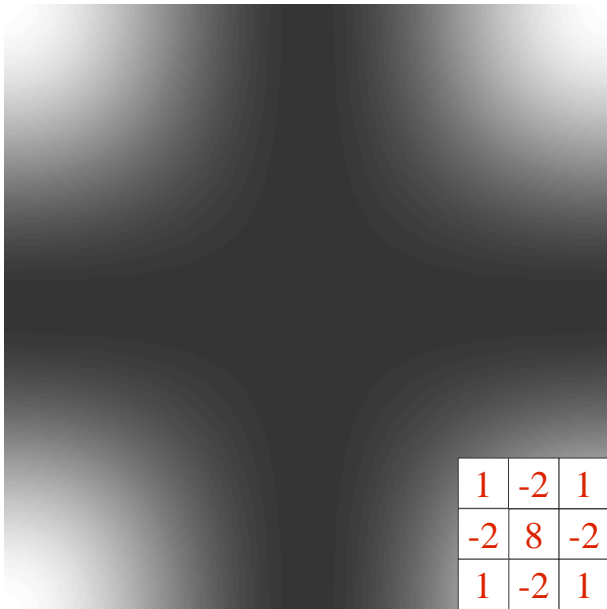
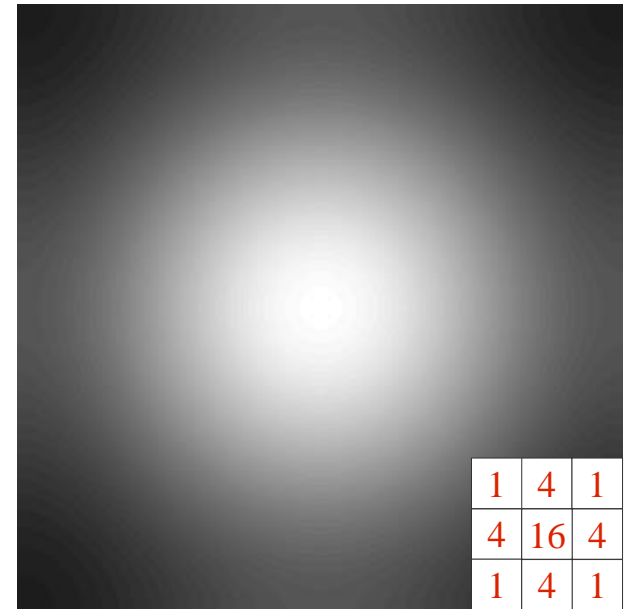
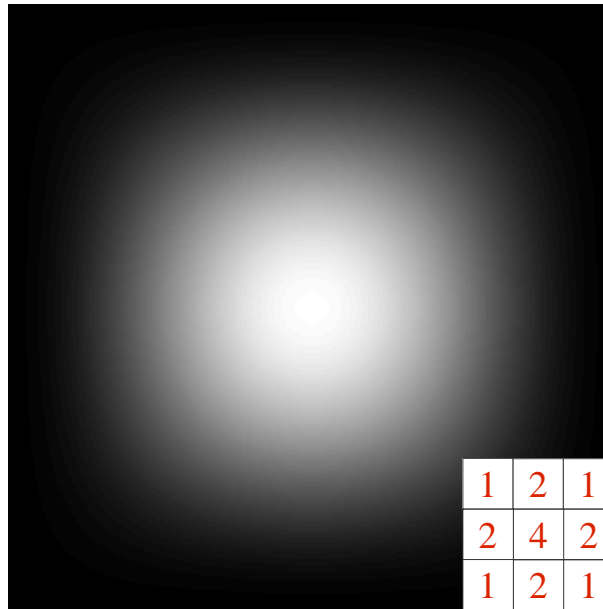
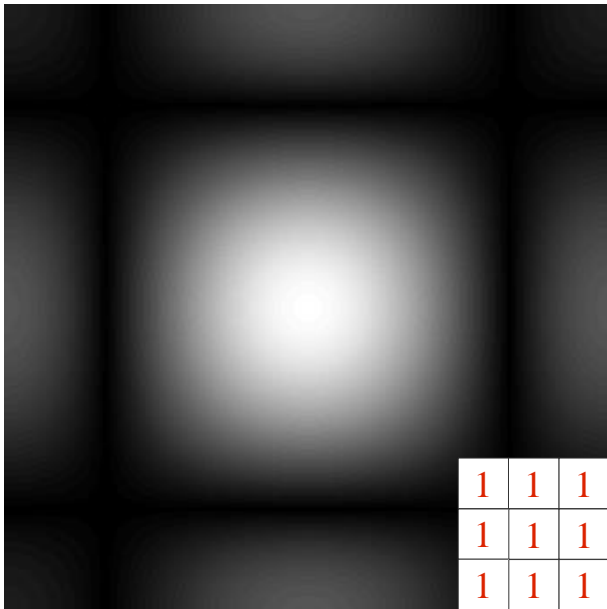
Periodikusság miatt
(N-1, N-1) is lehet

Szűrés





Szűrők frekvenciaképe

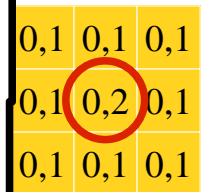
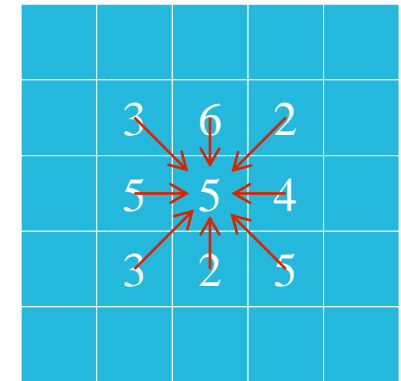
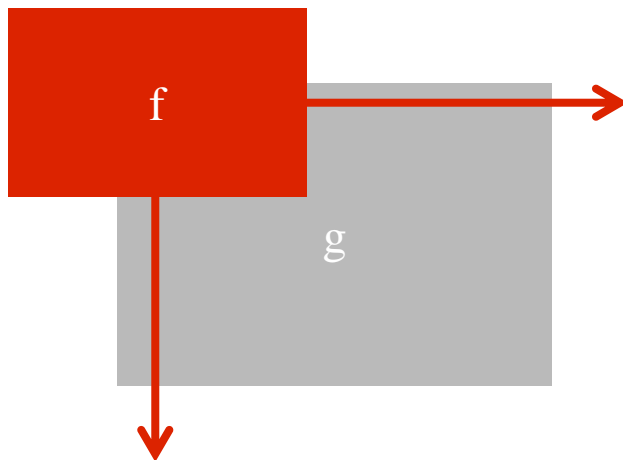


Konvolúció

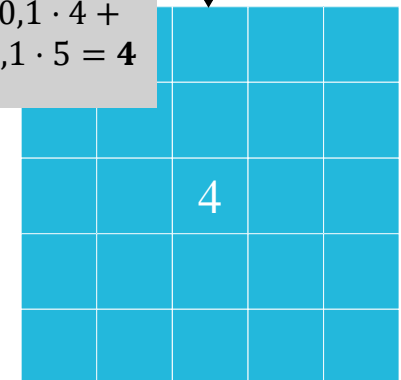


$$(f * g)(x, y) = \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} f(u, v) \cdot g(x - u, y - v)$$

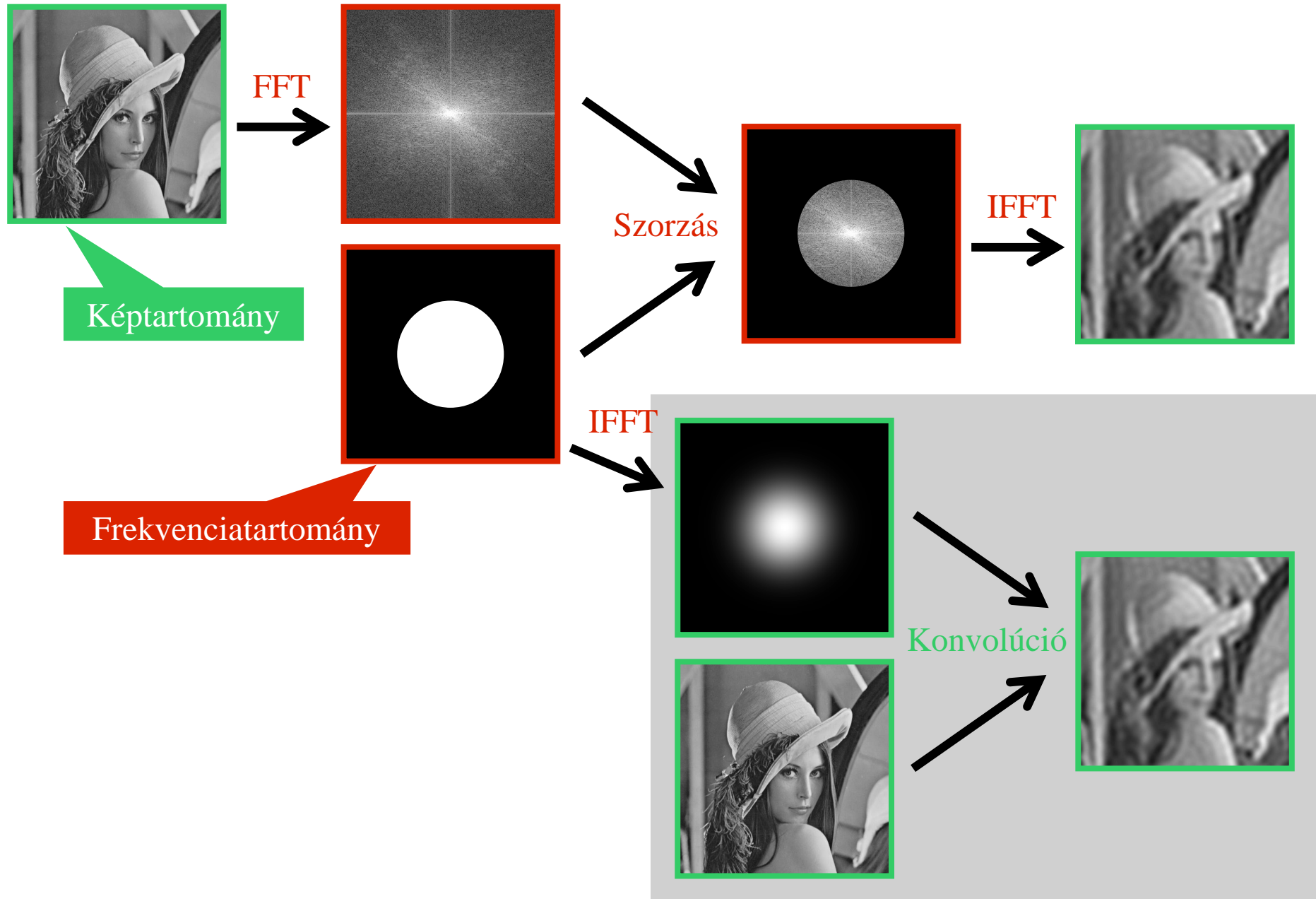
$$(k * I)(x, y) = \sum_{u=-1}^1 \sum_{v=-1}^1 k(u, v) \cdot I(x - u, y - v)$$



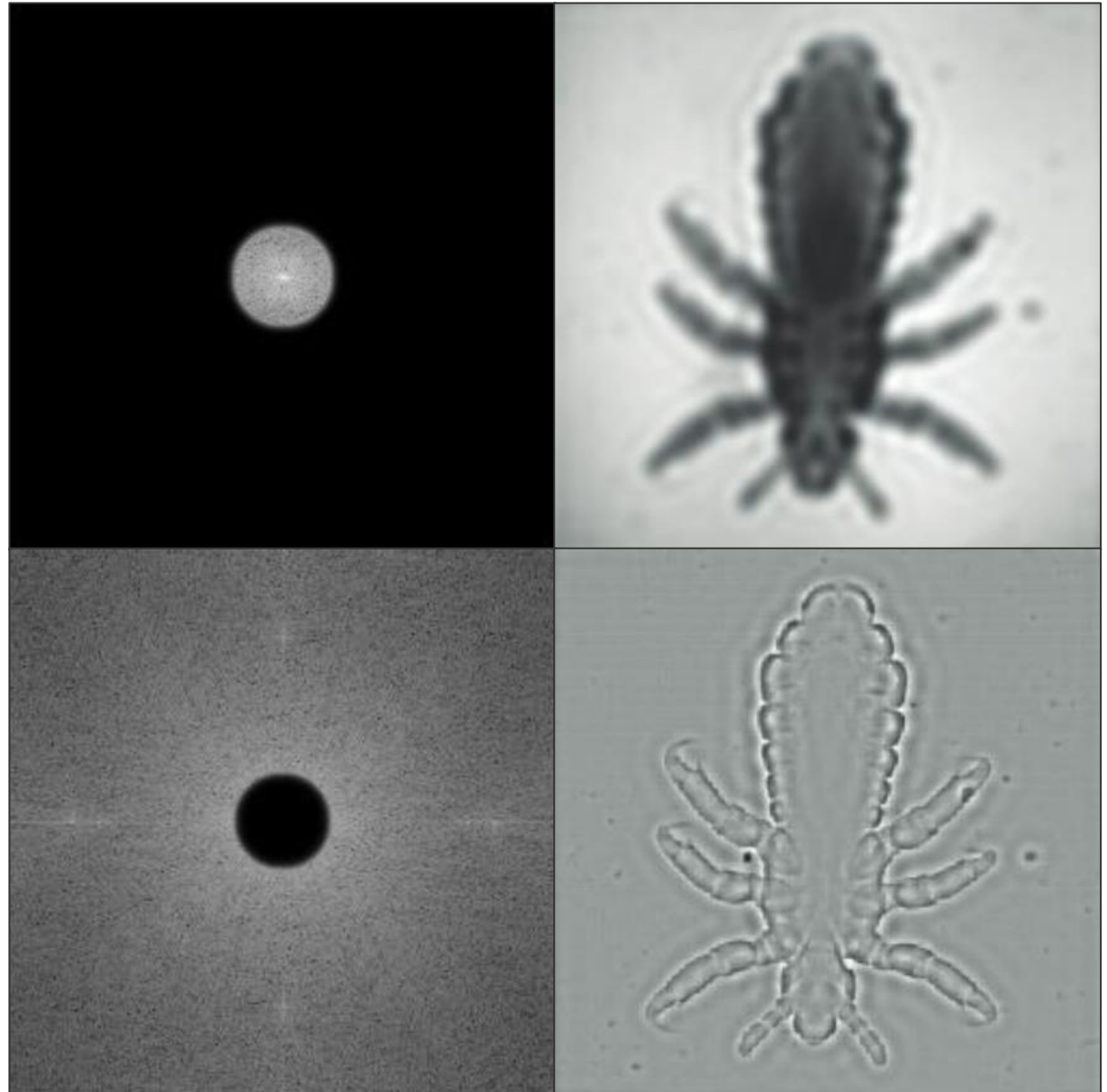
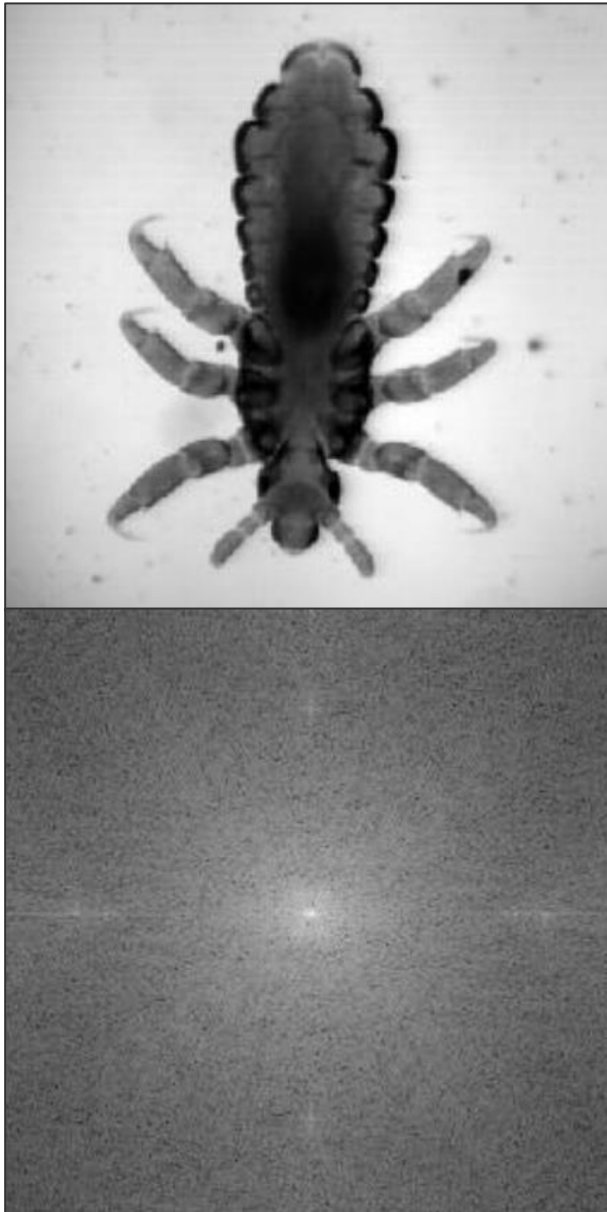
$$0,1 \cdot 3 + 0,1 \cdot 6 + 0,1 \cdot 2 + \\ + 0,1 \cdot 5 + 0,2 \cdot 5 + 0,1 \cdot 4 + \\ + 0,1 \cdot 3 + 0,1 \cdot 2 + 0,1 \cdot 5 = 4$$



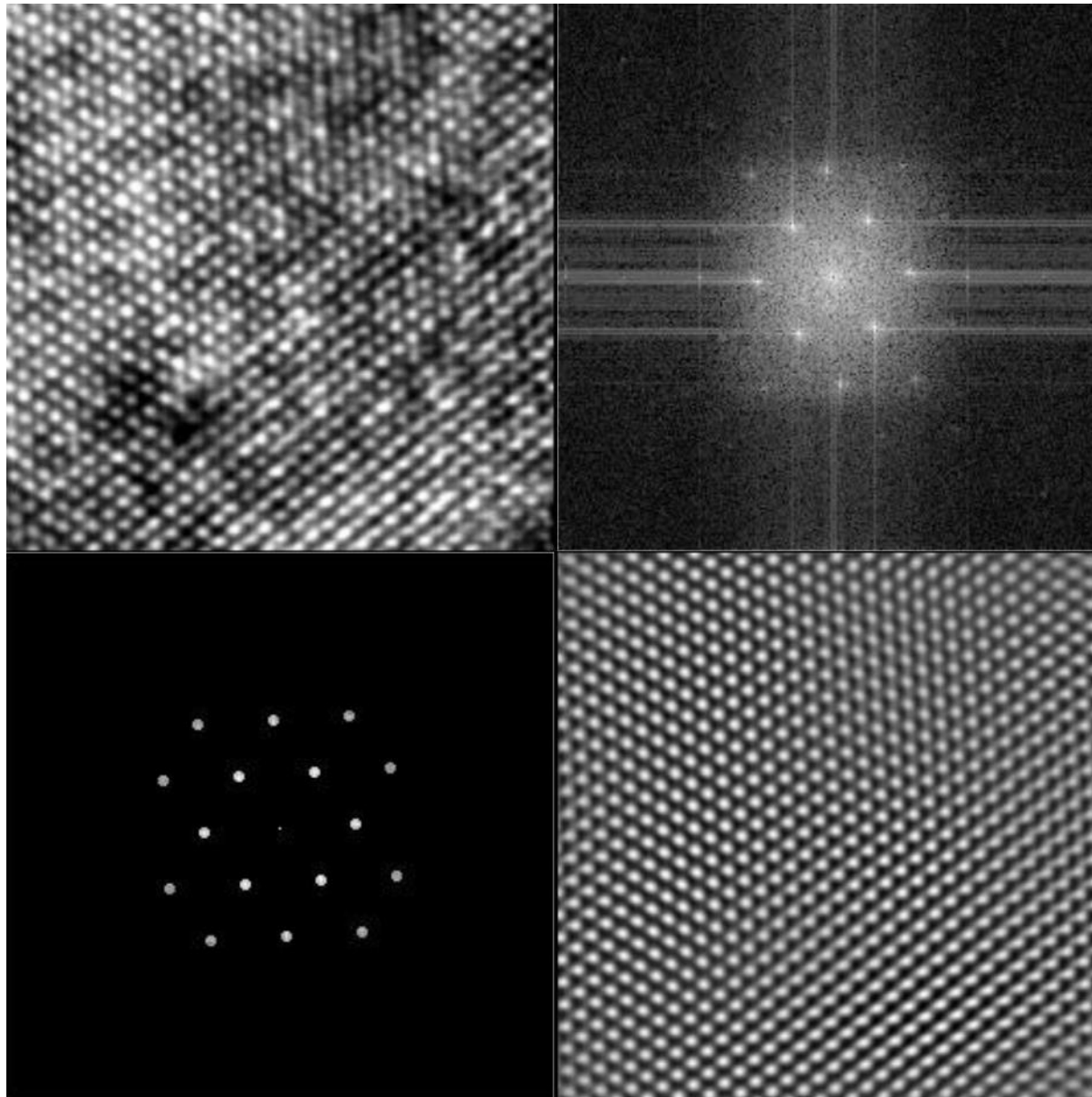
Konvolúció



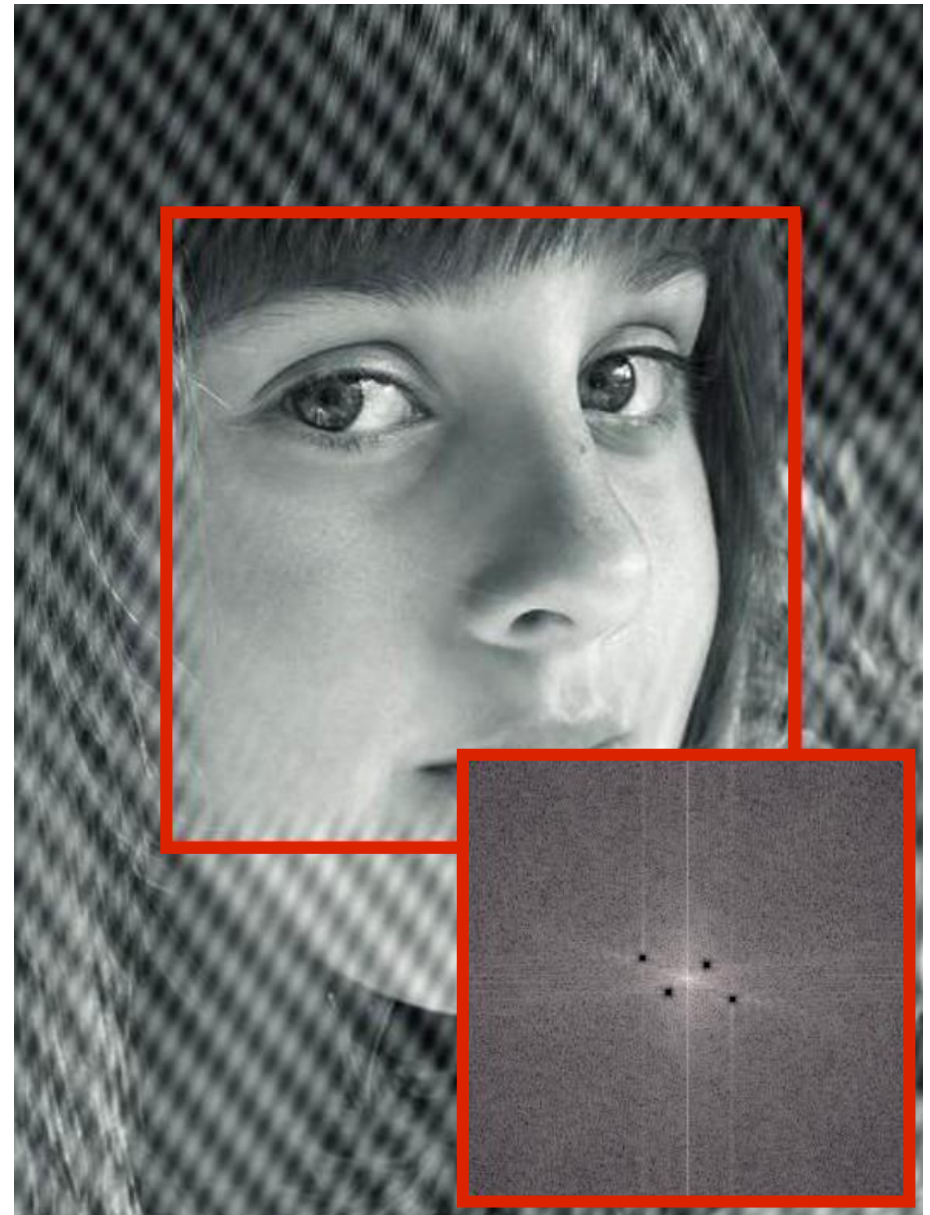
Szűrés



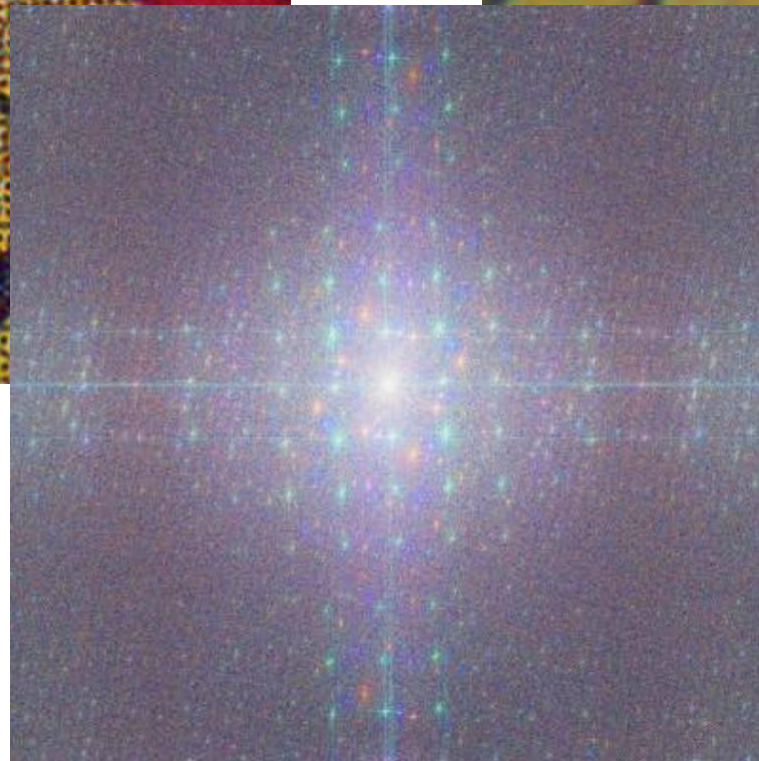
Szűrés



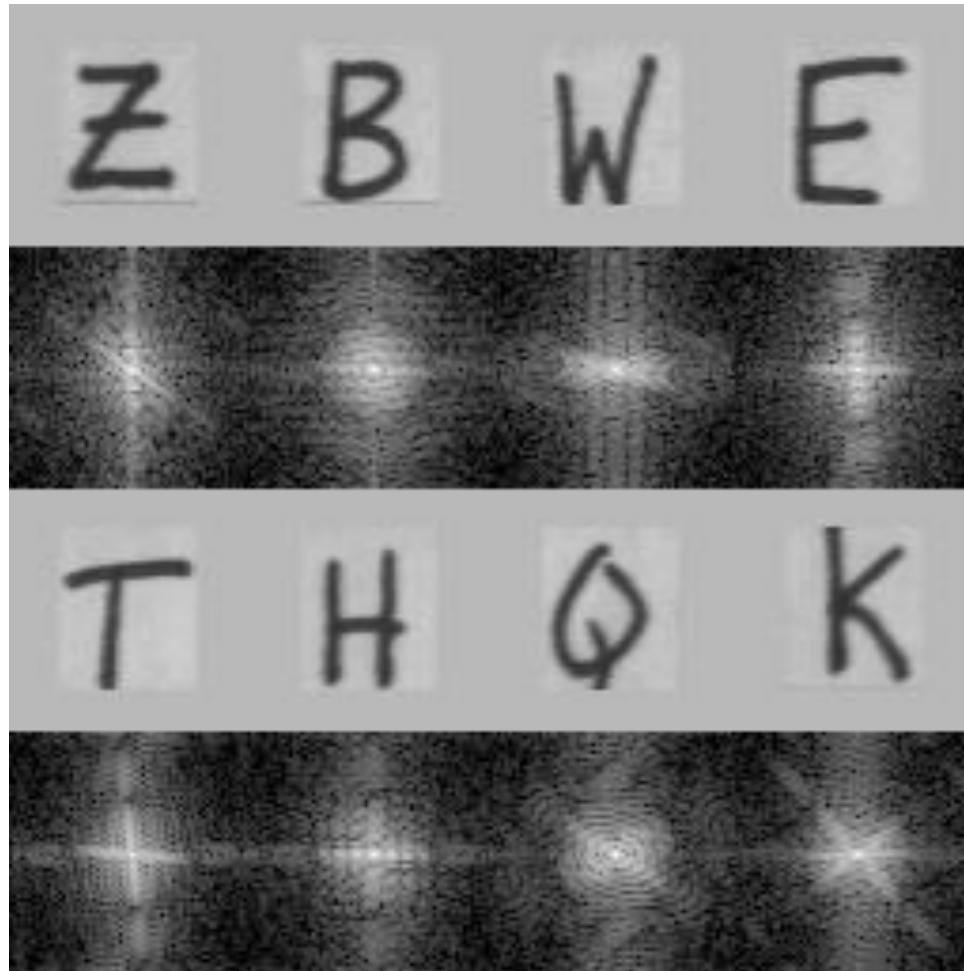
Periodikus zaj



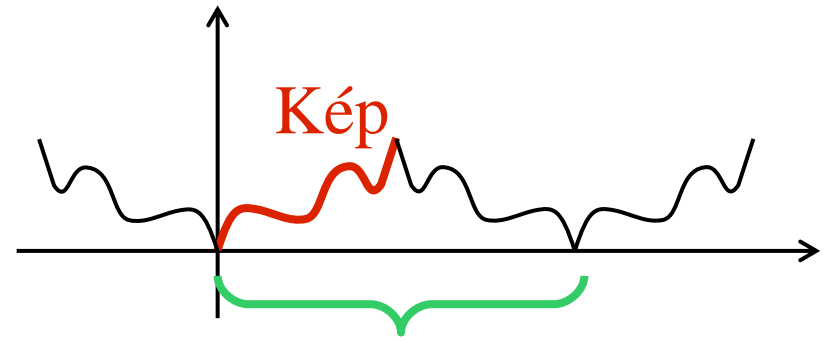
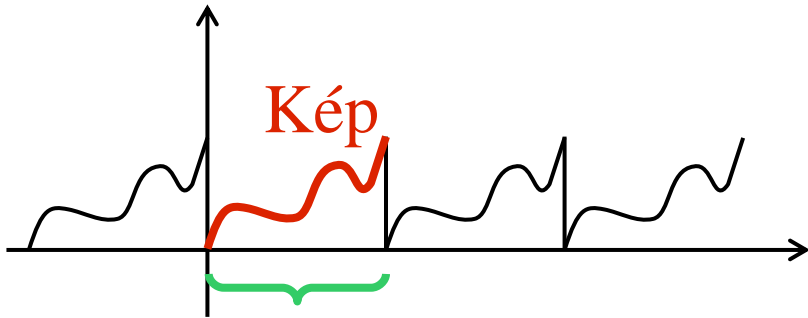
Halftone



Alakfelismerés



DCT



DCT



Valós értékek

Energiatömörítés

Egyszerűbb

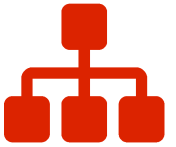
1D

$$C[k] = \sum_{x=0}^{N-1} i[x] \cos \left[\frac{\pi}{N} \left(x + \frac{1}{2} \right) k \right]$$

2D

$$C[k, l] = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} i[x, y] \cos \left[\frac{\pi}{N} \left(x + \frac{1}{2} \right) k \right] \cos \left[\frac{\pi}{M} \left(y + \frac{1}{2} \right) l \right]$$

FCT



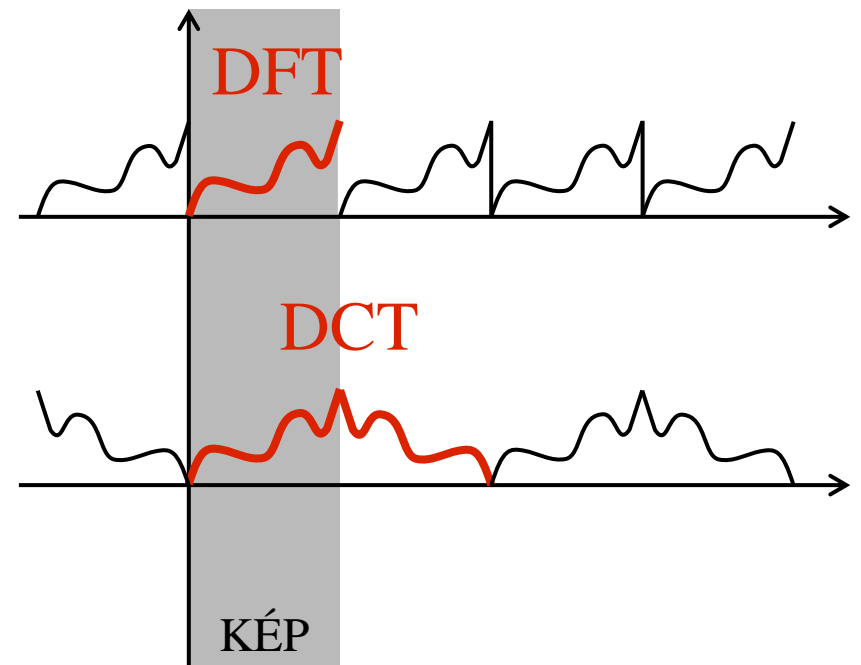
1. Szimmetrikus függvény

$$y[n] = \begin{cases} x[n] & , ha 0 \leq n < N \\ x[2N - 1 - n] & , ha N \leq n < 2N \end{cases}$$

2. FFT (2N adaton)

3. Valós rész

$$C[n] = \Re \left(e^{\frac{-i\pi k}{2N}} * Y[n] \right)$$



JPEG

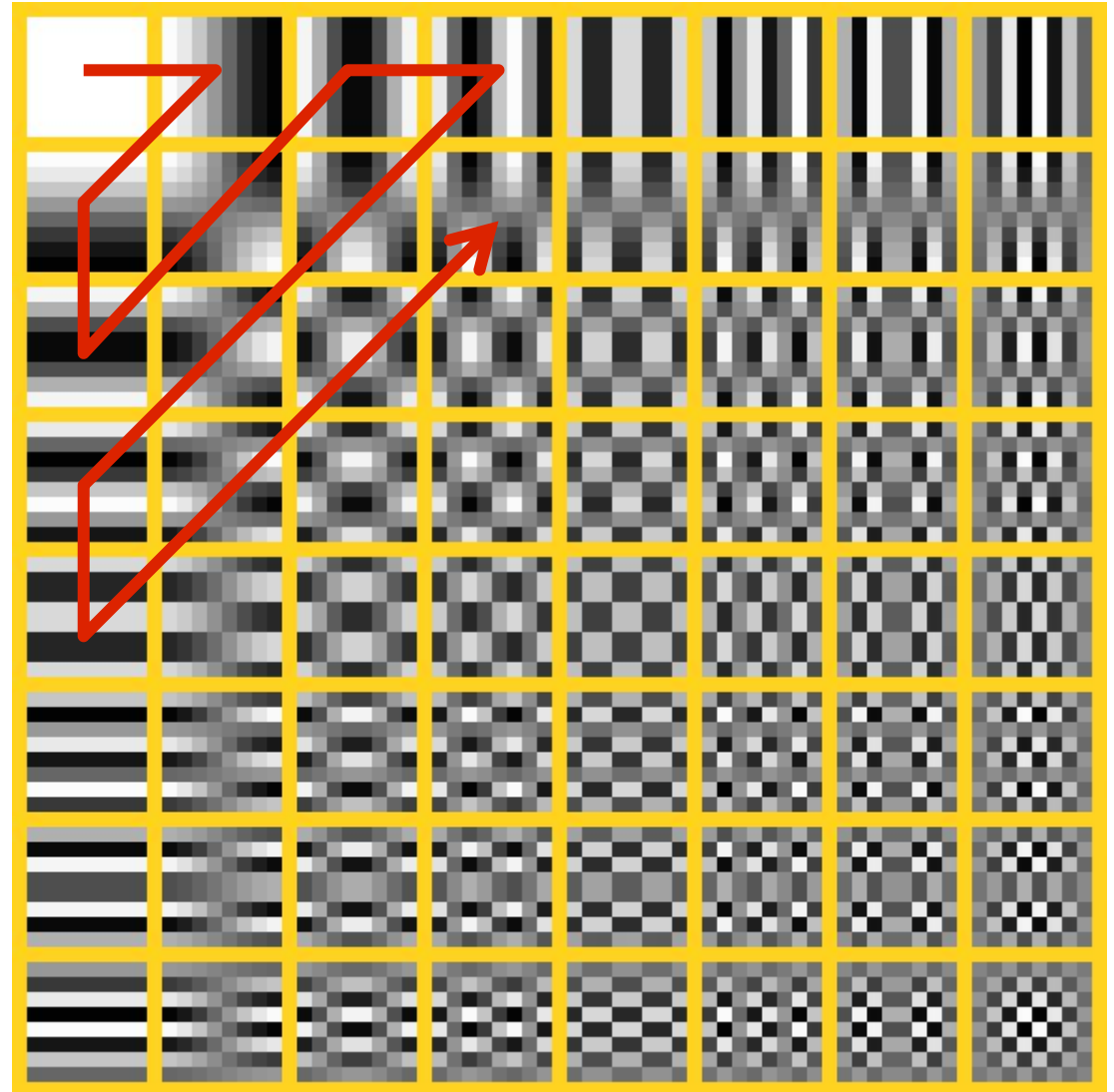


DCT-II

8x8 elemek

8x8 C-tábla

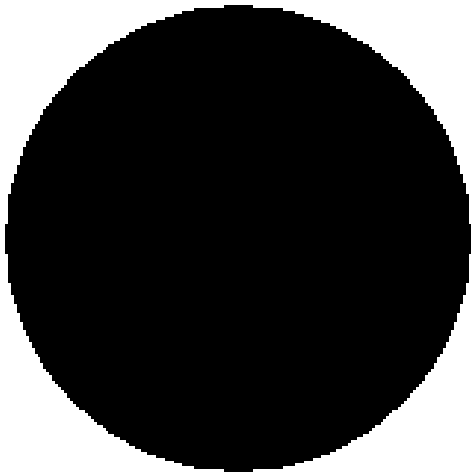
Csak az első pár elemet
tároljuk



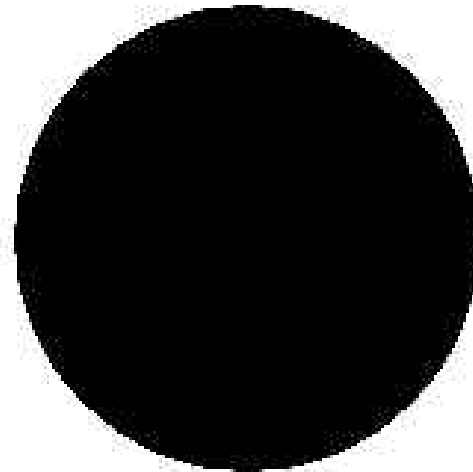
Tömörítés hatása



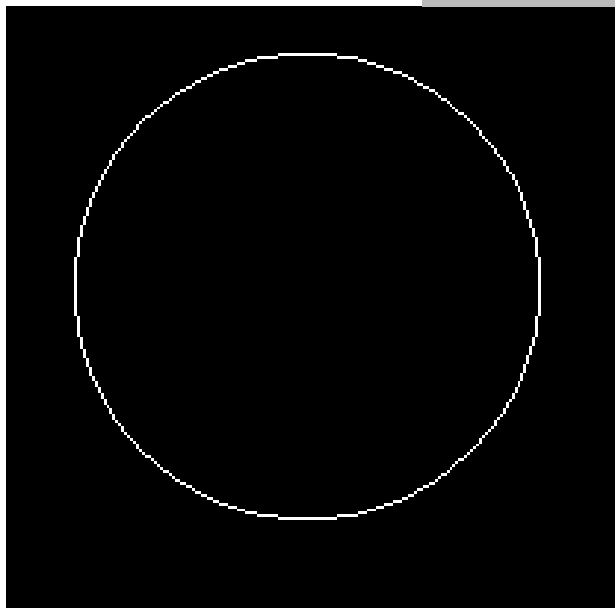
Veszteséges JPEG



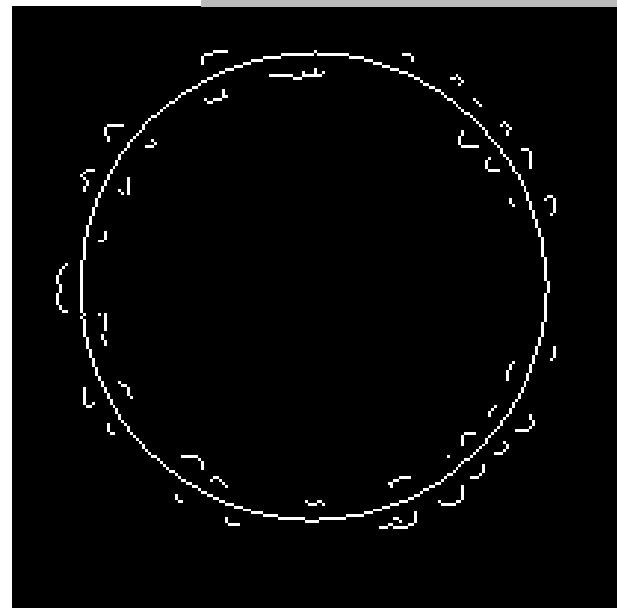
Eredeti



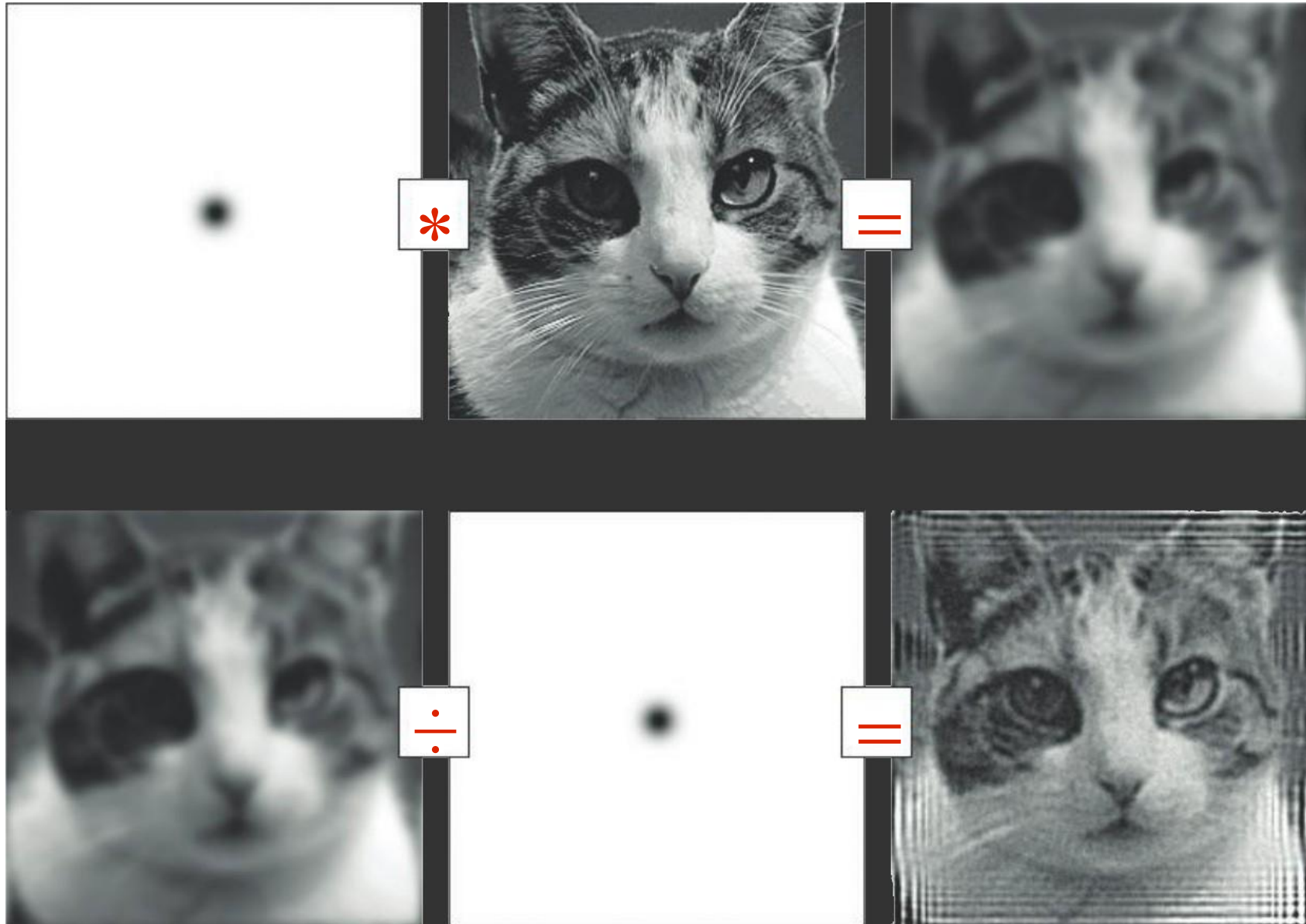
Veszteséges JPEG



Canny
élkereső



Dekonvolúció



Egyszerű dekonvolúció



$$g(x, y) = f(x, y) * h(x, y)$$

$$f(x, y) = g(x, y) \div h(x, y)$$

$$F(u, v) = \left[\frac{1}{H(u, v)} \right] \cdot G(u, v)$$

Wiener dekonvolúció



$$g(x, y) = f(x, y) * h(x, y) + \varepsilon$$

$$f(x, y) = g(x, y) \div h(x, y)$$

$$F(u, v) \approx \left[\frac{1}{H(u, v)} \right] \cdot \left[\frac{|H(u, v)|^2}{|H(u, v)|^2 + \frac{1}{SNR(u, v)}} \right] \cdot G(u, v)$$