

## Függvények határértéke

$$1. \lim_{x \rightarrow \pm\infty} \frac{2x^3 + x + 1}{x^2 - x + 10}$$

$$2. \lim_{x \rightarrow \infty} \frac{x\sqrt{x} + \sqrt{2x^3 + 3}}{x^2 + x + 1}$$

$$3. \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 + 1}}$$

$$4. \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x-1}$$

$$5. \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 - 1}$$

$$6. \lim_{x \rightarrow 0} \frac{1 - \cos x}{1 - \cos 2x}$$

$$7. \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x}$$

$$8. \lim_{x \rightarrow \infty} x \sin \frac{1}{x}$$

$$9. \lim_{x \rightarrow \infty} \arctg x \cdot \frac{\sin x}{x}$$

$$10. \lim_{x \rightarrow \infty} (x - \sqrt{x^2 - 8x})$$

$$11. \lim_{x \rightarrow \infty} (\sqrt{3x^2 + 1} - x)$$

$$12. \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 1} - x}{\sqrt{x+1} - \sqrt{x}}$$

$$13. \lim_{x \rightarrow 0^+} \frac{2\sqrt{x} + 3\sqrt[3]{x}}{6\sqrt[3]{x} - 8\sqrt{x}}$$

$$14. \lim_{x \rightarrow \infty} \frac{2\sqrt{x} + 3\sqrt[3]{x}}{6\sqrt[3]{x} - 8\sqrt{x}}$$

$$15. \lim_{x \rightarrow 0^{\pm}} e^{1/x}$$

$$16. \lim_{x \rightarrow \pm\infty} e^{1/x}$$

$$17. \lim_{x \rightarrow 0^{\pm}} x \cdot e^{1/x}$$

$$18. \lim_{x \rightarrow \pm\infty} x \cdot e^{1/x}$$

$$19. \lim_{x \rightarrow \infty} \frac{3 - a^x}{2 + 3a^x}, a \text{ pozitív}$$

$$20. f(x) = \frac{e^{1/x}}{1 + e^{1/x}}, \lim_{x \rightarrow 0^{\pm}} f = ? \quad \lim_{x \rightarrow \pm\infty} f = ?$$

$$21. f(x) = \frac{e^{1/x}}{1 - e^{1-x}}, \lim_{x \rightarrow 0^{\pm}} f = ? \quad \lim_{x \rightarrow \pm\infty} f = ? \quad \lim_{x \rightarrow 1^{\pm}} f = ?$$

$$22. f(x) = \frac{e^{1/x}}{e^x - e^{1/x}}, \lim_{x \rightarrow 0^{\pm}} f = ? \quad \lim_{x \rightarrow \pm\infty} f = ? \quad \lim_{x \rightarrow 1^{\pm}} f = ? \quad \lim_{x \rightarrow -1^{\pm}} f = ?$$

$$23. \lim_{x \rightarrow \infty} x^2 (e^{2/x^2} - 1)$$

$$24. \lim_{x \rightarrow \infty} e^x \cdot \ln(1 + 2e^{-x})$$

$$25. f(x) = e^x, \quad g(x) = f(f(f(1/x))), \quad \lim_{x \rightarrow 0^{\pm}} g(x) = ?$$

$$26. f(x) = x \sin \frac{1}{x} \text{ ha } x \neq 0 \text{ és } f(0) = 0, \quad g(x) = 0 \text{ ha } x \neq 0 \text{ és } g(0) = 1, \quad h(x) = \cos x$$

$$\lim_{x \rightarrow \infty} h(f(x)) = ? \quad \lim_{x \rightarrow 0} h(f(x)) = ? \quad \lim_{x \rightarrow 0} g(f(x)) = ?$$

27. Ábrázolja vázlatosan az alábbi függvényeket

$$a) f(x) = \frac{x-1}{x^2}$$

$$b) f(x) = \frac{(x-1)^2}{x^2}$$

$$c) f(x) = \frac{(x-1)^3}{x^2}$$

$$d) f(x) = \frac{e^x}{1 - e^{x-1}}$$

$$e) f(x) = \frac{e^{1/x}}{1 - e^{x-1}}$$