# Feladatok komplex számokkal

(1) $\left(1+i\right)^4\left(1-i\sqrt{3}\right)^6=?$

$1+i=\sqrt{2}e^{i\frac{\pi}{4}}$

$1-i\sqrt{3}=2e^{-i\frac{\pi}{3}}$

$\left(1+i\right)^4\left(1-i\sqrt{3}\right)^6=\sqrt{2}^4e^{i\pi}\cdot 2^6e^{-2i\pi}=-2^8$

(2) $\frac{\left(i+1\right)^{10}}{\left(1-i\right)^7}=\frac{\sqrt{2}^{10}e^{i\frac{10\pi}{4}}}{\sqrt{2}^7e^{-i\frac{7\pi}{4}}}=\sqrt{2}^3e^{i\frac{17\pi}{4}}=\sqrt{2}^3e^{i\frac{\pi}{4}}$

(3) $\left(\frac{1-i}{\sqrt{2}i}\right)^{13}=?$

$\frac{1-i}{i}=-i-1$

$\left(\frac{1-i}{\sqrt{2}i}\right)^{13}=\frac{-i-1}{\sqrt{2}}=e^{i\frac{5\pi}{4}}$

(4) $\sqrt[3]{8i}=\sqrt[3]{8e^{i\frac{\pi}{2}}}=\sqrt[3]{8}e^{i\left(\frac{\pi}{6}+\frac{2k\pi}{3}\right)}=2e^{i\left(\frac{\pi}{6}+\frac{2k\pi}{3}\right)}$,
ahol $k=0,1,2$

(5) $\sqrt[5]{-2-2i}=\sqrt[5]{\sqrt{8}e^{i\frac{5\pi}{4}}}=\sqrt[10]{8}e^{i\left(\frac{\pi}{4}+\frac{2k\pi}{5}\right)}$,
ahol $k=0,1,2,3,4$

(6) $\sqrt[7]{-1}=\sqrt[7]{e^{i\pi}}=e^{i\left(\frac{\pi}{7}+\frac{2k\pi}{6}\right)}$, ahol
$k=0,1,2,3,4,5,6$

(7) $z^2=\left(\bar{z}\right)^2$

$x^2-y^2+2ixy=x^2-y^2-2ixy$

$2xy=-2xy$

$xy=0\Rightarrow\begin{cases}x=0\Rightarrow z=iy\\ y=0\Rightarrow z=x\end{cases}$

(8) $z^3=\bar{z}$

$r^3e^{3i\varphi}=re^{-i\varphi}$

$3\varphi=-\varphi+2k\pi$

$r\left(r^2-1\right)=0\Rightarrow\begin{cases}r_1=0\\ r_2=1\end{cases}$

$\varphi=\frac{2k\pi}{4}=\frac{k\pi}{2}$

(9) $z^2+8z+17=0$

$z_{1,2}=\frac{-8\pm\sqrt{64-4\cdot 17}}{2}=\frac{-8\pm\sqrt{-4}}{2}=-4\pm i$

(10) $z^3=\frac{4+2i}{1-i}-3i$

$z^3=\frac{4+2i}{1-i}\cdot\frac{1+i}{1+i}-3i$

$z^3=\frac{2+6i}{2}-3i=1$

$z=\sqrt[3]{1}=\sqrt[3]{e^{i0}}=e^{i\frac{2k\pi}{3}}$, ahol $k=0,1,2$